Improved Timing for better performance of distributed data acquisition systems - a further refinement

Alexandru Marius PANAIT*

*Corresponding author
INCAS – National Institute for Aerospace Research "Elie Carafoli",
B-dul Iuliu Maniu 220, Bucharest 061126, Romania,
panait.marius@incas.ro

DOI: 10.13111/2066-8201.2022.14.1.9

Received: 03 November 2021/ Accepted: 23 February 2022/ Published: March 2022

Abstract: *Continuing the works debuted in the previous work "Improved Timing for better performance of distributed data acquisition systems" the present paper refines and revisits the concept of a local oscillator with GPS corrections as a source of long-term stable reference clock signals with direct application in distributed data acquisition systems. The system as described herein can generate precise and repeatable clock source signals for trigger and sync of various data acquisition equipment when used in mixed and distributed architectures. These timing signals are a vital part of data acquisition as oftentimes raw data is processed as time series and correlations are made based on timestamps. This paper details improvements in implementation of a cost-effective high precision time base (reference clock) using a GPS receiver as a primary time source and a local high precision real time clock as a secondary oscillator. The improvements to the previous version come both from hardware refinements and from updated software with reduced latencies and reconfigured structure that allows for compensation of internal delays and internal process offsets.*

Key Words*: time synchronization, distributed data acquisition systems, GPS DO, high precision timer*

1. DIGITAL DATA IS ALWAYS A TIME SERIES

Most of the time, modern data acquisition systems collect data in the form of a set of numeric values associated with a time stamp. The time stamp has various origins- it can be generated internally by the measuring equipment, or it can be synchronized/ transmitted from another networked equipment or it can be provided by a local master clock or time server. All data analysis has to do is correlate data – sometimes in very complex ways- but always referring to the same time base. Because of this, precise time stamping of data is an essential part of every data acquisition system's task.

If for whatever reason the time base is distorted, time interval measurement is also affected [1], [2], [3]. A good set of example data sampled with frame jitter and analyzed is presented in [4] where jitter trends are also discussed.[11]. Therefore, it is clear one must eliminate to the greatest possible extent all time base distortions such as jitter or wander, and filter out gross errors produced by arbitrary clock signal faults [11]. Jitter and wander both describe clock errors but while jitter is medium-high frequency deviations of the clock signal, wander is a low frequency variation -sometimes over increased periods of time, even weekly or monthly deviation of such timebases matter. Timing errors mostly represented by jitter are responsible for signal distortion and are a major identified source of measurement error [1] so accurately synchronizing multiple data sources and ensuring a stable time base for the data acquisition system is one of the primary concerns in any serious data collection activity.

As previously shown in [1].,[9] and [11], timestamping implementations fall in two categories: the centrally synchronized master-clock -driven scheme and the distributed “consensus of all subsystems” mean pulse technique. The first requires a dedicated time signal and an associated protocol while the latter can have time information simply mixed with “normal” data packets traffic. Hybrid schemes usually consist in a high precision master clock and a network server and NTP or SNTP protocols that distribute the time information to networked data acquisition systems. In any case, the basis of such systems is a reliable timing source, either local or locally managed remote located. Synchronization is handled differently based on the type of device and whether it is networked or not. In all cases, a basic ultra-stable clock source is necessary to be either directly referred to or is indirectly used by passing it to a timing server on the dedicated network [11].

Data is affected differently by time signal distortions: individual or so-called sample jitter is the apparent time-displacement of only one of the clock signals peaks – and associated data point in case of a single measurement-or it can be a much more serious event, a “frame jitter” -whenever using triggered measurement data [1],[2]. In the latter case, the (faulty) clock data point triggers the measurement of several data points, that will all be translated in time in relation with other data sets or points that would have been simultaneous had the clock glitch not happened. These phenomena bring about signal averaging errors and loss of coherence of data from multiple data sources, when the time signal is not centralized. Or even loss of coherence within the signal itself, by upsetting the temporal order or phase relations between frames- or data points, that make signal recovery difficult. If we are dealing with non-repeating random value signals it is to be expected some parts of the signal to be totally irrecoverable.

To simplify to the extreme, in all of the time series, time...matters a lot. All phase relationships and correlation between signals, as well as auto-correlations that compare a signal with a delayed copy of itself in order to detect periodic components faster and more precise than Fourier analysis are compromised by faults in clock signals. If the clocks are uncoordinated the errors are much greater and impossible to quantify with reasonable accuracy.

