Orbit Design and Trajectory Analysis of satellite in Low Earth Orbit

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Abstract: *An Attitude control system for a satellite in low earth orbit is designed in this project. As a reference, CARTOSAT-2 is selected to design the AOCS. CARTOSAT-2 is located in LEO so it suits the needs of the projects. It is mainly used for mapping of urban, rural areas and wetlands in India. To design a basic AOCS, a DC motor based model is considered. Transfer functions of DC motor and satellite dynamics model are calculated using Laplace transformation. Stability of the system is checked by plotting poles and zeroes of the systems. PID controller is used to improve the overall stability of the system by decreasing the errors in the output of the system. The systems are subjected to sine and step inputs and responses are plotted in the form of graphs. The plots are studied using SciLab as the tool to design the block diagram and the control system for the AOCS. Scilab along with FlightGear is used to visualize the response of the system.*

Key Words: *GMAT, STK, LEO, GEO, UTC, RAAN, SRP, GUI*

1. INTRODUCTION

Launching a satellite into orbit for communication was a major breakthrough. The satellites in Low Earth Orbit play a major role in applications like Remote Sensing, communication and imaging purposes. It is the closest orbit from our planet Earth to put satellites into. However, with the close proximity to Earth also give rise to certain major issues, which if not handled can be hazardous. Satellites have a number of sub-systems which tackle different problems. The Attitude and Orbit Control System is a sub-system which takes care of the satellite to determine the attitude and orbit and maintaining it to meet our requirements. The Attitude and Orbit Control System (AOCS) as the name suggests comprises two major parts: Attitude Control System and Orbit Control System [1]. The way a satellite is oriented in its orbit is known as the Attitude of a Satellite. The attitude of the satellite is interpreted with Pitch, Roll and Yaw. Appropriate attitude is essential for communication, the antennas in the satellite are needed to face the portion of Earth which is our base of communication. Communication with the satellite is necessary for the satellite to function normally as that way you can monitor the satellite and then give it certain commands to achieve the attitude required. If the perturbed

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force acts along any of the three coordinate axes, then the satellite will be rotated slightly along that axis.

We need to achieve zero error attitude which means that the antenna on the satellite is pointing towards the desired location on Earth. The sensors help us in identifying the error in attitude.

This error is then countered by the use of actuators, which provide thrust or torque to the satellite, which an opposite reaction is resulted and the desired orientation is achieved. Firstly, with the help of sensors the errors will be calculated and then the actuators will be initialized to obtain the appropriate orientation so as the antenna on the satellite faces the required location on Earth at all times [2].

We are dealing with satellites in the low orbit of the Earth, in terms of launching into orbit it is the most favorable as it is cheaper, requires less power and is faster. The problem arises when it is necessary to keep the satellite in its orbit for a long time. The LEO satellites are mainly subjected to perturbations due to non- sphericity of Earth and to atmospheric drag due to its close proximity to Earth. We need to make sure that these perturbations are not dominant, if so happens the satellite will lose its altitude and fall to Earth, which is the exact scenario we need to avoid [3].

The purpose of our satellite is to follow a certain trajectory and remain in orbit but in reality, it will drift from its original trajectory and orbit. The orbit control system is a part of AOCS which will help us in monitoring the satellite's orbit and see if it deviates from its original orbit. A satellite must have enough speed to retrace its path in order to stay in its orbit around Earth. Also a correct gravitational balance between the Earth and satellite is required for it to stay in its orbit.

We are dealing with Low Earth Orbit Satellites, which highly subjected to atmospheric drag that is why a greater velocity is required to keep the satellite in orbit and keep it from falling to Earth [3].

Keplerian Elements

Six numbers are required to characterize a satellite orbit. This arrangement of six numbers is known as the satellite orbital components, or in some cases "Keplerian" components (after Johann Kepler [1571-1630]), or just components.

These numbers characterize an oval, arrange it about the earth, and spot the satellite on the circle at a specific time. In the Keplerian model, satellites orbit in a circle of consistent shape and direction [4].

1. Eccentricity (*e*)

The state of the ellipse, depicting the amount it is lengthened contrasted with a circle.

2. Semi major axis (*a*)

The entirety of the periapsis and apoapsis separations isolated by two. For roundabout circles, the semi major pivot is the separation between the focuses of the bodies, not the separation of the bodies from the focal point of mass.

3. Inclination (*i*)

Vertical tilt of the ellipse as for the reference plane, estimated at the climbing hub. The tilting point is estimated opposite to the line of the crossing point between the orbital plane and the reference plane.

Any three points on an ellipse will characterize the ellipse orbital plane. The plane and the ellipse are both two-dimensional items characterized in three-dimensional space.

