Comparative Analysis of Low Earth Orbit Satellite Attitude Determination using Sensor Integration and Attitude Estimation using Kalman Filter

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Abstract: *The objective of the research is to undergo a performance comparison in terms of accuracy, convergence time, amount of memory, etc. between the satellite attitude determination and attitude estimation using non-linear filters. The fundamental approach towards it is to design an OBC (On Board Computer) that would help in achieving a controlled output for the chosen plant (MicroMAS1 satellite). The attitude determination algorithm is implemented through TRIAD algorithm, which takes sensor readings of body frame and inertial frame of reference. Then it is used to determine the rotation matrix (DCM) by converting the matrix form into vector form and again back to matrix form to determine the 3x3 matrix, which includes all the Euler angle equations to determine the pitch, roll, yaw characteristics of the system. The attitude estimation algorithms involves the use of nonlinear filters which provide an added advantage that energy can be transferred in a designed manner and extra degrees of freedom are available in filter design. The Unscented Kalman Filter (UKF) is preferred as it addresses the problem using a deterministic sampling approach. Moreover, the non-linear filters are used to remove the noise error and disturbance caused by engine. The design of satellite attitude models involves more of a mathematical approach that would be dealt with MATLAB and SIMULINK operations.*

Key Words: *Attitude determination, Non-linear Filters, Kalman algorithm, Euler angle*

1. INTRODUCTION

The orientation of the satellite in space is called attitude, and sensors must be used to control and stabilize the spacecraft and attitude information; with the help of reading sensors the Euler angles can be determined, i.e. pitch, yaw and roll. The attitude is measured with devices such as sun sensor, magnetometer, and earth horizon sensor. Generally, at least two or three sensors are placed in a spacecraft, because a single sensor cannot measure all of Euler's angles [1].

A. Attitude Determination from sensor

Sun Sensor: - It is a navigational instrument, which is placed in a spacecraft to detect the position of the Sun, which in turn helps to determine the sun vector in spacecraft coordinate; it is also used for attitude control of the spacecraft [1]. One of the main reasons why all

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spacecraft prefer the sun sensor is because it uses the sun as a reference which, compared to other astronomical objects, is brighter and also due to its relatively small radius observed by the space vehicle close to Earth. The sun sensor are used to orienting the attitude of satellite with the help of solar panels. The solar sensor is also needed to check if the solar panels used in a satellite are correctly oriented. There are two types of sun sensors i.e. continuous and discrete.

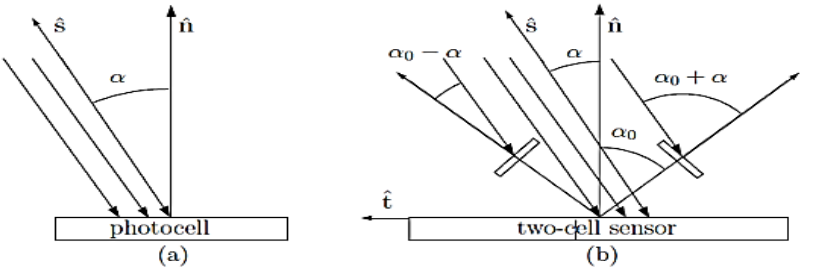

Fig. 1 Photocells for Sun Sensor; (a) Single Photo cell, (b) Pair of Photocell

Continuous sun sensors are defined as photocells which have a current output signal which is a continuous function of cosine angle between the direction of the sun and the normal to the photocell whereas Digital sun sensor are used to generate a discrete output that is made by the function of the sun angle [2].

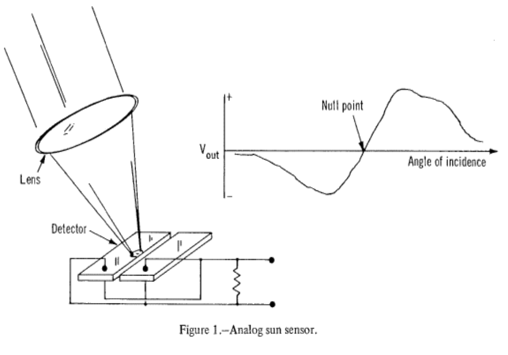

Fig. 2 Sun Sensor (Attitude Sensor)

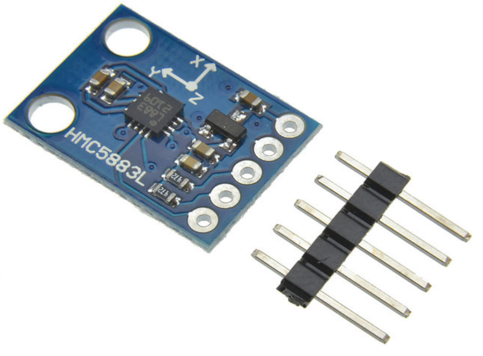

Fig. 3 Magnetometer

Magnetometer: - A magnetometer is a device for measuring the magnetic induction and also measures the local magnetic field along the three directions. The advantage of using magnetometer sensors over other sensors is that they can work normally under severe conditions [3]. A magnetometer helps in determining the earth's magnetic field vector in a body frame of reference and the designers help in determining the attitude of the frame of sensor with respect to the spacecraft body frame. The magnetometer helps in determining the magnetic field in the body frame and mathematical modeling of magnetic field of earth will determine the strength of magnetic field in the inertial frame of reference. It is widely used for the coarse attitude determination because of earth's variable magnetic field.

$$m_I^* = \frac{R^3 H_0}{r^3}\left[\, 3D_I^T \hat{\boldsymbol{r}}_I\, \hat{\boldsymbol{r}}_I - D_I \,\right] \tag{1}$$

$$m_I^* = \frac{R^3 H_0}{r^3}\begin{bmatrix} 3(d^T\hat{r})\hat{r}_1 - \sin\theta'_m \cos\alpha_m \\ 3(d^T\hat{r})\hat{r}_2 - \sin\theta'_m \sin\alpha_m \\ 3(d^T\hat{r})\hat{r}_3 - \cos\theta'_m \end{bmatrix} \tag{2}$$

where the vector $\hat{d}$ is the unit dipole direction, with components in inertial frame:

$$d_I = \begin{bmatrix} \sin\theta'_m \cos\alpha_m \\ \sin\theta'_m \sin\alpha_m \\ \cos\theta'_m \end{bmatrix} \tag{3}$$

The vector $\hat{r}$ signifies the unit vector toward the position vector of the shuttle. The constants R = 6378 km is the sweep of the earth and H_0 = 30,115 nT is a consistent portraying the Earth's attractive field, individually.

Earth Horizon Sensor**:** - It helps in determining the attitude information in LEO for the chosen satellite from the measurement provided by the thermopile; the thermopile measurement of the earth's emitted infrared rays locates the horizon of the earth [4].

The spacecraft attitude is determined by detecting the earth's infrared electromagnetic radiation and then determining the region covered by the earth in sensor's FOV to compute a nadir vector in the body frame of reference.