A quick test for errors in a signal analysis chain is to sum up two phases opposed but otherwise identical signals, as demonstrated with two simple sine waves in [1]. The result of such operation is formally and conventionally zero- but in practice residual values appear as the phase correlation between the two copies of the signal is affected by clock jitter and wander.

A good set of requirements for a complex data analysis system minimizes such errors by imposing all operations on a signal or set of signals to be governed by the same clock if possible (even if imperfect, it would affect all signals alike) and that the clock source to be as stable as possible – preferably precise (repeatable) if not accurate as well is possible. So at least for the most common immediate data processing requirements, jitter is much worse than wander [1],[2],[3].

This last seemingly simple observation has major implications in the architecture of a precise data source. We can envision a stable oscillator made up of two parts: an external high accuracy reference clock (GPS timing data) and a lower accuracy but high precision local oscillator- the quartz crystal real time clock. The dynamics of the two is influenced by the fact that while the local clock has lower long-term accuracy, it has immediate availability and could be thought as an instant access data source, while the GPS receiver has to maintain lock and process data from several sources and select the best suited to compute position and take time reference from, and yet again take some time to parse data streams to extract timing data. It is very accurate but is somewhat random access as the time to process data depends on a large number of external factors- from satellite position overhead to signal strength and interference as well as other internal processes that take place, vital to the operation of the receiver.

2. A BETTER CLOCK FROM TWO IMPERFECT TIME SOURCES

So far, we have shown that jitter, both the frame and the individual sample variety, is a source of measurement errors that cannot easily be eliminated after the data was collected. Clock wander also distorts the waveform but its relative impact is significant on longer data acquisition streams only. Unintended variations in sampling data rate induce distortions of the collected signal and, in the case of frame jitter, per- interval phase errors that lead to loss of coherence between samples of the same signal and between signals.

A model of these distortions is explored in [4] where a method for estimating the standard deviation of frame jitter in data acquired with oscilloscopes and other data collecting equipment.

Timing errors have a poor statistic representation- in fact they are either truly random or divergent series, qualities that do not allow for the classic statistic operators to be easily used to quantify and counter their effects.

To correctly asses the performance of time measurement devices a special class of statistic measures such as Allan variance and deviation, and modified Allan variance (MAVAR), total deviation and others are employed.

Oftentimes the collected data is used to reconstitute the original signal and sometimes signal spectra are analyzed using the Fourier transform- in its FFT incarnation for discrete numeric data. It has been shown in [5] and [6] that such methods fail if jitter and wander are not accounted for and data is considered to be correctly sampled. Variable data rate is even more affected by this error because if clock rate is not constant by method and it also varies due to timebase instability the error increases and is no longer even estimable by normal means. As modern modal analysis is made using integer multiples of a given frequency, if a signal is frame-shifted with regard to another supposedly synchronous one or component of one, the reconstruction fails because of possibly irrational coefficients slipping in the otherwise periodic signal. Precise numeric analysis does not recognize such signals as true periodic [5], [15]- as the answer to the question whether the sum of two periodic functions is also periodic or not depends on the precision used in numeric representations of the data points.

The device described here is inspired by [7]- a cheap GPS time receiver- but that did not incorporate a local high precision oscillator nor the Shortt-Synchronome hit and miss synchronizer nor the phase-locked 1PPS and 32 kHz time reference signal generators that transform the design into a proper scientific local time reference. A similar problem of synchronizing a local clock to a GPS source is treated in [8] -a cheap way to correlate individual seismic sensors distributed in various locations across the geographical fault or feature was found and a method to synchronize signals is described, with applications in geology – seismic surveys. The inspiration for the system's design is outlined in [9] where the most accurate pendulum clock ever built, that served as time standard from the 20's to the 40's when the quartz thermally stabilized oscillators appeared.

A physical representation of the etalon of time, closest to the literal definition of the second is a Cesium fountain clock- like the NIST-F1 primary time and frequency standard in the United States, part of an international group of atomic clocks that define UTC (Universal

Time, Coordinated). Its uncertainty is of the order of 3 x10^{-16}, equivalent to better than ±1 second in 100 million years [12].