4. Longitude of the ascending node (Ω)

On a level plane arranges the climbing hub of the ellipse concerning the reference casing's vernal point (symbolized by ♈). This is estimated in the reference plane and is appeared as the green point Ω in the chart.

5. Argument of periapsis (ω)

Characterizes the direction of the ellipse in the orbital plane, as a point estimated from the climbing hub to the periapsis

6. True anomaly (ν, θ, or f)

At epoch (M0) characterizes the situation of the circling body along the ellipse at a particular time (the “epoch”).

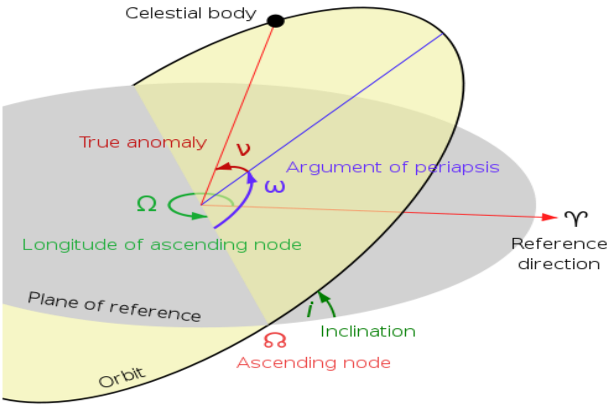

Fig 1: Keplerian elements

CARTOSAT-2

Cartosat-2 Series Satellite is the primary satellite carried by PSLV-C40 into the low earth orbit on 10 January 2010. This remote sensing satellite is similar in configuration to earlier satellites in the series and is intended to augment data services to the users [5].

For this project, CARTOSAT-2 will be used as reference satellite.

Table 1: Specifications of CARTOSAT-2

CARTOSAT-2 Satellite	**Specifications**
CAT No.	37838
DRAG	0.00001006
Inclination	19.7947
Right Ascension	62.7084
Eccentricity	0.0060638
Argument of perigee	17.8079
Mean Anomaly	342.4581
Semi Major Axis	7201.23183
Height above Equator	823.09
Period (in second)	6081.646441
Epoch Year	2011

The imagery sent by satellite is used for cartographic applications, urban and rural applications, coastal land use and regulation, utility management like road network monitoring, water distribution, creation of land use maps, change detection to bring out geographical and manmade features and various other Land Information System (LIS) as well as Geographical Information System (GIS) applications [6].

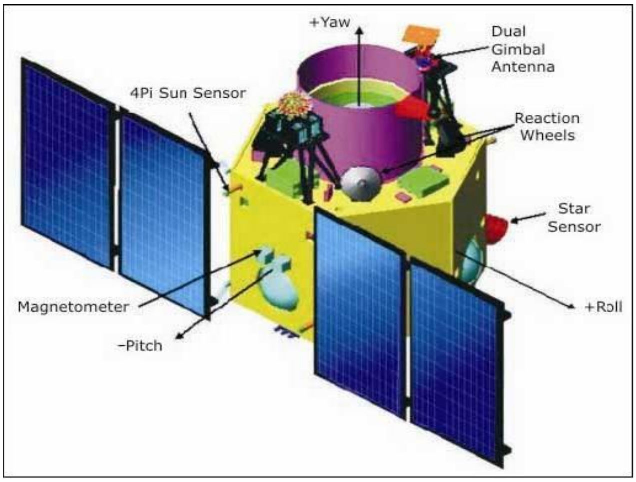

Fig. 2: CARTOSAT-2

2. MATHEMATICAL MODELLING

For designing the attitude control system of the system, mathematical modeling is done to calculate drag, transfer functions for the AOCS.

Atmospheric Drag

When a body moves through a fluid like gas or any liquid it experiences a resistance known as drag. A Satellite or a space craft experiences this drag when it moves through the different layers of atmosphere, greatest being at launch and reentry of the vehicle [7]. The effect of atmospheric drag on the space vehicle can be such that it can spiral back into the atmosphere with finally disintegrating or burning up. The deterioration of a spacecraft's orbit due to drag is called decay. The atmospheric drag can be calculated using the following formula;

$$D = (\rho \vartheta^2 CDA)/2$$

where, for Cartosat-2:
D = Drag Force
ρ = Density of fluid; 3.13* 10-8 Kg/m^3
v = velocity of the vehicle; 7.32 Km/s^2
CD = Drag Coefficient; 2
A = Cross sectional area of the vehicle; 2.4 * 2.5
Calculation:

$$D = 1/\,2 * 3.13 * 10 - 8 * 7.322 * 2 * 2.4 * 2.5$$

$$D = 0.00001006\ N$$

Attitude and Orbit control system

Due to unbalanced forces of the nature, the satellite in its predetermined orbit and altitude is disturbed from its actual path. This can be because of perturbations or any other internal factor of our designed systems. So to rectify this, and AOCS should be designed to account for the changes and bring the satellite to its original path. To design the AOCS, a motor, satellite and PID transfer functions are required [8]. The respective transfer functions are modelled as follows:

Transfer Function

DC Motor
The armature controlled DC Motor circuit is resolved using Kirchhoff's Voltage Law, then solved for induced EMF (E), Torque (T) and dynamic equation of the circuit.