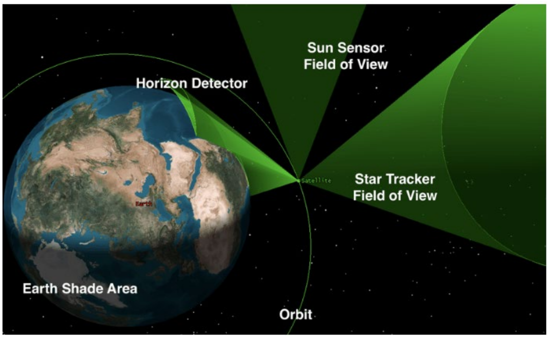

Fig. 4 Spacecraft Mission with horizon sensor

The most used sensor in a satellite is EHS because of its effectiveness and relatively low cost so it is the best option for an Earth-observing mission. The attitude knowledge relative to earth to provide the brighter target and thermopile which are used are inexpensive, low in size and weight. In a large spacecraft most of the time EHS sensors are placed on the spinning wheel while on smaller spacecraft they are body mounted due volume and power limitations.

2. LITERATURE REVIEW

Attitude estimation is a method from which we can determine the orientation of any space vehicle. Attitude cannot be determined through a single measurement, we have to calculate it from a set of measurements; this method of calculating the attitude is known as attitude estimation. There are a large number of ways through which we can estimate the attitude of the satellite. Furthermore, all the estimation techniques need observations as their input, which are taken from the sensors installed in the satellite. Every method has its own advantages and disadvantages. In this research work, our focus is on statistical filtering estimation. It is one of the precise methods of estimation. Kalman filter is one of the most common filters, which are used in statistical estimation. It is the only filter through which we can estimate the attitude as well as the angular rate at the same time. Kalman Filter works in a loop. It uses a number of equations and previous data values to estimate the true value of the space vehicle [5]. It is a calculation that uses a movement to estimate the observed overtime, which contains different noise and errors and produces the result of unknown variables, which will be more accurate than those dependent on a single estimate, by evaluating a joint probability appropriation over variables, each time. Furthermore, Kalman filter is largely used in the field of DSP in time analysis. Kalman filter works in loop, which means it needs only the estimated state from the previous step and the current measurements to calculate the current state. The algorithm of Kalman filter works in two step-processes. One is the prediction step and the other is the updated step. The predicted step helps to estimate the current state from the previous time step [5]. The predicted step is also termed as new state. In the updated step, the new state is combined with the current observations and we get updated estimation as a result.

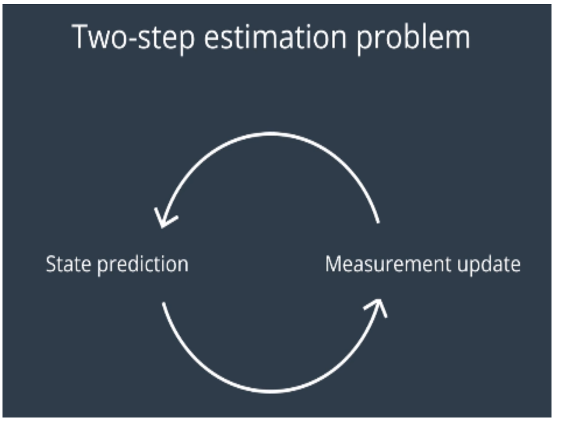

Fig. 5 Filter loop that goes on and on

Because of the recursive nature of the algorithm, it can run continuously by using the current information estimations and the determined state and its uncertainty matrix, there is no any need of extra past values. There is a myth regarding Kalman filter that it assumes all the errors and the measurements, and they are Gaussian distributed. Originally, the filter uses symmetrical projection hypothesis to show that the covariance is limited, and this outcome doesn't require any presumption [6], [7]. On the basis of extension and generalization, Kalman filter can be divided into two categories:

- EKF (Extended Kalman Filter)
- UKF(Unscented Kalman Filter)

The Kalman filter has a number of applications. It is used in tracking objects, navigations and control of vehicles, specially spacecraft etc. Furthermore, it is also used in many computer based applications like feature tracking, cluster tracking, etc.

3. METHODOLOGY

Determining Attitude from sensor integration

However, these sensors have limitations as sun sensors won't be able to work in the periods of eclipse in orbit i.e. they are inoperable and for a satellite in LEO during an eclipse event earth is huge and bright in the spectrum, magnetometers are unable to achieve highly accurate measurement of attitude because of variable magnetic field of earth [8]. An earth horizon sensor is more efficient and inexpensive and also determines the exact attitude, and also satisfies the attitude requirements in LEO for micro satellites, especially for earth missions. To determine the attitude of any satellite there must be a minimum of two sensors because an earth horizon sensor can only find the pitch and roll angle; so to find the yaw angle we must need another sensor. The satellite used in our project is a Nano satellite i.e. Micro sized

Microwave Atmospheric Satellite (MicroMAS1) compatible with 3U Cube Sat specification [9], [10]. It is used to collect microwave radiometry information and then to transmit the information to the ground station with the help of a radio frequency for testing and use in weather forecast.

A. Attitude Determination of MicroMAS1 (Nadir Angle Estimation for EHS)

The attitude is obtained by determining the hidden region of the earth, practically the visual field of the sensor, which is defined as the solid angle through which the sensor is sensitive to electromagnetic radiation; so, when the projection of the sensor's FOV collides with the projection of the earth, a region called the earth's horizon is formed, which is defined as their field of view partially covered by the earth. The horizon is the point where the sensor's line of sight is tangent to the earth's surface. From the figure bellow we can observe that there is an area which is hidden by the earth in the sensor's FOV which is defined the overlap region between the projection of the sensor's FOV and the earth onto the space vehicle centered unit sphere [11]. Both projections have a circle shape in which the FOV projection of the sensor has a radius of ε and the Earth's disk has a radius of ρ. The center of the sensor's field of projection and the center of the earth disk on the space vehicle centered sphere display the direction of the bore sight and the direction of nadir vector. S is the overlap area and α is the angle between the sensors boresight and the nadir vector.

Case 1: - when $\alpha \geq \rho + \varepsilon$

This shows that the circular projection does not overlap the sphere of the unit centered on the spacecraft, and that the sensor cannot detect the earth.