For lower equipment costs, we can use the same level of precision clocks available to end-users of GPS module receivers. A GPS receiver can be used to extract only time data it normally uses to compute position from the satellites. Each GPS satellite has onboard an assembly of several atomic clocks that work in consensus and the time stamp they generate is typically of a precision level comparable to the older NIST standard, of 1x10^{-13} seconds. The Ublox model NEO 7 is a reviewed and improved GPS module that boasts increased sensitivity and multiple fix (3 satellites at once), and cross-compatibility with the majority of the GPS constellations (the original GPS, GLONASS, QZSS). In the previous work [11] I described a low-cost receiver of a previous generation (Ublox NEO 6M) connected to a simple interface controller and using a local oscillator in the form of a simple real time clock module to build a precise GPS disciplined oscillator. For this revision not only the GPS module was upgraded to a faster and more sensitive one but also the real time clock is now a higher precision unit with deviations of max±1 second/day- the same DS 3231 SN that is used in frequency meters and time bases for digital oscilloscopes. The module comes with a temperature compensated quartz (TXCO) crystal oscillator and ensures an almost jitter-free experience.

A good way to obtain long term accuracy and short-term precision is to couple a low jitter local source oscillator with a high accuracy remote clock- such as the GPS receiver [9]. The main problem of such a design is that the "parsing" of the GPS data to extract time stamps from the raw data takes time, and also that applying the new values into the memory of the real time clock module also takes time.

To avoid problems with desynchronization I took inspiration from a very old design -the Shortt-Synchronome high precision pendulum clock- a precision time reference in use up to the invention of the Cesium fountain clock standards. It was in fact the first man-made timekeeping device more accurate than Earth itself-[13] it could be used to detect seasonal variations in Earth's rotation speed as in a famous 1927 experiment.

These high precision artifacts had a peculiar structure- a high precision free pendulum swung in a vacuum tank and was impulsed by a separate normal clock mechanism just when it started to lose momentum, using a special intermittent contact every 30 s so as to let it function without any perturbations.

The time was kept by using a special rolling contact that together with a set of electrically operated gravity levers adjusted the time of the secondary clock using the position of the master pendulum as reference. A second set of gravity levers gave the master pendulum a power restoring push if its amplitude decreased.

This is the true invention- the so-called "hit and miss synchronizer"- an electromechanical construct that compared the phases of the two pendulums after the primary pendulum got its impulse every 30 swings. If the secondary pendulum was lagging, a spring on it would catch a vane and get a push that shortened the swing time, bringing it in phase with the master (the action was called a" hit").

If by contrast the secondary pendulum was leading the master (a "miss") the leaf spring would miss the vane and there would be no accelerating/impulsing, until natural decay would bring the secondary pendulum to lag.

This type of adjustment encouraged small adjustments whenever necessary and allowed for free swinging of the master pendulum so that even if every 30 seconds small variations were sometimes observed, due to corrections the long-term accuracy was high., in fact so high that it compares favorably with almost any modern quartz frequency standard. The installation is shown in figure 1.

Figure 1. The Shortt-Synchronome No 32 free pendulum high precision time reference, used as the US Frequency Standard from 1924 to 1929. The master pendulum is encased in a vacuum cylinder to the left, and the slave clock has electric contacts to distribute 1PPS (one pulse per second) standard scientific time signals. It was subsequently used to determine the gravitational constant by Heyl [14]. The basic principle is behind any PLL oscillator of today and its "asynchroneous synchronization" technology, called the "hit and miss synchronizer" is the basis of a proper GPS-DO (GPS disciplined oscillator)

The idea is basically the same for the GPS disciplined oscillator presented here. A form of software hit and miss synchronizer would try and adjust time every hour (as the precision of a local oscillator using a temperature-compensated quartz crystal is better than the Synchronome slave pendulum) and make the necessary adjustments. A prior determination of the time needed for the program to do a time adjust on the real time clock module will be made using a millisecond timer onboard the controller itself. The time source is in this case a 16 Khz quartz fork crystal – lower precision than any of the elements priorly mentioned. This still works because of the nature of the hit and miss synchronizer- the error compensation will be observed and the needed offset will be adjusted as required to completely null out in the long run. To get the required precision faster, a prior RTC module clock will be set to the programming PC's timing, and this will be disciplined by a consensus of coordinated clocks- the Meinberg NTP networked time protocol. This allows for a minimization of jitter by averaging.

