A versatile analytical representation of the different aircraft wings

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Abstract: *This paper presents an analytical method to describe the shape of an airplane wing. The analytical shape of the wing section implies analytical expressions for the wing parameters: maximum thickness, maximum camber, maximum camber position, minimum thickness position, and so on. The analytical expressions of the wings in a first representation allow simple relationships with the geometric parameters of the wing. The defined expressions for the wings parameters relevant for aircraft design are versatile.*

Key Words: *wing geometric characteristics, lacunary polynomials, lift force, drag force*

1. INTRODUCTION

An important part of the construction of an aircraft is the shape of the wings. Their aerodynamics involve numerous evaluations and simulations through fluid dynamics. The reduction of the evaluation times is achieved by a parametric function. This is represented by the ratio of two polynomials.

We have constructed a simple mathematical wing model. We replace the discrete wing shape with a continuous form, which is described by lacunary polynomials.

2. WING CHARACTHERISTICS

The aircraft design needs a careful evaluation of the wings [1–5].

The defined expressions for the wings parameters relevant for aircraft design are versatile.

The maximum drag force are defined by the aerodynamic profile. The wings shape have a tremendous impact on aircraft performance [5–7].

This shape is defined by the outline of the section in the vertical location of the wing. The significant profile elements are: leading edge, trailing edge, chord, surfaces (Extrados and Intrados), depth, skeleton, profile thickness, arrow, profile, and so on [8–10].

For constructors the wings characteristics need numerous evaluations and optimizations. So the reduction of the computer efforts is important.

The chord represents the distance between leading edge and trailing edge. Extrados means the upper convex profile wing. Intrados means the bottom profile wing. The depth is Euclidean length between the two edges. The skeleton is the geometrical location of equidistant points between intrados and extrados [8, 10].

The thickness profile is the distance in the vertical plane between the extrados-intrados corresponding points.

The profile arrow is the distance between profile chord and skeleton.

From a geometrical point of view the profiles classification is: symmetric, biconvex, plan-convex and concave-convex.

The Intrados wing profile is a plane of length *L* [7,9].

The Extrados profile is a convex one described by the function

$$f(x) = \frac{wx}{x^2 + a^2} \tag{1}$$

where *w* and *a* are parameters, *L* is the wing chord.

The numbers of elementary operations are as follows: one division, one addition and three multiplications.

From the equation $f(x)=L$ we obtain the maximum position (*emaxpos*) for the Extrados airfoil value:

$$emaxpos = \frac{w + \sqrt{w^2 - 4L^2a^2}}{2L} \tag{2}$$

with equivalent restriction conditions:

$$a > \frac{w}{2L} \ \ or \ \ w < \frac{2L}{a} \ \ or \ \ L > \frac{w}{2a} \tag{3}$$

The maximum airfoil thickness is: $Emax=f(emaxpos)$. For the plane – convex profile the camber line equation is $(f(x)\text{-}x)/2$, which for this case reduce to $f(x)/2$.

From the definition the camber line is equal to the skeleton one. In this case the maximum camber position and the maximum thickness position coincide.

Another type of PLANE CONVEX AIRFOIL profile of finite length is given.

For X the wing we take a profile described by the following Extrados expression:

$$f_1(x) = \frac{w}{(x-b)^2 + a^2} \tag{4}$$

where *w*, *a* and *b* are positive parameters.

The numbers of elementary operations are: one division, two additions, one substitution and five multiplications.

From equation $f_1'(x) = 0$ we obtain the maximum position ($emaxpos_1$) for Extrados airfoil value: $emaxpos_1 = b$

The maximum airfoil thickness in this case is:

$$E_{max1} = f_1(emaxpos1) = \frac{w}{a^2} \tag{5}$$

3. SURFACES

The Extrados airfoil surface is:

$$\text{Extrados air foil surface } = \int_0^L \frac{wx}{x^2 + a^2} dx \tag{6}$$

Then after integration we have:

$$\text{Extrados air foil surface } = \frac{w}{2} ln \frac{a^2 + L^2}{a^2} \tag{7}$$

The Intrados surface is the projection of the Extrados surface, which is less than the Intrados one.

For wing surfaces we have the product of two terms: wing length and arc length in vertical cross section.

We use the upper (Extradoswingsurface) part or the bottom (Intradoswingsurface) part of this cross section

