Multi objective optimization of process parameters of AA2014 Friction Stir Weldments using Genetic Algorithm

L. Suvarna RAJU*,¹, Borigorla VENU^{1,3}, G. MALLAIAH²

Corresponding author ¹Department of Mechanical Engineering, Vignan's Foundations for Science, Technology & Research (Deemed to be University), Guntur-522213, India, drlsrajuvu@gmail.com, venuborigorla@gmail.com ²Department of Mechanical Engineering, Kamala Institute of Technology & Science, Singapur, Huzurabad, Telangana 505468, India, mallaiahg.kits@gmail.com ³Department of Mechanical Engineering, VLITS, Guntur, India

DOI: 10.13111/2066-8201.2020.12.3.15

Received: 23 April 2020/ Accepted: 20 July 2020/ Published: September 2020 Copyright © 2020. Published by INCAS. This is an "open access" article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/)

Abstract: The influence of tool pin profile and process parameters on microstructure and mechanical properties of AA2014 weldments was studied. Tool pin profiles such as a Straight Cylindrical Threaded (SCT) and Taper Cylindrical Threaded (TCT) profiles are used for experimentation. The process parameters such as constant tool rotational speed of 900 rpm, welding speed and tool tilt angles at 30, 40, 50, and 60mm/min and 1°, 2°, respectively, are used to fabricate the weldments. A set of experiments was conducted with two different tool pin profiles and mechanical properties were evaluated. The better mechanical properties such as tensile strength of 367N/mm², impact strength of 10J and hardness of 139HV were obtained by using TCT pin when compared to SCT pin. The observed mechanical properties have been correlated with microstructure. The mechanical properties were analyzed by ANOVA and regression analysis. Objective functions and constraints are developed for the three responses in terms of factors. The factors are optimized using Genetic Algorithm (GA). From the GA results, it is observed that the welding speed of 58mm/min and tool tilt angle of 1.95° are found to be the better combination for carrying out the experiments using TCT pin profile.

Key Words: Friction stir welding, mechanical properties, microstructure, Regression ANOVA, Genetic Algorithm

1. INTRODUCTION

Present century, the demand for manufacturing was growing very fast towards the truncated cost and decreasing weight of materials had meaningfully increased in many applications like environmental safety, industrial sector [1-2]. Friction Stir Welding (FSW) is used to join similar & dissimilar aluminum alloys and currently more focus is given to join specific Aluminum alloys. The use wide range of Aluminum alloys in specific applications provides benefit of cost and increased performance [3]. Generally, AA2014 alloys are the most significant industrial materials due to their heat treatable, high strength, at room and elevated

temperatures on aircraft, transportation (truck body) applications [4]. Joining of AA 2XXX in a permanent manner is usually complicated by fusion welding processes because of AA2XXX alloy often produces defects like porosity, hot-cracks an these defects significantly reduce mechanical properties. The problems occurring in fusion welding techniques can be overcome by using solid state environmental welding technique named as FSW shown in Figure 1(a) [5]. FSW has already exhibited its supremacy in joining of aluminum and magnesium alloys [6, 7]. A non-consumable tool, which comprises shank, shoulder and probe, is used in this process which is shown in Figure 1 b & c. During the process the end of the probe is plunged into the rolling sheet until the shoulder bottom surface touches to the top surface of the workpieces. This action creates frictional heat in between the tool and workpiece. The frictional heat that gets generated in the process actually softens the neighboring material and this softened material is transported from leading edge to trailing edge of the tool and this plasticized material get forged in to a solid phase joint [8, 13].