Case 2: - when $\alpha \leq \rho - \varepsilon$

This shows that the sensor's FOV projection is fully covered by the earth projection. So we can conclude that the sensor can detect the earth horizon sensor within the range $(\rho - \varepsilon, \rho + \varepsilon)$

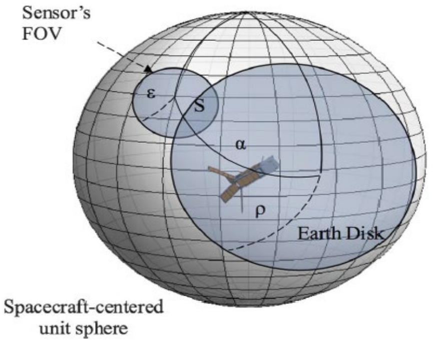

Fig. 6 Projection of the senor's FOV and earth on the spacecraft centred unit

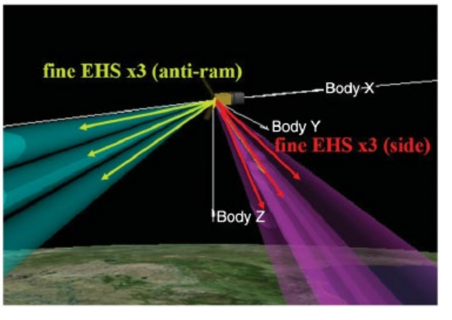

Fig. 7 EHS configuration

The overlap region S can be found out mathematically as it is a function of $(\alpha, \varepsilon, \rho)$

$$S(\boldsymbol{\alpha},\varepsilon,\boldsymbol{\rho}) = 2\Big(\boldsymbol{\pi} - \cos(\boldsymbol{\rho})\,\mathbf{cos}^{-1}\Big(\frac{\cos(\varepsilon)-\cos(\boldsymbol{\rho})\cos(\boldsymbol{\alpha})}{\sin(\boldsymbol{\rho})\sin(\boldsymbol{\alpha})}\Big) - \cos(\varepsilon)\,\mathbf{cos}^{-1}\Big(\frac{\cos(\boldsymbol{\rho})-\cos(\varepsilon)\cos(\boldsymbol{\alpha})}{\sin(\varepsilon)\sin(\boldsymbol{\alpha})}\Big) - \mathbf{cos}^{-1}\Big(\frac{\cos(\boldsymbol{\alpha})-\cos(\varepsilon)\cos(\boldsymbol{\rho})}{\sin(\varepsilon)\sin(\boldsymbol{\rho})}\Big)\Big) \tag{4}$$

Assumption: - (1) within the sensor's FOV, the earth's infrared radiation wavelength is uniform (2). The horizon is sharp and circular [12].

B. Nadir Vector Estimation

In MicroMAS1, the coordinate system is defined as the x coordinate in the nominal velocity vector, y direction coordinates towards the side of the space vehicle, the z direction points to the nadir. Figure. 8. shows the two cones intersection having different axis direction defined by the cone angle and a sensor boresight vector which is defined as the nadir angle which we have found earlier in nadir angle determination [13]. The vectors S_1 and S_2 represent the sensor boresight; Φ_1 and Φ_2 represent the nadir angle and the vector P and P' are the intersection of the two S_1 and S_2 centered cone representing the nadir vectors.

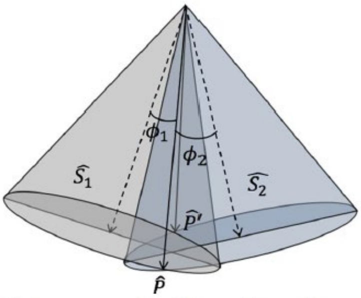

Fig. 8 Pictorial Representation of Nadir Vector

$$\hat{\boldsymbol{P}}.\hat{\boldsymbol{S}}_1 = \cos(\Phi_1) \tag{5}$$

$$\hat{\boldsymbol{P}}.\hat{\boldsymbol{S}}_2 = \cos(\Phi_2) \tag{6}$$

$$|\hat{\mathbf{P}}|=1 \tag{7}$$

The analytical form of the nadir vector are the solution of the set of three equations where the first and the second equations ensure that the angle between the solution and the boresight vector equals to the nadir angle, and the third equation is the normalized condition. The above three equations consist of three variables which are three nadir vector components, when both sensors detect the horizon then the system of equation has at least one solution [14], [15].
Let the nadir vector solution $P=(P_x, P_y, P_z)$ and the horizon sensor boresight vector $S_1 =(S_{1x},S_{1y},S_{1z})$ and $S_2=(S_{2x},S_{2y},S_{2z})$ and Φ_1 and Φ_2 be the nadir angle.

$$(A_X^2+A_Y^2+1)P_Z^2 + 2(A_XB_X + A_YB_Y)P_Z + (B_X^2+B_Y^2-1) = 0 \tag{8}$$

$$\left.\begin{aligned}
\mathbf{P}_x &= \frac{(\cos\Phi_1 - \mathbf{S}_{1z}\mathbf{P}_z)\mathbf{S}_{2y} - (\cos\Phi_2 - \mathbf{S}_{2z}\mathbf{P}_z)\mathbf{S}_{1y}}{\mathbf{S}_{1x}\mathbf{S}_{2y} - \mathbf{S}_{1y}\mathbf{S}_{2x}}\\
\mathbf{P}_y &= \frac{(\cos\Phi_1 - \mathbf{S}_{1z}\mathbf{P}_z)\mathbf{S}_{2x} - (\cos\Phi_2 - \mathbf{S}_{2x}\mathbf{P}_z)\mathbf{S}_{1x}}{\mathbf{S}_{1y}\mathbf{S}_{2x} - \mathbf{S}_{1x}\mathbf{S}_{2y}}\\
\mathbf{P}_z &= \frac{\sqrt{(\mathbf{a}_x\mathbf{b}_x + \mathbf{a}_y\mathbf{b}_y)^2 + 2(\mathbf{A}_x^2 + \mathbf{A}_y^2 + 1)(\mathbf{B}_x^2 + \mathbf{B}_y^2 - 1)}}{(\mathbf{A}_x^2 + \mathbf{A}_y^2 + 1)}
\end{aligned}\right\} \tag{9}$$

here,

$$\left.\begin{aligned}
\mathbf{A}_x &= \frac{\mathbf{S}_{2z}\mathbf{S}_{1y} - \mathbf{S}_{1z}\mathbf{S}_{2y}}{\mathbf{S}_{1x}\mathbf{S}_{2y} - \mathbf{S}_{1y}\mathbf{S}_{2x}}\ ;\ \mathbf{B}_x = \frac{\cos\Phi_1\mathbf{S}_{2y} - \cos\Phi_2\mathbf{S}_{1y}}{\mathbf{S}_{1x}\mathbf{S}_{2y} - \mathbf{S}_{1y}\mathbf{S}_{2x}}\\
\mathbf{A}_y &= \frac{\mathbf{S}_{1z}\mathbf{S}_{2x} - \mathbf{S}_{1x}\mathbf{S}_{2z}}{\mathbf{S}_{1x}\mathbf{S}_{2y} - \mathbf{S}_{1y}\mathbf{S}_{2x}}\ ;\quad \mathbf{B}_y = \frac{-\cos\Phi_1\mathbf{S}_{2x} + \cos\Phi_2\mathbf{S}_{1x}}{\mathbf{S}_{1x}\mathbf{S}_{2y} - \mathbf{S}_{1y}\mathbf{S}_{2x}}
\end{aligned}\right\} \tag{10}$$

As stated above that attitude determination can be done through sensor readings for the body and the inertial reference frame.