We take the Laplace Transform of the four equations which are all in frequency domain. Using the four equations, a block diagram is prepared [9].

We can find the Transfer function of the system by Block diagram reduction method. The governing equation of a DC Motor is,

$$\mathrm{Tm}(s) = \mathrm{TL}(s) + \mathrm{Td}(s) \tag{1}$$

On deducing the above equation we get,

$$\mathrm{Tm}(s) = s.\ \theta(s)[Js + B] \tag{2}$$

$$\mathrm{Tm}(s) = \frac{V_a(s) - K_b.s\theta(s)}{R_a + L_a(s)}.\ K \tag{3}$$

where, $V_a(s) = V_o(s).s$ and $K_b = K^2$
Now equation (2) equals equation (3),

$$\theta(s)\{s(Js + b).(R_a + sL_a)\} = sV_o(s) - K^2 s\theta(s) \tag{4}$$

The final Transfer function,

$$\frac{\theta(s)}{V_o(s)} = \frac{K}{[(Js+b).(R_a + L_a(s)] + K^2} \tag{5}$$

where,
θ is the output deflection in motor
Kb and K are the constants
Ra is the resistance in the motor
Va is the voltage input to the motor
La is the Inductance in the motor
J is the moment of Inertia
b is the damping factor

A DC motor controls the space craft attitude control system and that orients the space craft to its desired variable, in our case, the Yaw angle.

We preferably take a closed loop, feedback control system, so that it stays stable and does not diverge. Using Simulink and flight gear we have tried to explain the simulations performed by us.

MATLAB helps to perform a stability analysis. We can find poles of a system and depending on their sign convention we can decide whether the system is stable or not. In this case the poles for the used DC Motor have negative real part which tells us that the system is stable [10].

Likewise if we look at the poles of the transfer function of satellite, we find a value equal to Zero which depicts that the system is marginally stable and has the potential to become completely unstable after certain time.

Satellite Transfer function

The transfer function for a satellite can be found using a simple Laplace transform and rearranging of equation as follows:

$$J\left(\frac{d^2}{dt^2}\,\theta(t)\right)+B\left(\frac{d}{dt}\theta(t)\right)=Tm \quad Governing\ Equation$$

$$J\ddot{\theta}+\ B\dot{\theta}\ =\ Tm \tag{6}$$

$$Js^2\,\theta(s)+Bs\,\theta(s)=\ Tm \tag{7}$$

$$\frac{\theta(s)}{Tm}={}^{1}\!/_{(Bs+\,Js^2)} \tag{8}$$

Transfer Function of satellite

Next, we can also determine, as to whether the system lies in the negative or the positive plane by plotting a root locus for the system.

Desirably the locus must lie on the negative side of the plane. For the DC Motor in this case the root locus lies on the negative side of the plane where as for the given satellite, we have one point lying on the (0,0) point, depicting that the system is marginally stable but we must converge towards negative side as much as possible.

Also, we can do a Gain Phase margin called the bode plot and check for the behavior of our simulation at different frequencies.

No system is stable at all frequencies. In case of given satellite, both the gain and phase margin are decreasing and depicts the instability of a system [11].

We can also perform a feedback loop and analyze a step response which is nothing but a unit input. In this case, 0.01 has used as the unit input. In case of DC motor, we see certain oscillation as until this point the controller has not been introduced into the system.

It is desirable that the peak overshoot be minimized Looking at the step response of the satellite, we observe very few oscillations but at the same time we observe a very slow converging time.

The CARTOSAT-2 being in LEO is subjected to perturbation due to Aerodynamic drag. Attitude Control is used to eliminate this unwanted drift from the desired orbit. In our case we use a DC Motor to orient the satellite in the desired orbit.

PID Controller

A PID controller, also known as Proportional Integral Derivative is a very well-known and simplest controller which keeps memory of the present, past and future error. It is a form of feedback controller. The plant is what we want to control, we will give an actuating signal to it and a controlled variable will be the output.

This will be then compared to the commanded variable (desired output) by sending a feedback signal, which will result to an error of the system. A zero error system is what we desire [12].

However, if that is not the case, a controller will be used to convert the error term into suitable actuated command, which with time will be driven to give us zero error. This is known as the Proportional.

The Integral keeps record of the past error. Moreover, the Derivative, keeps the rate of change of the system, and predicts the future error.