Clock wanders occur over a period greater than 0.1 s (10 Hz), and vary over time. Therefore, wander measurements must be performed over a longer period of time (e.g., 24 h or longer). Unlike jitter testing, wander testing requires a very stable reference clock (i.e., a Cesium or Rubidium clock) [1],[2],[3].

I feel it is necessary to include a definition of real time systems: they are time-deterministic systems using a special structure that ensures each instruction/command takes a (rigorously fixed) number of processor cycles.

While most real time systems are also fast, that is not a requirement. For motion control in loops where necessary data is also collected and sampled, processed and made into commands to the effectors, a real time architecture is required.

Also, when correlating data from several sources, a real time system is a must for accurate representation of the investigated phenomena as the phase correlation between signals must be accurate. In practice, at the Trisonic Wind Tunnel there is a somehow more relaxed approach to precise timing and synchronization as the main research focus has been on stable phenomena so far. Recently several tests of highly unstable phenomena and transient regimes became of interest so the new instrumentation has to be capable of accurate synchronization and timing [11].

So, using the long-proven model of Shortt-Synchronome style clocks rather than the more modern phase locked loops that require a greater uniformity of components, we can design a sufficiently accurate and reasonably precise local time reference with the now customary 1PPS and 32 kHz sync signals available for distributed data systems sync.

The real time clock signals are phase-locked though by design of the clock module, and very stable for short periods of time.

The difference between the Shortt-Synchronome and modern PLL loop is the discrete or continuous nature of the output adjustment. In a way, they counter wander (Shortt-Synchronome) and jitter (traditional PLL). Using both in a device ensures precise generation of the timing signals.

Having all that prior information we can now show the basic structure for a GPS-DO – a GPS disciplined oscillator – or a local low jitter oscillator that is periodically reset with a high accuracy (but also high jitter) GPS time source.

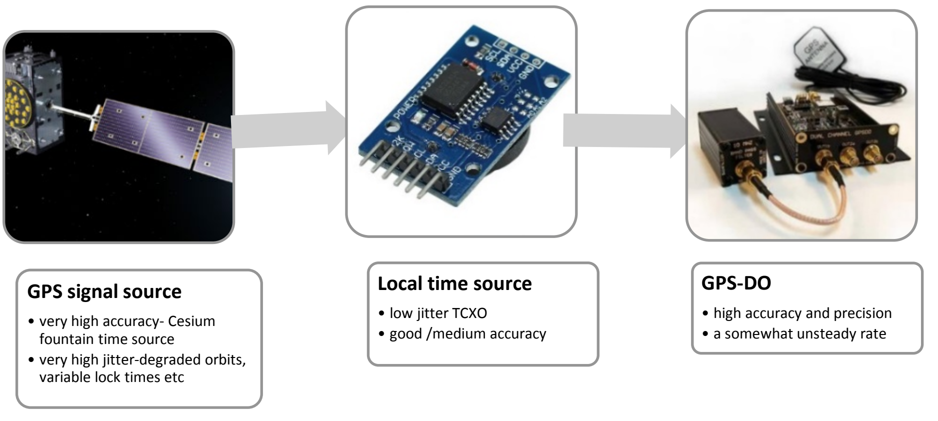

Figure 2. Basic structure of a GPS-DO

3. LOCAL TIME REFFERENCE ARCHITECTURE AND AN AFFORDABLE STRATUM 0 LOCAL TIME SOURCE

In modern research environment the use of a synchronized time reference for all networked computers is a basic requirement. A similar requirement on data timestamping is present even in the most basic long term data collection systems, for example [8] in long term high density seismic surveys.

Wherever the direct use of NTP (Networked Time protocol) is possible, this can provide with a perfectly valid time source with a precision of better than 10^{-32} seconds. Simpler implementation ensures only a readable value of up to a second but that is only limited by the local clock granularity or temporal resolution. With better local oscillator higher precisions are attainable.

NTP protocol is a special networked time transmission protocol that organizes its time sources in layers, with zero or reference being a reference clock- either directly a Cesium fountain clock or older cesium beam etalon, or even a GPS receiver with a high precision local oscillator, and the server directly connected to it relegated to Stratum 1, and so on to lower precision and delay values.

The superior strata servers are understood to have a good Internet connection, stable and wideband to properly sync and link to peers and superior strata clocks. For non-networked applications or where computers are not allowed to be linked to the Internet a local high precision, stratum 0 source is necessary, such as the previously discussed GPS disciplined local oscillator.