Now, the challenge is that the problem becomes either undetermined or over determined. This happens because three parameters are needed for the attitude determination of a particular space vehicle and one sensor only provides two parameters, which make the system undetermined.

If two sensors are used then the system is over determined as the number of unknowns is three and the number of known quantities is four.

Therefore, different approaches are used to determine the attitude of the satellite like Quest method, Triad algorithm, least square method etc.

The main target is to determine the attitude of a particular chosen satellite which can be achieved by finding the matrix of rotation response i.e. DCM [16].

The rotation matrix is the deviation between the body and the inertial reference frame, which is expressed as: [S_i= inertial frame with respect to Sun; M_i= inertial frame with respect to Magnetic Field; S_b= body frame with respect to Sun; M_b= body frame with respect to Magnetic Field]. Where, $S_b=R*S_i$ and $M_b=R*M_i$; Here, R= Rotation matrix.

The Rotation matrix R can be found using the Triad Algorithm, which involves the use of the inertial frame values obtained from the sensor and the body values.

Suppose, the sensor readings are in the form of two vectors (a & b) such as shown below: $a=a_b+a_i$(inertial and body components); $b=b_b+b_i$(inertial and body components) Where, $a_b=[a_{bx}\ a_{by}\ a_{bz}]^T$ & $a_i=[a_{ix}\ a_{iy}\ a_{iz}]^T$; $b_b=[b_{bx}\ b_{by}\ b_{bz}]^T$& $b_i=[b_{ix}\ b_{iy}\ b_{iz}]^T$

Design Steps

Step: 1 - Inertial/Satellite Body frame (3X1 Matrix)

$t_{1b}=a_b$ (can be expressed as a 3x1 matrix)

$t_{1i}=a_i$ (can be expressed as a 3x1 matrix)

Step: 2 – Conversion from one frame to another frame

$t_{2b}=(a_b x b_b)/Ia_b x b_b I$ (can be expressed as a 3x1 matrix)

$t_{2i}=(a_i x b_i)/Ia_i x b_i I$ (can be expressed as a 3x1 matrix)

Step: 3 – Vector representation in one frame to others

$t_{3b}=t_{1b} x t_{2b}$ (can be expressed as a 3x1 matrix)

$t_{3i}=t_{1i} x t_{2i}$ (can be expressed as a 3x1 matrix)

Step: 4 – Rotation Matrix

Two rotation matrices are constructed with the help of above [3×1] matrices as the columns of the rotation matrices[14].

$R_1 = [t_{1b}\ t_{2b}\ t_{3b}]$ & $R_2=[t_{1i}\ t_{2i}\ t_{3i}]$

Now, the desired rotation matrix is found by multiplying the R_1 matrix by the transpose of the R_2 matrix, as below:

$$R=R_1R_2^T \quad \& \quad R=[T_{1B}\ T_{2B}\ T_{3B}][T_{1I}\ T_{2I}\ T_{3I}]^T \tag{11}$$

This is the final equation (11) by which the rotation matrix is determined which internally? means that the attitude of the satellite is determined.

C. Estimating Attitude from the filter (Multidimensional Kalman filter)

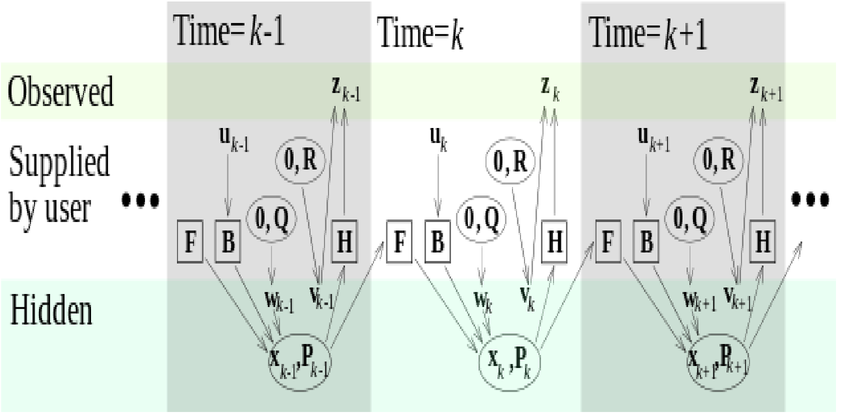

Fig. 9 Kalman filter's prediction step in various time stages

The Kalman filter model assumes the true state at time k; it? is derived from the state at (k-1) according to this equation [17]:

$$X_k = \mathbf{A}X_{k-1} + \mathbf{B}\mu_k + \boldsymbol{\omega}_k \tag{12}$$

here:
A is the adaptive matrix for previous state X_{k-1}; **B** is the adaptive matrix for control matrix μ_k.
$\boldsymbol{\omega}_k$ is the noise in the process with covariance factor $\mathbf{Q_k}$ $\boldsymbol{\omega}_k \sim \mathbf{N(0,Q_k)}$
At time k, an observation or measurement Z_k of the true state is made according to: $\mathbf{Z_k = H_k X_k + V_k}$
here; $\mathbf{H_k}$ is the adaptive matrix which changes true state into observed state. $\mathbf{V_k}$ is the observation noise.

4. KALMAN FILTER ALGORITHMS

The extended Kalman filter (EKF) is a compressed variation of Kalman channel that changes about a guess of the current mean and covariance. An extended Kalman filter normally combines both noisy data and dynamic-model predictions. Basically, in repeated dribble structure it gives an estimated largest state approximate $\hat{x}$ of an adherent state-space exemplary. But normally, the filter slender the abridged active model about the end estimated state vector through Jacobian matrix.

Extended Kalman filter is a rehashed advancement attempting to inexact the precise condition of a noticeable abbreviated framework where just a couple of loud calculations are appropriate. It is also the most extensively used algorithm combined in both theory and absolute form by spacecraft association.