We can control the contribution of these three into our system, which is what tuning the controller means.

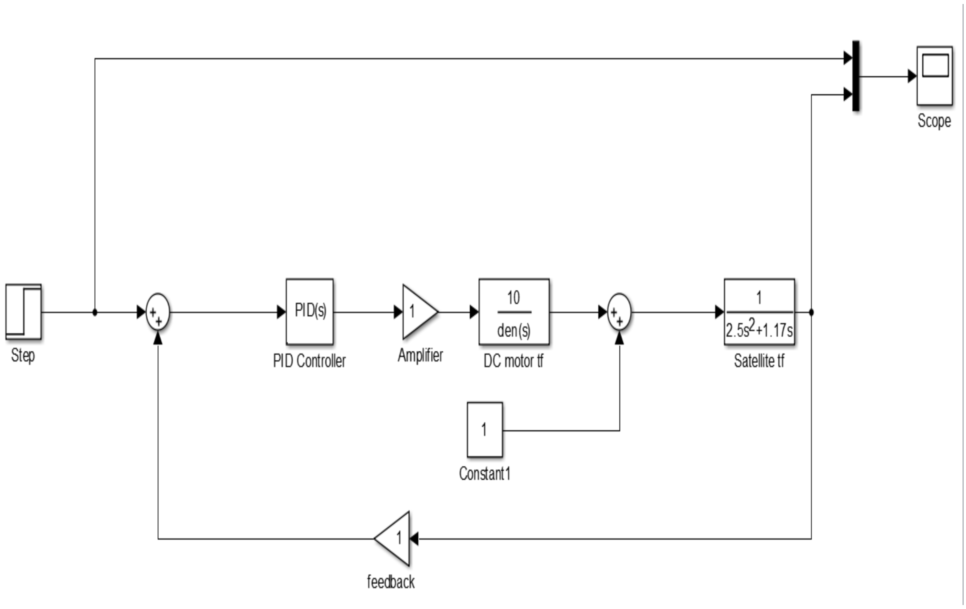

Fig 3: Block diagram of Attitude Control system

Feedback Control Design

Feedback plays an important role in our system as it produces an error by comparing the Controlled variable and Commanded variable. We wish to get zero error. However, this is not possible in a real time system. Therefore, the error will be processed by the controller and give the desired input to the plant to get desired output, and consecutively yielding to zero error [11]. Feedback control design is essential, as our system is prone to perturbations due to aerodynamic drag.

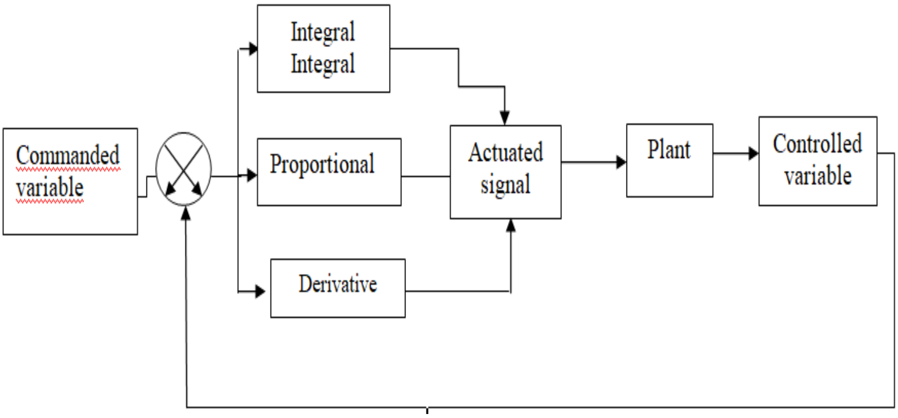

Fig 4: PID controller functionality blocks

Tuning a control system

The linear system we are dealing with is basically controlled using a controller. In our case, we use a Proportional Integral Derivative. Each of the three terms stores memory of the present, past and predicted future errors. We can monitor the contribution of each of the above three to obtain the desired input to the system. Controlling the contribution of PID Controller is a phenomenon known as Tuning a Control System [12].

Laplace Transform

Laplace transform has a critical role to play in space applications. It allows us to evaluate and understand the stability and frequency response of a system. Additionally, it provides a method for easily solving differential equations. Laplace transform coverts continuous time signals into frequency domain.
Mathematically,

$$L[f(t)] = \bar{f}(s) = \int_0^\infty e^{-st} f(t)dt \tag{9}$$

where, 's' is complex number frequency parameter.
$\{s = \sigma + j\omega\}$; σ & $j\omega$ are real numbers.

A major application of Laplace transform in this project was used to form the transfer function of the used DC motor and the satellite from their respective governing equations [10].

Additionally, Laplace transform helped in understanding the stability of the system from the received outputs.