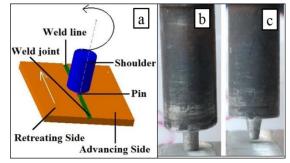


Figure 1. a) Schematic diagram of FSW Process b) SCT profile c) TCT profile

Many of the researchers and studies are proven the microstrure ,mechanical properties and weld quality of the weldments are depended on the tool design and process parameters such as tool rotational speed (TRS), welding speed(WS), Tilt Angle(TA) and axial force [9-12]. TRS and WS are most influenced parameters in welding process and it regulate material plastic flow during FSW as well as it affects the amount of plastic deformation and material stirring. The better combinations of TRS, WS, and TA are developed to increase plastic deformation and helps to fabricate the defect free weldments. Improper combinations of these parameters creates a defect in the weldments [14, 15]. The studies from various experiments found that the threaded tool pin designs enhance the material mixing [16]. Equally, the fixing of the process parameters is a significant phenomenon in FSW process such as the TRS, WS and position of the weld tool [17,18]. In FSW process, process parameters are optimized by various methods such as ant colony algorithm, practicle swarm optimization algorithm and satistical analysis etc.

To optimize the process parameters Meng et. al., conducted a number of trail experiments and optimized the shoulder and pin diameter of the tool with constraint-based on a GA for maximizing the tensile strength of the FSW AL-Li alloy weldments [19]. Based on RSM Elangovan et. al., they developed a mathematical model correlating the process parameters and tensile strength of AA6061 [20]. From the existing works, it is identified that the tool pin profile is a critical factor which influences the heat and stirring required to the weldment in addition to that process parameters such as TRS, WS, and TA to produce a quality of weldments [14]. The present work aimed to improve the weld quality using Non-Traditional optimization technique GA. Hence, GA is used to predict best set of process parameters to exhibit better mechanical properties of AA2014 weldments. The effect of WS and TA on the weld quality was evaluated by Analysis of Variance (ANOVA).

2. METHODOLOGY

This study consists of four major sections:

- Tool design
- Experimentation and analysis of mechanical properties and microstructure,
- Statistical analysis
- Optimization of process parameters.

Tool design

In FSW, geometry of the tool significantly effects the quality of the joint. Especially, shoulder and pin profile of the tool governs the heat and material flow during the process [21]. Many researches proved flat, concave and convex shape shoulders are better for large extent to heat due to their nature in the FSW process. In the current work, the SCT and TCT tool pin profiles with flaten shaped shoulder was used to prepare the AA 2014 weldments.

> Experimentation and analysis of mechanical properties and microstructure

Experimentation was done by the FSW setup and mechanical properties evaluation and microstructure study was carried out by the various destructive methods.

Statistical analysis

Stastical analysis of the experimental results was done by using ANOVA and regression analysis for checking the process acceptance.

> Optimization of process parameters

The process parameters were optimized to exhibit better mechanical properties of AA2014 weldments using GA.

3. RESULTS AND DISCUSSION

3.1 Microstructure study

The microstructures of weld region of weldments were perceived using an optical microscope, and the relevant micrographs are presented in Figure 2.

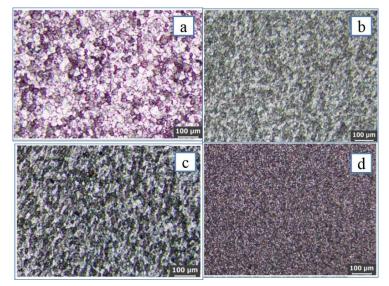


Figure 2. Microstructure images obtained at various conditions (a-d weldments fabricated by TCT)

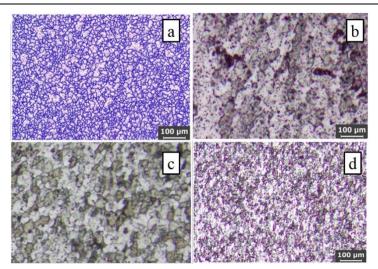


Figure 3. Microstructure images obtained at various conditions (a-d weldments are fabricated SCT)

