Aerodynamic performance prediction of Darrieus-type wind turbines

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Abstract: *The prediction of Darrieus wind turbine aerodynamic performances provides the necessary design and operational data base related to the wind potential. In this sense it provides the type of turbine suitable to the area where it is to be installed. Two calculation methods are analyzed for a rotor with straight blades. The first one is a global method that allows an assessment of the turbine nominal power by a brief calculation. This method leads to an overestimation of performances. The second is the calculation method of the gust factor and momentum which deals with the pale as being composed of different elements that don’t influence each other. This method, developed based on the theory of the turbine blades, leads to values close to the statistical data obtained experimentally. The values obtained by the calculation method of gust factor - momentum led to the concept of a Darrieus turbine, which will be tested for different wind values in the INCAS subsonic wind tunnel.*

1. INTRODUCTION

The power generation technologies aim at increasing the use of unconventional and clean energy. In this respect activities related to research and development, manufacturing and operation of wind plants are stimulated.

Considering that the wind potential in Romania is small as extent and intensity, INCAS aims to take advantage of the experience gained in aerodynamics, aero-elasticity and mechanical engineering in order to achieve wind turbines suitable for the wind potential available in our country. A combined aero-rotor was designed, consisting of a Savonius rotor and a Darrieus turbine which ensure its operation at low wind speeds of 4-5 m/s. The Savonius rotor built from lightweight materials and having small dimensions decouples by means of a centrifugal clutch when the Darrieus turbine reaches the nominal working regime, allowing for a substantially increased couple [3].

A five-bladed Darrieus turbine will be tested in the subsonic wind tunnel. Testing will be done for different construction and arrangement of blades [1-2].

2. CALCULATION METHODS FOR A ROTOR WITH STRAIGHT VERTICAL BLADES

The global method

For turbines with straight vertical blades, the rotation axis is perpendicular to the wind direction. The maximum power coefficient estimation using the global method is inspired by Betz's model, in which a tube of current of turbine rotor size is considered [3-4].

Let's consider the turbine area divided in two: S' and S''. Passing through the two surfaces, the current modifies its speed twice. The force exerted by the fluid on S', and S'' respectively, is

$$F' = Q\left(V_\infty - V_2\right) = \rho\, S'\, V_2 \left(V_\infty - V_2\right) \tag{1}$$

and

$$F'' = Q\left(V_2 - V_3\right) = \rho\, S''\, V_2 \left(V_2 - V_3\right) \tag{2}$$

Using the linear nature of the velocity transformation law, $V_2 = k\, V_\infty$ and $V_3 = k\, V_2 = k^2 V_\infty$, the average power supplied by S' and S''

$$P_m = F'\, V'_m + F'' V''_m \tag{3}$$

where V'_m and V''_m are the average speeds across the respective surfaces:

$$V'_m = 0.5\left(V_\infty + V_2\right) = 0.5\, V_\infty (1 + k) \tag{4}$$

$$V''_m = 0.5\left(V_3 + V_2\right) = 0.5\, k\, V_\infty (1 + k) \tag{5}$$

After the necessary replacing the provided average power is obtained:

For a $k_{max} = \dfrac{1}{\sqrt[4]{5}}$ (determined experimentally):

$$P_{max} = 0.5\, \rho\, S'' V_\infty^3\, \frac{4\sqrt[4]{125}}{25} \tag{6}$$

in this case the power coefficient is

$$C_p = \frac{4\sqrt[4]{125}}{25} = 0.53 \tag{7}$$

This method of calculation overestimates the performances.

The method of the gust factor and momentum

This method involves an analysis of the blade as being composed of distinct elements, which don't influence each other from the aerodynamic point of view. The velocity induced on each element is determined using the momentum equation; the aerodynamic forces on the element are calculated using the lift and drag coefficients of the considered profile section.

For a gust element we have: F – the element aerodynamic centre, R – the distance from to the Darrieus rotor axis, θ – the angle between the Ox axis and the R radius, z – the F height and δ – the angle of the normal element in F to the blade element and the horizontal Oxy plane (for cylindrical rotors $\delta = 0$). Position of F with respect to the $Oxyz$ axis system is determined by the R, θ, z coordinates.

Let V be the absolute wind speed, W its relative speed against the considered gust element and U the appropriate transport velocity U,

$$U = \omega\, r \tag{8}$$

it results

$$W = V - U = V - \omega\, r \tag{9}$$

where:

ω – turbine rotor angular velocity.

The relative velocity components along the specified axes are

$$W_r = V \sin\theta, \qquad W_t = \omega\, r + V \cos\theta, \qquad W_v = 0 \tag{10}$$

The director cosines of the normal to blade element in the considered system of axes are: $\cos\delta, 0, \sin\delta$.