The first incarnation of the concept is presented in Figure 3 [11].

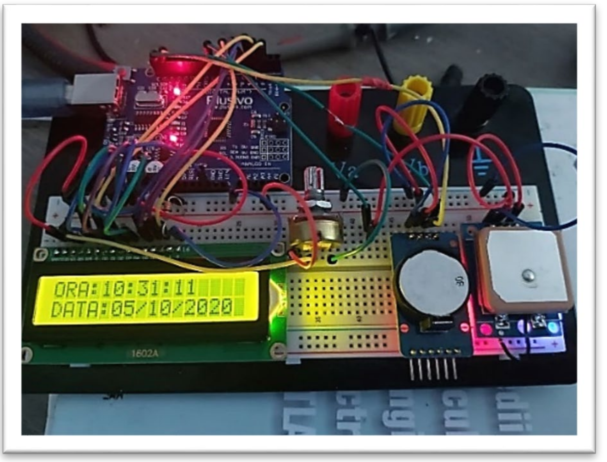

Figure 3. The former Stratum 0 Time Reference Clock.

The old version 1 prototype implemented a simple synchronization routine that was carried out once every two days; it did not take into account variations in the functioning of the local oscillator and did not verify accuracy regularly. The concept of hit and miss synchronizer was not implemented fully but only hinted at. Total free running tests for the first prototype was a week. The 1PPS and 32 kHz outputs were not used.

The improved prototype, shown in Figure 4 shows a good stability and reasonable accuracy; its drift and jitter parameters are currently estimated- we still need a highly stable atomic clock to compare as the device is above or equivalent to the precision class reached by networked time servers (NTP/PTP). Outputs are available in square pulse form, with an adjustable fill factor and frequencies of 1 Hz and 32 KHz, and in text mode format on the serial port on the controller, or via I2C from the real time clock. Also, there is a direct display readout for less demanding synchronization tasks and verification of proper clock operation. Devices similar to this have been in operation both in specialized laboratories and amateur radio transmission shacks for a very long time.

The operation of the prototype is as follows: initially time is read from the GPS receiver and written to the real time clock module's memory. This initial operation, the clock set, is carried out at the initial commissioning. After this step the real time clock will hold the proper time as long as the battery lasts (around 6 years).

The second step is operation of the free running real time clock module that also outputs the 1 pulse per second (1PPS) signals with periodic checks against the GPS receiver. The GPS parsing takes around 40 milliseconds on the used hardware controller and is integrated as an offset to the set time whenever necessary. The attainable precision using an intermittent GPS should be better than 100 microseconds [8], as typical with GPS disciplined RTC devices deployed in the field.

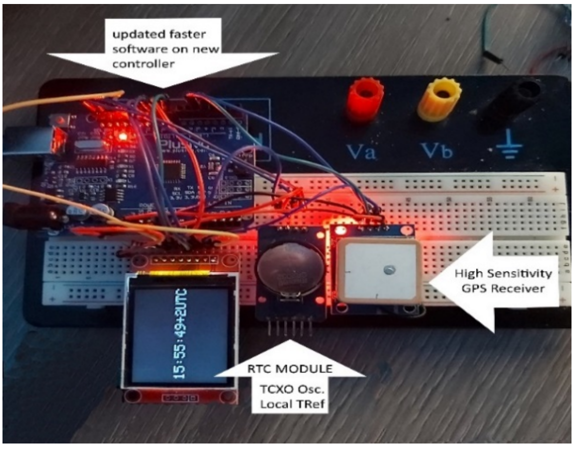

Figure 4. The improved prototype for stratum 0 clock shows updated controller and a better GPS unit along with a revised display unit.

Because of the delay caused by the NMEA sentence parsing (the extraction of useful timing data from a general data string containing position data -discarded in this application-, control data, headers, etc provided by the GPS receiver) the 1PPS precision pulse duty is relegated to the real time clock module. This is checked every second for sync errors and whenever they are over a user-specified value, a clock reset operation is carried out by setting the time from the GPS receiver to the RTC. The reset procedure and the parsing procedure are both highly deterministic – they have the same duration, allowing for compensation of delays induced by set/reset and sync operations. As the primary source for jitter and wander of clocks of this design is thermal variations in ambiental temperature, a thermally compensated RTC chip was chosen. Further improvement would see a large thermal mass fitted over all the active components of the RTC as well as a sealed temperature-stabilized container for thermal stability. The prototype was fitted with a passive ceramic-metallic fractal antenna and did not benefit from the increased sensitivity an active antenna would bring. Still because of the high-performance U-Blox Neo 6 receiver it still functioned satisfactorily indoor by the window with the antenna pointed to the open sky.