A. EKF Algorithm

The genuine nonlinear framework elements and estimation portrayed by consistent model [17]:

$$\mathbf{X^{\cdot} = f(X) + w} \qquad \& \qquad \mathbf{Y = h(X) + v} \tag{13}$$

From the constant model access condition (1) and (2) is changed into the discrete-time model is

$$\mathbf{X_k = f(X_{k-1}) + w_{k-1} \quad \& \quad Y_k = h(x_k) + v_k} \tag{14}$$

Here the subscript of the factors indicates the time step, while w_{k-1} and v_k are limited accepted as Gaussian disseminated noises with mean zero and covariance Rw and Rv individually to such an extent that $w_{k-1} \sim N(0,R_w)$ and $v_k \sim N(0,R_v)$. At that point, the assessed state is got through the accompanying advance:

Step 1: Set the underlying state gauge $\hat{x}_0 = \hat{x}_{0|0}$ and variance $P_0 = P_{0|0}$

Step 2: Repeat

Prediction state (prior estimate)

Jacobian of $f(x_{k-1})$: $F_{k-1} = \frac{\partial f}{\partial x}\Big|x_{k-1|k-1}$

Predicted state estimate: $\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1})$

Predicted covariance estimate

$$P_{k|k-1} = F_{k-1} P_{k-1|k-1} F^T_{k-1} + R_w \tag{15}$$

Update state (Posterier estimate)

Jacobian of $h(x_k)$: $H_k = \frac{\partial h}{\partial x}\Big| \hat{x}_{k|k-1}$

Kalman gain: $K_k = P_{k|k-1} H^T_k [H_k P_{k|k-1} H^T_k + R_v]^{-1}$

Updated state estimate: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k [y_k - h(\hat{x}_{k|k-1})]$

Updated covariance estimate:

$$P_{k|k} = [I - K_k H_k] P_{k|k-1} \tag{16}$$

B. Unscented Kalman filter

The unscented Kalman filter (UKF) is the nonlinear variant of Kalman filter which uses a deterministic sampling approach in estimating the output. When the functions i.e; f and h, are highly nonlinear, then the EKF can give poor performance therefore we use UKF for a better output result.

UKF utilizes an alternate way to pick an insignificant arrangement of test focuses around the mean. These sample points are called sigma points [17]. These sigma points help to calculate the new mean and the covariance. In additon, this technique expels the requirment of Jacobians's calculation. This makes this technique less complex. Like EKF, UKF proceeds in the same two steps, except the method of selection of the sigma points.

UKF Algorithm

Consider the following nonlinear system, depicted by the observations and noises:

$$\mathbf{X_k = f(x_{k-1}) + w_{k-1} \qquad \& \qquad Y_k = h(x_k) + v_k} \tag{17}$$

The initial state $X_\circ$ is a random vector with known mean $\mu = E[X_\circ]$ and covariance as $P_\circ = E[(X_\circ - \mu_\circ)((X_\circ - \mu_\circ)^T]$. The unscented transformation scheme is applied to augmented state:

$$\mathbf{X_k^{aug} = [x_k^T \quad w_{k-1}^T \quad v_k^T]^T} \tag{18}$$

Let Xk-1be a set of 2n+1 sigma points and their associated loads:

$$X_{k-1} = (X_{k-1}^j, W^j) \qquad ; j=0\ldots2n$$

Consider the following selection of sigma points:

$X_{k-1}^0 = X_{k-1}; -1 < W^0 < 1;$ $\qquad X_{k-1}^i = X_{k-1}^a + \left(\sqrt{\frac{n}{1-W0}\, Pk - 1}\right)$

for all i=1, 2, …… n; $W^j = \frac{1-W0}{2n}$ $\qquad$ for all j= 1,2,…..n

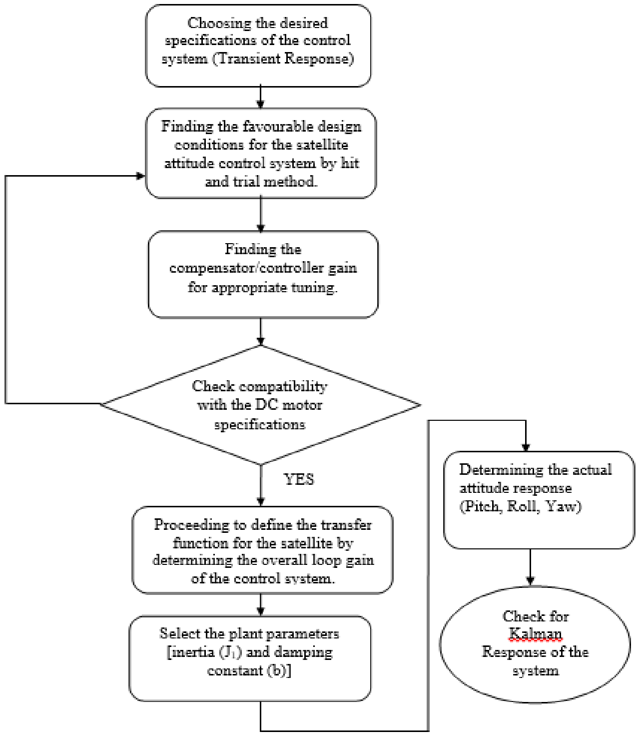

Fig. 10 Flow chart for satellite attitude determination

Loads must follow this: $\sum_{j=0}^{2n} Wj = 1$. Here, W^0 control the position of the sigma points, $W^0 >= 0$ points tends to move further from the starting point, $W^0 <=$ points to be closer to the starting point. Each sigma point is propagated through nonlinear model: $X_k^{f,j} = f(x_{k-1}^{j})$

The new values of mean and covariance values: [17]

$$\left.\begin{aligned} \mathbf{Xkf} &= \sum_{j=0}^{2n} \boldsymbol{Wj}\, \mathbf{Xkf,j} \\ \mathbf{Pkf} &= \sum_{j=0}^{2n} \boldsymbol{wj}\,(\mathbf{Xkf,j} - \mathbf{Xkf})(\mathbf{Xkf,j} - \mathbf{Xkf})^{\mathbf{T}} + \mathbf{Qk1} \end{aligned}\right\} \tag{19}$$

The Methodology includes various steps, which are as follows:

1. The implementation of the various algorithms for determining the satellite orientation response and estimating the response is done and compared based on performance parameters. For example, the TRAID algorithm and the concepts of sensors have been used for attitude determination of the satellite and concepts like Kalman filtering are used for estimating the attitude of the satellite by statistical approach.
2. To perform a comparative analysis between the algorithms in terms of parameters like computation time and accuracy, a thorough research has been done and then we worked upon the results.
3. The nonlinear approach contains the best algorithms to estimate the attitude of the satellite and hence this phase is deeply studied and carried out in the project for sensor integration with the help of a controller (OBC). The use of nonlinear filters like UKF, EKF is taken into consideration because of their cost effectiveness factor [17].

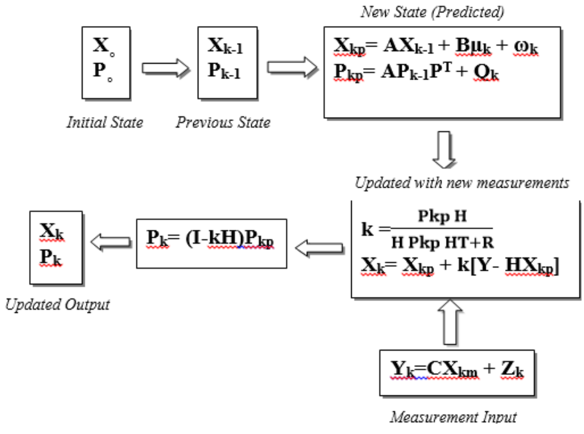

Fig. 11 Algorithms Flow chart for UKF

Here: **X**: State Matrix; **P**: Process Covariance Matrix (Error in estimate process); K: Kalman Gain I: Identity Matrix; ω: Predicted State Noise Matrix; R: Sensor Noise Covariance Matrix; μ: Control variable Matrix; Q: Process Noise covariance Matrix [17].