Bode Plot And Root Locus:

There are multiple frequency response methods, one of them is the BODE PLOTS used to visualize the frequency response of the system across the entire spectrum. It basically gives a measure of change in gain and phase in the output, by changing the frequency of the input, hence bode plots are also known as gain phase margin [11].

Bode plots work in certain conditions, like the input frequency must be positive and the roots and poles must lie in the negative half of the plane. In this case, to check the behavior of the simulation at different frequencies, bode plot is generated and it can be observed that for the given satellite, both the gain and phase margin are decreasing and depicts the instability of a system. The design and stability of a system is always of major concern and so, root locus is one such a method to analyze the stability of a given system. In Root locus, poles of the transfer function of a system are plotted in the s-plane with changing values of the unknown parameter. The stability of the system then can be determined by analyzing how the poles are moving in the plane.

It is desirably that the locus be on the negative side of the plane. For the DC Motor in this case the root locus lies on the negative side of the plane where as for the given satellite, we have one point lying on the (0,0) point , depicting that the system is marginally stable but we must converge towards negative side as much as possible [12]. The methodology opted for this project is to choose a satellite in low earth orbit, study its parameters, design a circular orbit, study the parameters, include the perturbations effects, then design an orbit for maximum efficiency.

3. METHODOLOGY

Attitude and Orbit Control system

Due to unbalanced forces of the nature, the satellite in its predetermined orbit and altitude is disturbed from its actual path.
This can be because of perturbations or any other internal factor of our designed systems. Therefore, to rectify this, AOCS should be designed to account for the changes and bring the satellite to its original path.