The W_n normal component (perpendicular to the gust element) of the W speed:

$$W_n = V \sin\theta \cos\delta \tag{11}$$

To calculate the aerodynamic forces acting on the blade element the W_u component of the relative wind speed shall be considered

$$W_u^2 = W_t^2 + W_n^2 \tag{12}$$

the following relation is thus obtained

$$W_u^2 = (\omega r + V \cos\theta)^2 + (V \sin\theta \cos\delta)^2 \tag{13}$$

For (α) local incident angle, defined by the relation

$$\mathrm{tg}\,\alpha = \frac{W_n}{W_t} \tag{14}$$

It results

$$\mathrm{tg}\,\alpha = \frac{V \sin\theta \cos\delta}{\omega r + V \cos\theta} \tag{15}$$

With:

C_n – normal aerodynamic coefficient,

C_t – tangential aerodynamic coefficient,

C_z – lift coefficient

C_z – rag coefficient the following can be written:

$$C_n = C_z \cos\alpha + C_x \sin\alpha \tag{16}$$

$$C_t = C_z \sin\alpha - C_x \cos\alpha \tag{17}$$

The α incident angle varies over a complete rotation of the considered blade element, being dependent on the angle θ.

The elementary normal force (dN) and the tangential force (dT) acting on blade element depends on the coefficients C_n and C_t

$$dN = q\, C_n\, c\, ds \tag{18}$$

$$dT = q\, C_t\, c\, ds \tag{19}$$

where:

q – represents the dynamic pressure,

$$q = \frac{1}{2} \rho W_u^2 \tag{20}$$

c – length of profile chord,

ds – width of considered blade element.

Having dz – the height of considered blade element,

$$dz = ds \cos\delta \tag{21}$$

to result

$$dN = q\,C_n\,\frac{c\,dz}{\cos\delta} \tag{22}$$

$$dT = q\,C_t\,\frac{c\,dz}{\cos\delta} \tag{23}$$

Noting with dF – the elementary aerodynamic force on wind direction, we have:

$$dF = dN\cos\delta\sin\theta - dT\cos\theta \tag{24}$$

Depending on (dN) and (dT), this becomes

$$dF = q\,c\left(C_n\sin\theta - C_t\frac{\cos\theta}{\cos\delta}\right)dz \tag{25}$$

The dF elementary aerodynamic force varies over a blade complete rotation around the axis Darrieus rotor. If the chord c is constant along the length of the blade, the aerodynamic force F exerted on the Darrieus rotor on wind direction is calculated with:

$$F = \frac{n_p c}{2\pi}\int_{-H}^{H} q\left(C_n\sin\theta - C_t\frac{\cos\theta}{\cos\delta}\right)d\theta\,dz \tag{26}$$

where: n_p – number of wind turbine blades.

For power calculation, considering the relation

$$P = M\,\omega \tag{27}$$

the momentum on the considered blade element about the turbine axis of rotation is:

$$dM = r\,dT = \frac{q\,c\,C_t}{\cos\delta}\,r\,dz \tag{28}$$

The momentum given by the dN component is null.

Over the whole Darrieus rotor we have the momentum expression:

$$M = \frac{n_p c}{2\pi}\int_{-H}^{H}\frac{q\,C_t}{\cos\delta}\,r\,d\theta\,dz \tag{29}$$

The relation for power calculation becomes:

$$P = \frac{n_p c}{2\pi}\int_{-H}^{H}\frac{q\,C_t}{\cos\delta}\,\omega\,r\,d\theta\,dz \tag{30}$$

λ – the end speeds ratio,

$$\lambda = \frac{\omega R}{V_1} \tag{31}$$

where V_1 – wind speed

The power coefficient C_p is given by the relation

$$C_p = \frac{P}{\frac{\rho}{2}V_1^3\,S} \tag{32}$$

As

$$P = \frac{n_p c}{2\pi}\int_{-H}^{H}\frac{q\,C_t}{\cos\delta}\,\omega\,r\,d\theta\,dz = \frac{\rho\,n_p c}{4\pi}\int_{-H}^{H}\int_0^{2\pi}W_u^2\,C_t\,\frac{\omega\,r}{\cos\delta}\,d\theta\,dz \tag{33}$$

it results,

$$C_p = \frac{n_p c}{2\pi S}\int_{-H}^{H}\int_0^{2\pi}\frac{W_u^2}{V_1^3}C_t\frac{\omega r}{\cos\delta}d\theta\,dz \tag{34}$$

C_m – momentum coefficient

$$C_m = \frac{M}{\frac{\rho}{2}V_1^2\,S\,R} \tag{35}$$

where:
S – surface described by the rotor, perpendicular to the wind direction.

To calculate the momentum coefficient we consider the following relation existing between it and the power coefficient:

$$C_m = \frac{C_p}{\lambda} \tag{36}$$

It follows that knowing one of the two coefficients we can immediately determine the other.