The revised prototype ran continuously over a full week at the end of the run no apparent errors were recorded down to the available granularity of one second. Real precision is on the order of 10^{-22} seconds, limited by the PC hardware the NTP service runs on (a 2009 generation 2 Intel i5 v 2500).

To extract meaningful data from the NTP service the application was ran for at least two hours until the monitor indicated minimal delays in regard to the source. The clock is adjusted using skewing (small steps and “second lengthening”) avoiding large adjustments (called skipping) as much as possible. The analysis is shown in NTP Server Monitor by Meinberg- figure 5.

Both the hardware and software used are adaptations of the open-source projects released under LGPL license, for example [7] to the available hardware at the moment of drafting this paper. The finished work includes both community and original code and several modifications and adaptations were made to fit the intended purpose, and should be regarded as LGPL compliant- full schematics and code source will be available upon request and filed under LGPL v3. Precisions in similar systems with a local oscillator and on-off GPS to save power were rated at precisions of up to 100 microseconds per day [8].

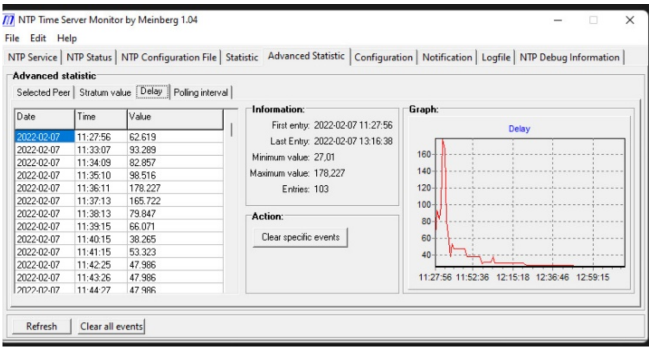

Figure 5. Delays in NTP indicated time shows a stabilization after approximately two hours of continuous running.

Using the 1PPS signals several data collecting nodes in a distributed DAQ system can be synchronized far better than the coarse time resolution of one pulse per second might suggest: all data nodes have precise internal clocks capable of subdivisions , but need to be put in sync at the beginning of the data collecting phase, and can use the 1PPS signal as trigger; then, once triggered, they can adjust their variable delay PLLs so that the next pulse finds them in sync; if not, the second on-board high precision timer is used to compute a time difference that is added or subtracted so that the next pulse acquires perfect synchronism.

From the free running tests as well as from literature, the best time adjustment for the GPS disciplined RTC is found to be one using both periodic and aperiodic checks, much like the so-called astronomical regulator clocks used to in the past (Shortt-Synchronome style precision pendulum regulators) [9].

After initial set-up, a second counter is used to check for time deviations on the local working RTC every 30 seconds for example – and adjustments are made as necessary – but also if the deviation is above a certain user-specified threshold, supplemental checks are scheduled, faster than 30 seconds. It might sound puzzling at first but over-correcting a free-running clock is as harmful as under-correcting it- the Allan variance should be as close to linear as possible.

Frequency jumps and unsteady clock rate caused by overcorrecting hurt more the signal stability than a somewhat small running rate error for the base clock. A running rate error leads to signal compression or expansion- harmonic deformation – but sudden frequency jumps lead to a garbled, interrupted or totally broken signal like shown in Figure 2, panel 3.The official clock correction method uses induced jitter to correct for wander, basically trading one type of clock error for smaller quantities of both (jitter and wander)- but clock corrections must take into account the characteristics of each particular oscillator and refrain from overcompensation [10].

From these requirements we deduce the need for quality local oscillators (with low jitter) to ensure minimal short-term signal shape corruption and a well-tempered policy of corrections to keep long term stability within reasonable values.

The project will continue with a long-term stability assessment and building two more prototypes of the same structure, to compare performance and try the improved precision timing technique using “consensus of several independent clocks”. It is also interesting to compare performance of said prototypes with a PTP based PC client using a GPS dongle as a secondary time source.