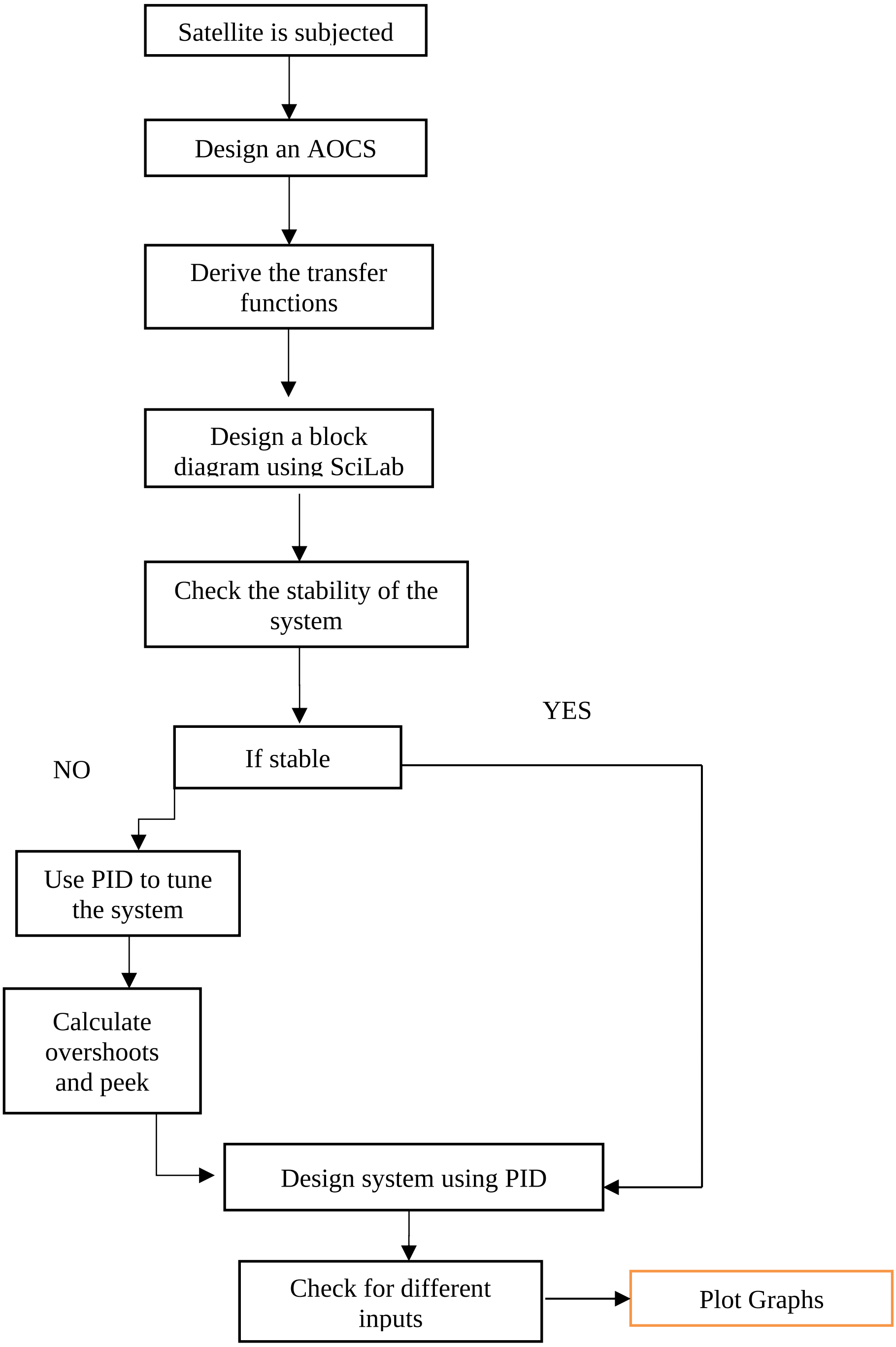

Fig. 5: Flow chart for satellite attitude and orbit control

4. RESULTS AND SIMULATION

Attitude and Orbit control system

For designing the AOCS for given CARTOSAT-2 satellite in LEO, the stability of the system must be checked first. For this purpose the transfer functions of motor and satellite were defined earlier in the mathematical modelling chapter.

To check the stability of the system, the poles and zeroes of the system must be calculated which was done using SciLab function and Xcos model.

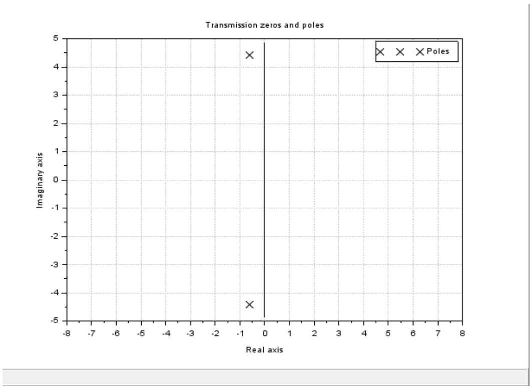

Fig. 6: Poles and zeroes of motor dynamics

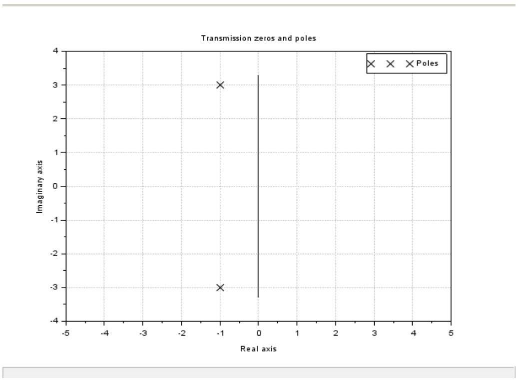

Fig. 7: Poles and zeroes of satellite dynamics

From Figs. 6 and 7, it can be concluded that both systems are stable as, both poles are located on the left side of the plot in the graphs.

This is in accordance with the stability criteria.

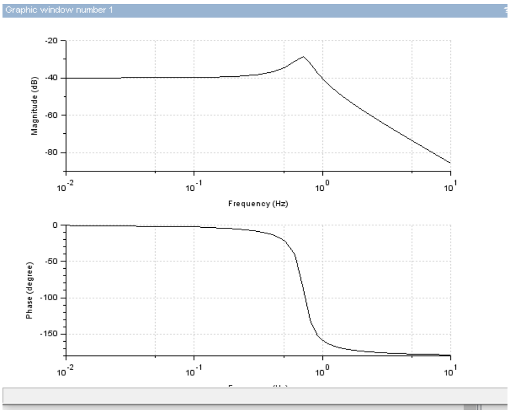

Fig. 8: Bode Plot motor

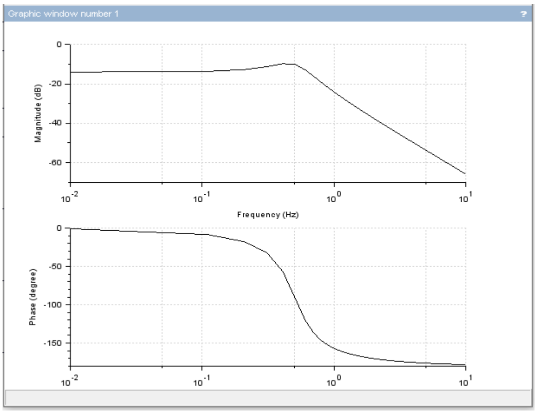

Fig. 9: Bode plot satellite

From Figs. 8 and 9 it can be concluded that, the given system bode plots are found to be stable in very less time which is desirable for the functioning of the system but there can be seen some fluctuations which will be rectified by using a PID controller with control system.

System Check with various inputs

After checking the stablility of the system, the system is put to test under various inputs and outputs are plotted using SciLab and XCos.