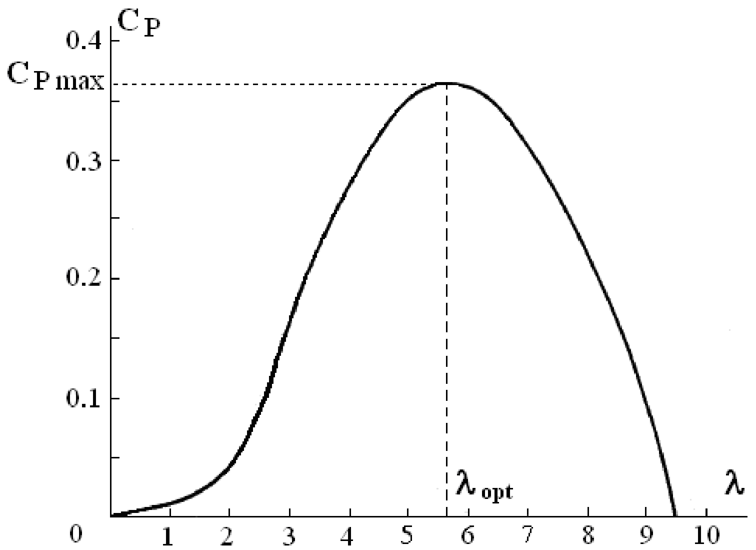

Fig.1 – Power coefficient variation C_p depending on the end speeds ratio λ

The dependence $C_p(\lambda)$ is significantly influenced by the solidity ratio σ, which is an important design parameter generally defined as the ratio of blades total surface to the area described by them in their rotational motion [6-8].

$$\sigma = \frac{n}{S}\int_0^b c(y)\,dy \tag{37}$$

where:
b – blade span;
$c(y)$ – blade chord in section y.

If vertical axis turbines, the solidity ratio is usually defined as

$$\sigma = \frac{n_p\, c}{R} \tag{38}$$

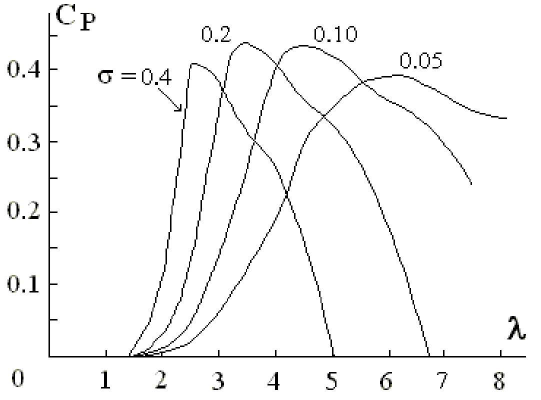

Fig.2 – Influence of σ solidity ratio on the $C_p(\lambda)$ curve

The prediction based on the calculating method for the gust factor and momentum gives closer results to the obtained experimental data in comparison with other calculation methods.

The INCAS concerns have highlighted the possibility of obtaining experimental data for the verical shaft rotor in the subsonic wind tunnel.

Combined results for a combined Darrieus-Savonious rotor allowed to design an installation for water extracting.

Fig.3 – Combined Darrieus-Savonius rotor tested in the subsonic wind tunnel

The calculation method for the gust factor and momentum was utilized to conceive a multi-blades Darrieus rotor intended to produce electricity.

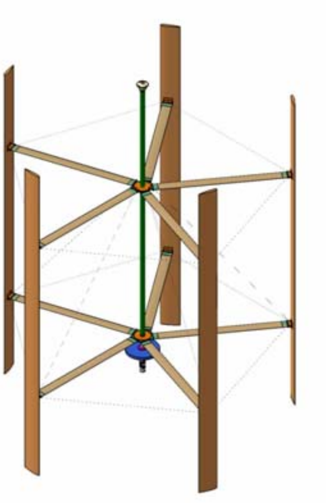

Fig.4 – Darrieus rotor

3. CONCLUSIONS

The prediction of the aerodynamic performances of a Darrieus turbine enables the wind installations manufacturer to estimate the structural characteristics of the rotor appropriate to the wind potential of the operation area. Applying the lifting surface analysis to the quasi-steady and nonsteady movement conditions the essential characteristics of a Darrieus turbines are highlighted.

The calculation method for the gust factor and momentum ensures getting a range of types and sizes depending on the average wind speeds, without requiring a laborious numerical calculation. Based on the designed Darrieus turbine performances the required generator can be established. The forces calculated by the gust factor and moment method allow sizing the turbine base support.

The text referred to a combined Darrieus Savonius rotor tested in the subsonic wind tunnel and a 5-blade Darrieus turbine design.

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