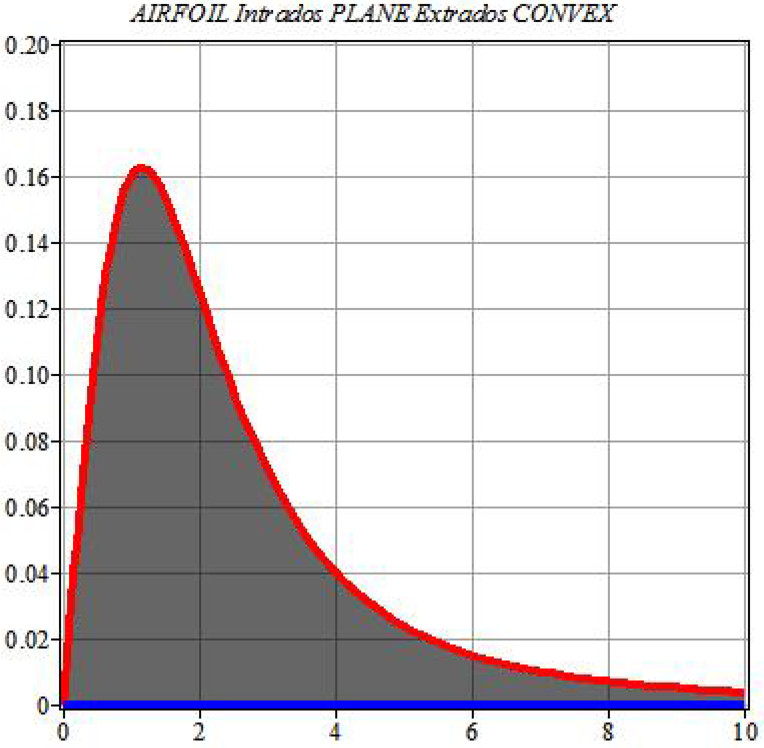

Fig. 1 Airfoil. Intrados PLANE. Extrados CONVEX

So the total wing surface is:

S total = Intradoswingsurface + Extradoswingsurface

For the second case $f_1(x)$ – Extrados wing surface (surface Extrados1) is obtained by the integral:

$$\text{Surface Extrados1} = \int_0^L \frac{w}{x^2 - 2bx + a^2 + b^2} dx \tag{8}$$

Integration gives an analytical expression:

$$Surface\ Extrados1 = w.\frac{arctan\left(\frac{b}{a}\right) + arctan\left(\frac{L-b}{a}\right)}{a} \tag{9}$$

with condition $L > b$.
Supplementary conditions are:

$$-\pi < \frac{b}{a} < \pi \tag{10a}$$

$$-\pi < \frac{L-b}{a} < \pi \tag{10b}$$

4. LIFT AND DRAG FORCE

The forces exerted on the wings are lift and drag. The lift one is perpendicular to free-stream airflow.

The drag one is parallel to the free-stream airflow. For subsonic speed we have a laminar regime.

The lift and drag forces are proportional with the wings surfaces exposed to the air flow, air flow speed and air density.

This force ("Portance") is a contact force caused by the aircraft-flow interaction. (Similar is in hydrodynamics for liquids).

We have two type of forces: laminar and turbulent.

We consider the laminar case, which is more favorable. This is observed experimentally by the smook method.

The flow on Extrados is accelerated.

The reduction of the air pressure on Extrados produces a lift force (in opposition with the aircraft weight).

The aerodynamic force (R) is decomposed in the lift force (R_z – perpendicular on the flow) and the drag force (R_x – parallel to the flow):

$$\overrightarrow{\boldsymbol{R}} = \overrightarrow{\boldsymbol{R_x}} + \overrightarrow{\boldsymbol{R_z}} \tag{11}$$

This force ($\vec{R}$) is applied in a point related with the profile and relative flow.

$$R_x = \frac{1}{2}\rho S V^2 C_x \tag{12}$$

where: ρ – is the fluid density;
S–is the wing surface;
V – is the relative flow speed;
C_x – is the lift coefficient.

The lift force:

$$R_z = \frac{1}{2}\rho S V^2 C_z$$

C_z - is drag coefficient.

C_x and C_z are addimensional and are obtained experimentally.

5. CONCLUSIONS

We represent the shape of the wing by a parametric function: the ratio of two polynomials.

Thus, we obtain an analytical expression for the wing parameters. These expressions involve simple arithmetic operations. So we reduce the computing time.

We replace a discrete wing shape with a continuous shape, which is described by the lacunar polynomials. The Extrados foil length is evaluated numerically.

We intend to obtain analytical expressions with tolerances for the Extrados surface.

For the $f(x)$ and $f_1(x)$ expressions of the two Plane Convex wing profiles we have the table with necessary number of elementary operations:

Table 1. The number of elementary operations for calculation of the functions $f(x)$ and $f_1(x)$

	$f(x)$	$f_1(x)$
Division	1	1
Addition	1	2
Substraction	0	1
Multiplication	3	5
Total number of operations	5	9

For $f(x)$, surface evaluations we need ***ln*** functions and for $f_1(x)$ the surface evaluations we need ***arctan*** functions.

Example: for $L = 1.5$; $a = 0.5$; $b = 1.5$

Table 2. Numerical and analytical evaluations of surface extrados1 (8)

$\int_0^L f_1(x)\mathrm{dx}$	Numerical evaluation	Analytical expression evaluating
CPU time	0.14	0.31

The value of the $\int_0^L f_1(x)\mathrm{dx}$ has been evaluated numerically and analytically and the results are similar (Table 2).

It results a reduction of CPU time four times.

The aircraft design requires a careful evaluation of the wings [1, 4, 9]. The shape of the wings has a tremendous impact on the performance of the aircraft.

For builders, the wings characteristics require numerous evaluations and optimizations.

So reduction of the computer efforts is important. Also, the number of parameters is reduced.

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