The system is checked for the follwing inputs:

1. Sine Input
2. Step Response

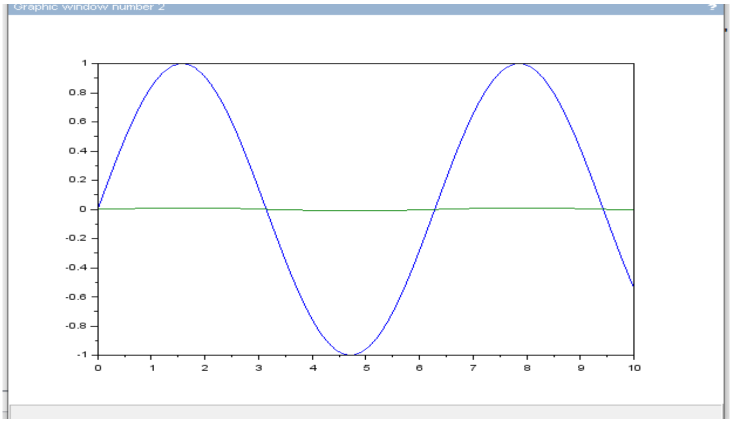

Fig. 10: Sine resposne of motor

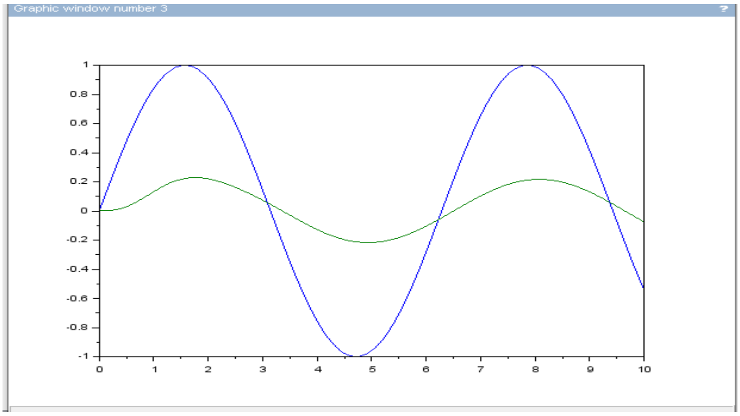

Fig. 11: Sine response of satllite

From Figs. 10 and 11 it can be infered that the system reponds quite well to the the sine put, the system stablizes very fast in case of the motor and there are very few fluctuations but in the case of satellite some fluctuations can be seen which will be rectified by using PID tuning.

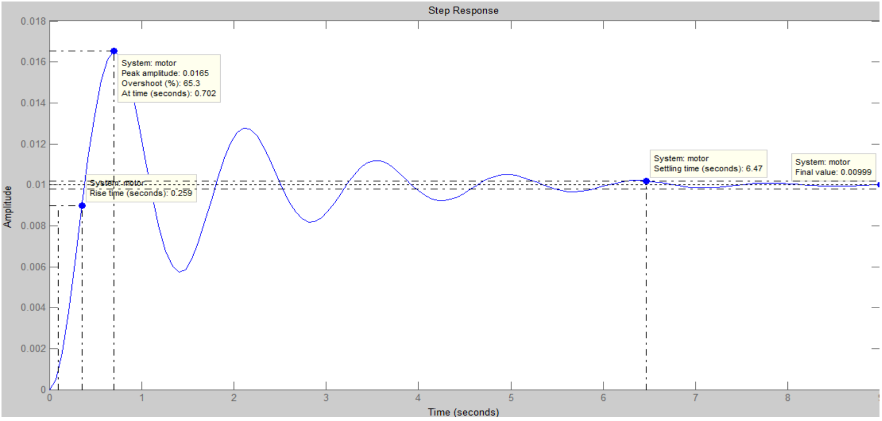

Fig. 12: Step response motor

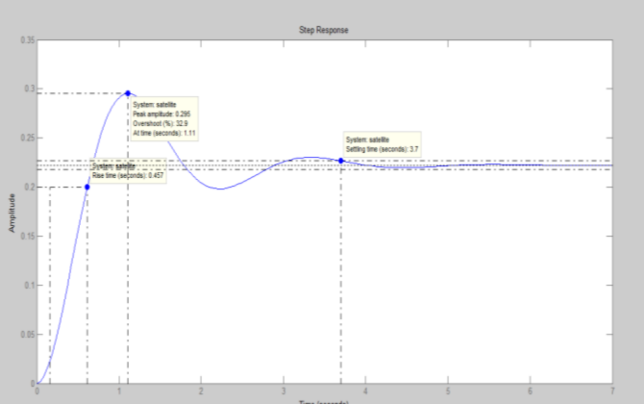

Fig. 13: Step response satellite

In Fig. 12, it can be seen that the system takes about 9 seconds to settle which is desirable as when rotating the satellite in its orbit, it should be subjected to sudden change as it damage the structure because of torque produced in the movement as shown in Fig. 13. The system is very quick to respond and settles very fast in 3.5 seconds only, which is desirable. The overshoot, settling time and rising time are represented for the system.