Active GPS antennae will be used to increase sensitivity and improve cold start times for the GPS disciplined clocks.

4. CONCLUSIONS

The improved prototype showed a faster GPS lock and set/stabilization due to a better GPS receiver and revised software. The Synchronome hit-and-miss synchronizer was implemented in software and optimized so that the routine does not engage when it is not needed, to avoid clock skips (improves wander values) and lower jitter levels are ensured by using a temperature stabilized crystal oscillator for the reference local quartz clock.
The 1PPS and 32 kHz outputs ensure high precision trigger pulses that can be used directly as triggers for data acquisition systems and the stabilized clock source can be used as source for local time synchronization. The NTP protocol is not implemented in the software so far, but can be implemented in a future revision. As it stands of now, the real time clock can be set once using a special function that captures system time.

REFERENCES

[1] A. N. Kalashnikov, R. E. Challis, M. E. Unwin and A. K. Holmes, Effects of frame jitter in data acquisition systems, in *IEEE Transactions on Instrumentation and Measurement*, vol. **54**, no. 6, pp. 2177-2183, Dec. 2005, doi: 10.1109/TIM.2005.858570.

[2] T. Dabóczi, Uncertainty of signal reconstruction in the case of jittery and noisy measurements, *IEEE Transactions on Instrumentation and Measurement*, vol. **47**, no. 5, pp. 1062–1066, Oct. 1998.

[3] T. M. Souders, D. R. Flach, C. Hagwood, and G. L. Yang, The effects of timing jitter in sampling systems, *IEEE Transactions on Instrumentation and Measurement*, vol. **39**, no. 1, pp. 80–85, Feb. 1990.

[4] A. N. Kalashnikov, R. E. Challis, M. E. Unwin, and A. K. Holmes, *"Quantification of frame jitter in data acquisition systems*," in Proc. 2003 IEEE Int. Symp. Intelligent Signal Proc., ch., Budapest, Sep. 2003, pp. 15–20.

[5] R. N. Bracewell*, The Fourier Transform and Its Applications (revised ed.),* 1986, McGraw-Hill, 1st ed. 1965, 2nd ed. 1978.

[6] M. Rahman, *Applications of Fourier Transforms to Generalized Functions*, WIT Press Southampton, Boston, ISBN 978-1-84564-564-9, 2011.

[7] * * * *A cheap and accurate clock based on GPS - Arduino Project Hub*, retrieved 27-11-2020, an open-source project released under LGPL.

[8] R. Tian, J. Zhang, S. Zhang, L. Wang, H. Yang, Y. Chen, Y. Jiang, J. Lin and L. Zhang, A High-Precision Energy-Efficient GPS Time-Sync Method for High-Density Seismic Surveys Applied Sciences, *Applied Sciences*, An Open Access Journal from MDPI, 2020.

[9] M. A. Panait, *Shortt-Synchronome Time synchronization in distributed data collection systems- an old solution to a new problem,* Aerospace 2020 Virtual Conference, Online, 15th of October 2020.

[10] R. H. Miles, *Synchronome - Masters of Electrical Timekeeping*, London: AHS. ISBN 978-0901180551, (2019).

[11] M. A. Panait, Improved Timing for better performance of distributed data acquisition systems, *INCAS Bulletin,* Volume **13**, Issue 1/2021, https://doi.org/10.13111/2066-8201.2021.13.1.12.

[12] * * * https://www.nist.gov/pml/time-and-frequency-division/time-realization/primary-standard-nist-f1, retrieved 02..02.2022 – a description of the primary time and frequency standard

[13] J. Betts, (May 22, 2008), *Expert's Statement, Case 6 (2008–09) William Hamilton Shortt regulator,* Export licensing hearing, Reviewing Committee on the Export of Works of Art and Objects of Cultural Interest. UK Museums, Libraries, and Archives Council. Archived from *the original (DOC)* on October 25, 2009. *Retrieved 02.02 2022, Wayback Machine*

[14] * * * NIST Virtual Museum, US National Institute of Standards and Technology website, https://www.nist.gov/image/p15421coll3891fulljpg, copyright free. Primary source Wikipedia on Frank Hope-Jones article, retrieved 02 02 2022.

[15] * * * Lecture Notes for EE 261- *The Fourier Transform and its Applications*, Prof. Brad Osgood, Electrical Engineering Department, Stanford University