5. SATELLITE ATTITUDE CONTROL SYSTEM (ACS)

The ACS is a control system having closed loop which is used for the attitude determination of the satellite with the help of an OBC (On Board Computer) which is the controller configuration used to obtain a controlled output to any given input. The controller is used to make the plant stable by eliminating the disturbance torque and making the output converge in minimal time. The Satellite attitude control system includes:

Input Block -> Plant -> OBC -> Feedback -> Sensor -> -Scope (Output)

Principle of operation

The input is fed as a step input to determine the change in the controlled output with respect to time. This is done by introducing a DC motor to the system, which helps in providing torque value to the plant (satellite system). The satellite system combined with the disturbance torque (due to the perturbations in space) is fed to the scope to get the determined output. The tuning of the system is done through a PID controller, which is needed to make the response of the system stable and meeting the design requirements.

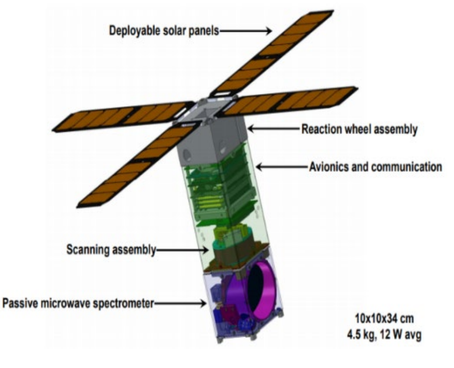

Fig. 12 MicroMAS1 (Satellite Model) [17]

Design Requirements such as: Reduces the
Rise time (< 1 second)
Settling time (< 10 seconds)
Percentage overshoot (< 10%)
Steady state error

The Proportional Gain (K_p), Integral gain (K_i) and Derivative gain(K_d) values are calculated through Ziegler-Nichols method and then the values of P, I and D are put into the PID tuner and the system is made stable with the desired design requirements.

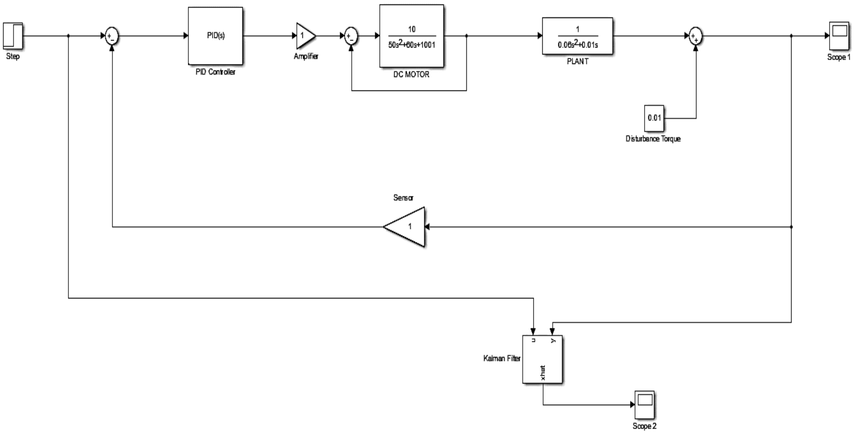

Fig. 13 Satellite Attitude control system Block Diagram

The other phenomena which come into the picture are the system noise, disturbances, noise error, etc; which are dealt with the Kalman Block which takes the input in the form of a state space system. The major concern is that the system along with the other components should be continuous in nature to get more accurate understanding of the behavior of the output plots with respect to time.

The transfer function of the satellite is shown below:

Torque equations,

$$T=I*W \tag{20}$$

where, I= satellite inertia; T= input torque provided to the satellite; w= satellite's angular velocity

Now, the transfer function is the ratio of response at output end to the response at input end of the system i.e., w(s)/T(s)=1/I(S), where, I(s) is of the form of J_1s^2+B & J_1 is the satellite inertia and B is satellite damping [16].

Therefore, the transfer function becomes

$$\mathbf{w(s)/T(s)=1/(Js^2+Bs)} \tag{21}$$

The moment of inertia of the MicroMAS1 satellite is calculated as 0.06 kg.m^2 as the dimensions of the satellite are 10cm × 10cm × 34cm and the satellite mass is approximately 4kg. The damping constant optimized results is assumed to be 0.01Nms. The system function of the DC motor is defined as

$$\mathbf{Tf(s)=K/[(Js+B)*(R_a+L_a*s)+K^2]} \tag{22}$$

The DC motor used to provide torque to the plant (satellite) is a brushless DC motor, which has the following design specifications:

K=DC motor constant = 0.01 Nm/s;

R_a=DC motor resistance= 1 Ohms;

L_a=DC motor inductance= 0.5 H;

J=DC motor inertia = 0.01 kg.m^2;

b=DC motor damping = 0.1Nms

6. RESULTS AND DISCUSSIONS

Table. 1 Ephemeris Data for particular Satellite [17]

S. no	Satellite name	Orbit	Physical dimensions (meter)	Mass (kg)	Altitude (km)	Mission life	Average Power (watt)
1	MicroMAS1	Low earth orbit (LEO)	10cm × 10cm × 34cm	4.25	500	1 years	10

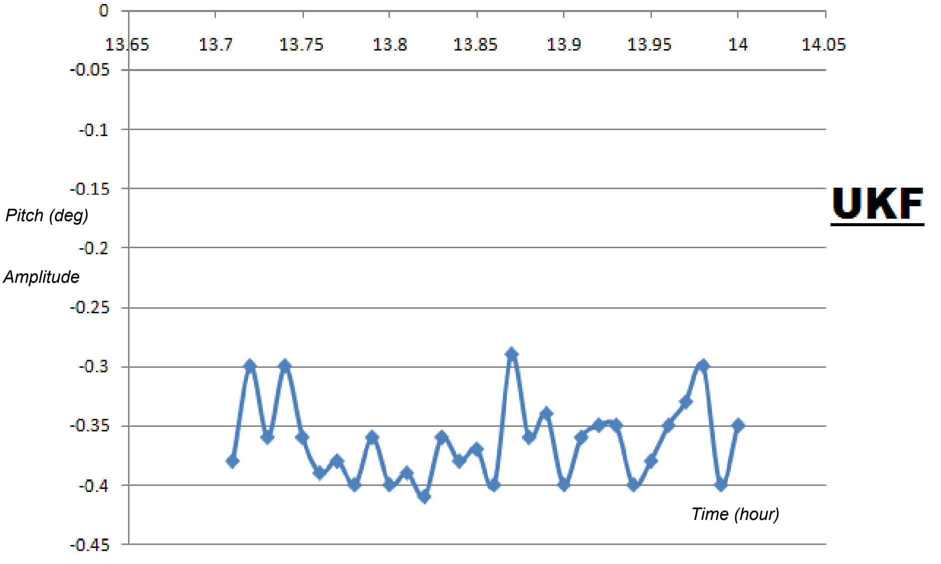

Fig.14 Pitch Response (Time Vs Amplitude)

The pitch response displays the change in pitch (in deg) amplitude with respect to time (in hour) of the chosen satellite MicroMAS1.