Interfacing ScilabXCos model with FlightGear V2.12

To check the response of the system on satellite is not possible before developing a prototype, which is on the costlier side so as an alternative, FlighrGear an open source flight simulation software was used to simulate the system developed in SciLab. Aerospace block set along with FlightGear was used to connect the system by inputting body reference frame values and the initial height and altitude of the satellite in LEO. The yaw, pitch and roll were fed to the FlightGear by the XCos model developed earlier.

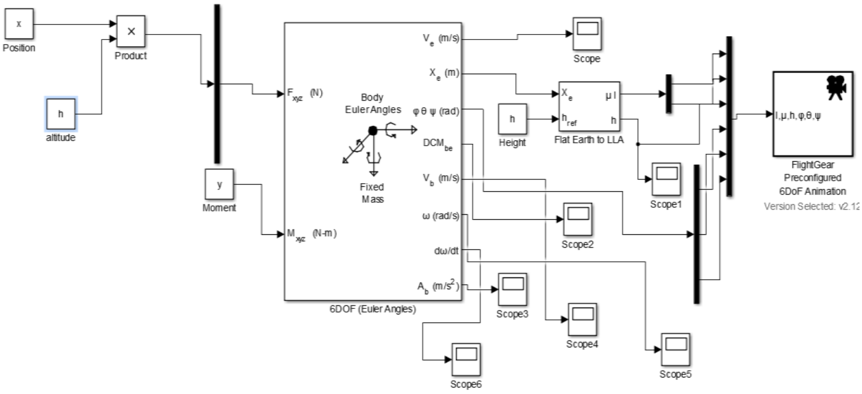

Fig. 14: Block Diagram of Flight Gear

Fig. 14 shows the control system modelling for FlightGear interface with the SciLab –Xcos control system model. One of the major block of importance here is FlighGear Preconfigured 6DOF module which connects the two tools at work here. Flight parameters are the input to the system like yaw, pitch, roll and moments in all the three directions. Initial height for the satellite is the input to the Flat earth to LLA block. This block is also important as FlightGear doesn’t understand what is x, y and z parameters, it works only on longitude, latitude and altitude parameters.

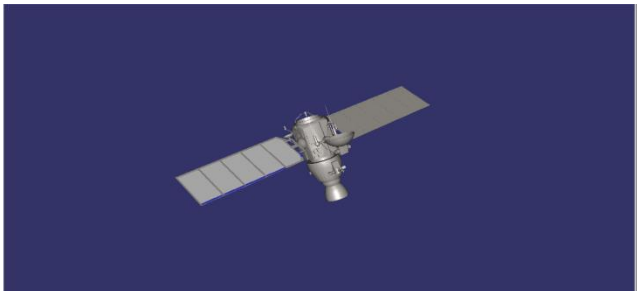

Fig. 15: FlightGear simulation of CARTOSAT-2

In Fig. 15, the satellite represented is CARTOSAT-2 and it was designed in CATIA V5 which was then converted to vrml format by sung AC3D design software. The response of the system was seen on the satellite and it was stable.

5. CONCLUSIONS

In this project the orbit for a satellite launched in low Earth orbit is designed using software like STK, GMAT and MATLAB. For reference, keplerian elements of ISS were studied to get an idea about the keplerian elements in Low Earth Orbit.

In the initial stage the trajectory analysis CARTOSAT-2 and ISS was done on three different software namely GMAT, STK and MATLAB by obtaining the two-line element file for the same and putting it in the respective software.

After analyzing CARTOSAT-2 and ISS, we started with the most basic orbit i.e. circular orbit without the perturbation effects. Graphs were plotted for keplerian elements, velocities, altitude vs earth elapsed days in GMAT.

In GMAT, the circular orbit with effects of perturbations was simulated and a graph was plotted between changes in semi major axis and elapsed days.

After collecting data from the above simulations, in MATLAB environment we calculated keplerian elements for an orbit with perturbation effect.

As a result, we were able to design an orbit for the satellite with more lifespan and less perturbation effects. The final simulation was done with these parameters.

The future scope of this project can incorporate jet actuators into our satellite to control our satellite's linear motion in addition to its orientation. This would prove invaluable in correcting orbital trajectory after collisions or gradual orbit decay. The most important extension that can be made to our control system however would be to incorporate control in multiple dimensions. Our work is a very good starting point though and demonstrates the basic principles involved in controlling a satellite.

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