This is the simulation result with the help of UKF algorithm, which is used for estimating the satellite's attitude by removing the noise errors.

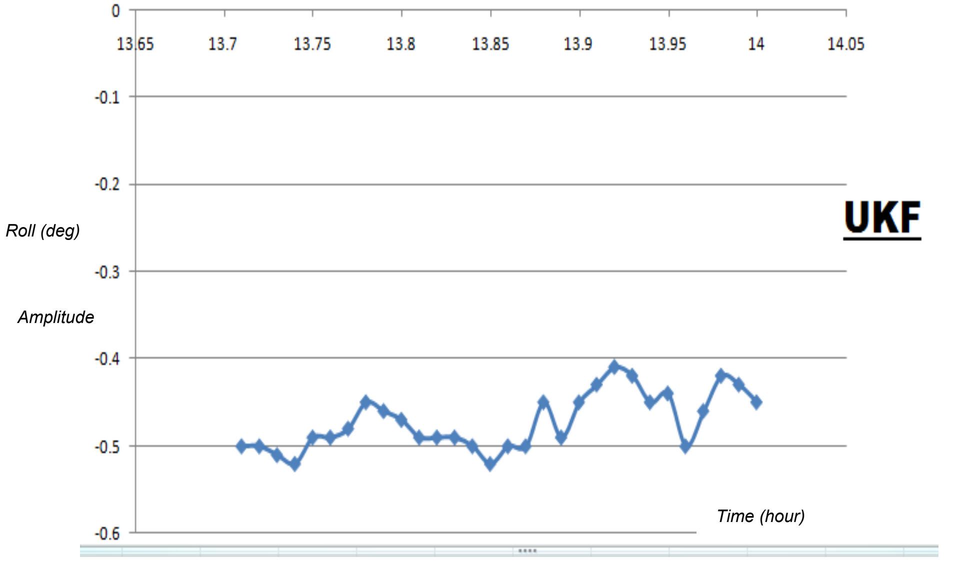

Fig. 15 Roll Response (Time Vs Amplitude)

The roll response displays the change in roll (in deg) amplitude with respect to time (in hour) of the chosen satellite MicroMAS1.

This is the simulation result with the help of UKF algorithm which is used for estimating the satellite's attitude by removing the noise errors.

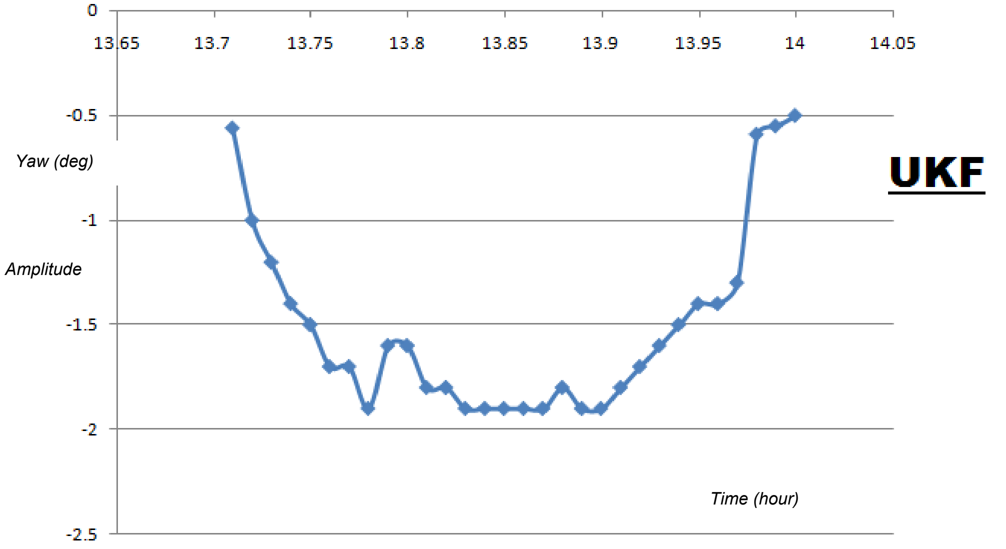

Fig. 16 Yaw Response (Time Vs Amplitude)

The yaw response displays the change in yaw (in deg) amplitude with respect to time (in hour) of the chosen satellite MicroMAS1.

This is the simulation result with the help of UKF algorithm which is used for estimating the satellite's attitude by removing the noise errors.

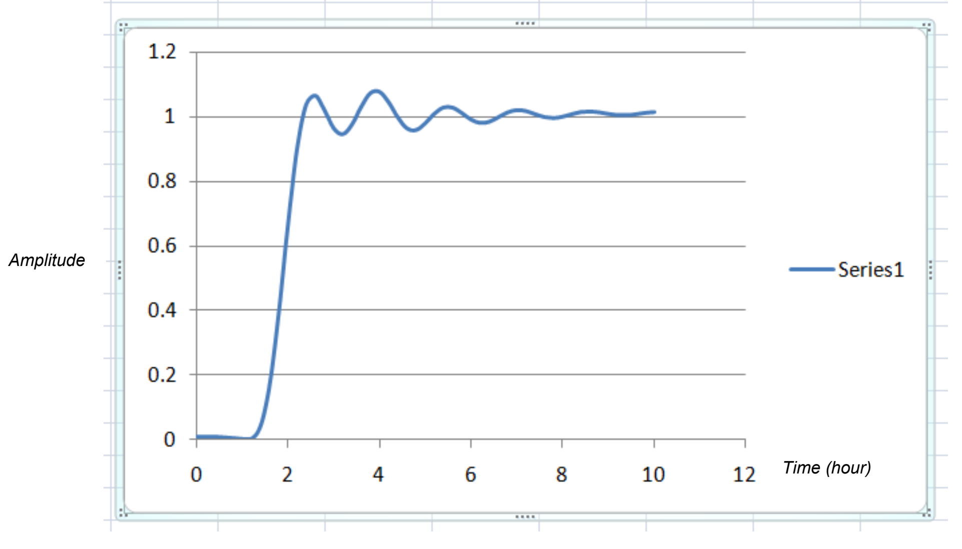

Fig. 17 Output Response of ADCS (Time Vs Amplitude)

This is the output response of the SADCS which is a stable output response of a step input meeting all the desired design specifications of rise time < 1 second, settling time < 10 seconds and a maximum overshoot < 10%.

These design requirements achieved helps in controlling the satellite's attitude more conveniently and with fewer errors.

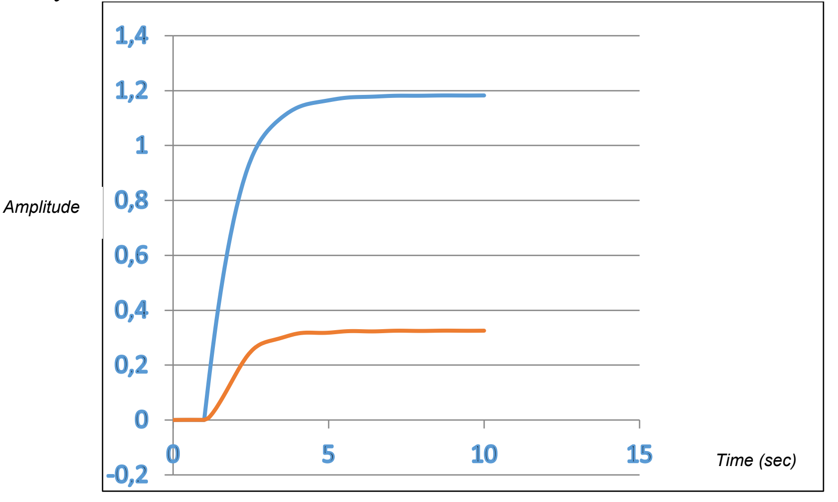

Fig. 18 Kalman O/P Response (Time Vs Amplitude)

This is the output response for the Kalman Filter which takes one input as the initial step and the other input as plant's output and the disturbance torque. The output of the Kalman Filter is a step response with lesser amplitude than the initial step input due to the noise error and disturbance error. This is a stable output response. Table 2. shows the comparison between the attitudes of the satellite for the transient response of the ACS without the controller and with the PID controller configuration.

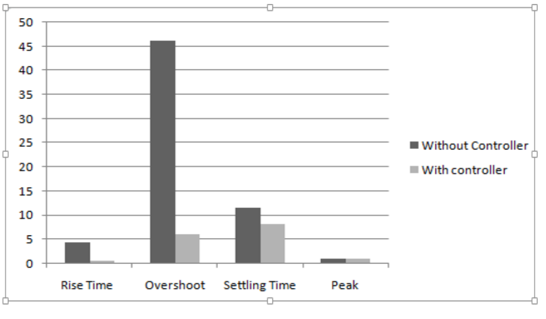

Fig. 19 Satellite Transient Response

Table 2. MicroMAS1 Satellite Attitude Response

Specifications	Satellite Attitude Dynamics	
	Without Filter	**With Filter**
Rise Time(Sec)	4.44	0.692
Overshoot (%)	11.6	8.17
Settling Time (Sec)	46.3	6.17
Peak Time(Sec)	1.12	1.08

The satellite transient response indicates that, by introducing a controller configuration, the system response to any action is fast and the settlement of the system after completion of the response to any action is also reduced with a reduced peak and overshoot response.

These design characteristics make the system more reliable and accurate and thus a stable attitude response can be achieved.

6. CONCLUSIONS

The results obtained from the simulation of various satellite attitude determination algorithms provide us a clear view about the approach we need to apply in our analysis. It can be seen that the nonlinear observation is the most appropriate in terms of all the performance comparison parameters like accuracy, reduced errors, etc. Moreover, the calculations done for the attitude determination of the chosen satellite are validated through software which provides the various plots for the velocity and position in body frames as well as inertial frames of reference. It can be concluded from the various plots that a filtering technique must be implemented which can reduce the error and divergence in the various plots. The Kalman Filtering technique is used to control the output as it will filter out the errors and provide the removal of noise components from the step input provided. The OBC i.e. the controller configuration, which will help us provides a desired output as the attitude of a satellite. This controller configuration is based on Kalman filtering (UKF, EKF, etc.).

REFERENCES

[1] M. Ben-Ari, *A tutorial on Euler angles and quaternions*, Version 2.0.1, 2014-17.

[2] J. Diebel, *Representing Attitude: Euler Angles, unit quaternions and rotation vectors*, Stanford University, 2006.

[3] S. A. Rawashdeh, *Passive Attitude Stabilization for Small Satellites*, University of Kentucky, Master's Theses, 624, 2010.

[4] M. A. Si Mohammed, H. Boussadia, A. Bellar and A. Adnane, Performance comparison of attitude determination, attitude estimation and nonlinear observer algorithms, *Journal of Physics: Conference Series*, Volume **783**, 13th European Workshop on Advanced Control and Diagnosis (ACD 2016) 17–18 November 2016, Lille, France.

[5] C. Hall, *AOE Spacecraft Dynamics and Control, Chapter 4 - Attitude Determination*, March 18, 2003.

[6] * * * Staff of Princeton Satellite Systems, *Attitude and Orbital Control using the Spacecraft Control,* Toolbox v4.6, December 2000.

[7] A. Sofayli and E. M. Jafarov, *Integral Sliding Mode Control of small satellite attitude motion by purely Magnetic Actuation,* Preprints of the 19th World CongressThe International Federation of Automatic ControlCape Town, South Africa, August 24-29, 2014.

[8] A. Zribi and M. Chtourou, M. Djemel, *A New PID Controller Design for Non linear Processes*, 2017.

[9] M. Shahrokhi and A. Zomorrodi, *Comparison of PID Controller Tuning Methods,* 2017.

[10] M. B. Rhudy and R. A. Saluero, *A Kalman Filtering tutorial for undergraduate students,* 2017.

[11] R. V. Garcia and M. C. F. P. S Zanardi, *Unscented Kalman filter applied to the spacecraft attitude estimation with Euler angles,* 2016.

[12] S. Mohan and N. Naik, *Introduction to Kalman filter and tuning its Statistics for near Optimal Estimates and Cramer Rao bound*, IIT Kanpur, Feb 2015.

[13] H. Hamzah and S.Yaacob, Nonlinear observers for attitude estimation in gyro less spacecraft via EKF algorithm, *International Journal of scientific and Research Publications*, Oct 10, 2014.

[14] A. Odry, I. Kecskes and P. Odry, *A novel Fuzzy- Adaptive EKF for real time Attitude Estimation of Mobile Robots,* DOI: 10.3390/s20030803, Feb 2020.

[15] T. Nguyen, K. Cahoy and A. Marinan, *Attitude determination for small satellite with infrared earth horizon sensor*, https://doi.org/10.2514/1.A34010, 2018.

[16] S. Chouraqui, M. Benyettou, A. Si Mohammed, *Sensor Vector Modelling for small satellite attitude determination*, 2016.

[17] W. J. Blackwell and G. Allen, C. Galbraith, T. Hancock, *Nano satellite for earth environmental monitoring the MicroMAS Project*, DOI: 10.1109/MicroRad.2012.6185263, 2012.