From this investigation it can be concluded that the weld region of all the weldments customarily contain uniformly distributed, broken up aluminum and second phase (copper) particles. However, the size of the particles is different, and it is found to be influenced by the WS and TA. The weld region of AA2014 alloy fabricated using a TCT with a WS of 60 mm/min and TA of 2° shows very fine aluminum and second phase particles as shown in Figure 2 (d), compared to other microstructures are shown in Figures 2 of (a), (b) and (c). Similarly, a SCT tool provides smashed aluminum particles are presented in Figures 3 of (a), (b), (c), and (d). From the results it is observed that the two different tool pin profiles SCT and TCT exhibits finer precipitate particles due to their stirring and material flow nature. In addition to that TCT produce finer eutectic aluminum particles in the weld zone compared to SCT, because TCT had superior contact between tool and workpiece, which gives sufficient amount of heat to produce finer eutectic aluminum particles. From the microstructure study, it is found that the size of the fragmented Al particles is increasing with the increase of the heat input and also revealed that there is a considerable variation in the dispersion of Al particles by varying the process parameters and tool pin profile.

3.2 Mechanical Properties

The tool pin profile and process parameters influence the mechanical properties such as Tensile Strength (TS), impact strength (IS) and Hardness (H) of friction stir welded AA2014 weldments which are presented in Table-1. From the results, it was found that the weldments fabricated by utilizing TCT profile with 900 rpm of TRS, 60 mm/min of WS and TA of 2° exhibited good mechanical properties compared to SCT profile. TCT profiles were produce required amount heat generation which causes the free flow of a material to weldment. The existence of excellent and uniformly dispersed particles in the weld region may also be one of the reasons for obtain the better mechanical properties of weldments.

Tool Pin Profile	WS (mm/min)	TA (degree)	TS (MPa)	IS (MPa)	H (HV)
TCT profile	30	1	131.94	8.02	119.67
	40	2	323.33	9.16	124.54
	50	1	156.45	8.24	134.3

Table 1. Mechanical properties of weldments

	60	2	366	9.31	138.4
	30	1	112.56	7.62	83.4
	40	2	330.97	9.13	115
SCT profile	50	1	144	7.82	88.1
	60	2	346.66	9.29	121.8

4. STATISTICAL ANALYSIS

Statistical analysis is the process of generating statistics from experimental data and analyzing the results to deduce or infer meaning.

4.1 Analysis of experimental data

The phases of Mathematical model development are as follows.

- > Taguchi Method adopted for developing the experimental design.
- > Effects of input process parameters on TS, IS, and H.
- Data scrutiny (ANOVA, main effects, S/N Ratios)
- Developing and Evaluating the linear regression models (R², adjusted R², and Predicted R²)

4.2 Major effects of the process parameters

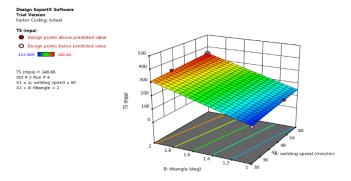
The analysis of mean effects is utilized to estimate the influence of considered factors on the mechanical properties of weldments. The mean effect and the Signal to Noise ratio of TS, IS, and H for specific level are evaluated and the average of all the output's that were attained with that specific level and are presented in Table-2. Analogous analysis is found in related research works concerning process properties for diverse materials [22-24].

4.3 ANOVA for mechanical properties

In this paper, ANOVA is utilized to find out the effect of WS, TA on the mechanical properties of weldments. The percentage contribution of each process parameter is determined and same is used to measure the equivalent effects on the joint strength. The implemented plan is evaluated at 95% confidence level [25-27].

5. INTERACTION EFFECT OF PROCESS PARAMETERS ON MECHANICAL PROPERTIES OF WELDMENTS

In FSW, the process parameters influence the joint strength. For a better quality of welds, selection of process parameters is a key factor. In the current study, the influence of process parameters on mechanical properties of weldments and their relationship is established by the Design of Experiments (DOE) software. Figure 4 & Figure 5 illustrate the effect of process parameters vs. material properties. Consequently, from the main plots of the parameter interaction effects on weld quality (Figure 4(a), Figure 4(b) & Figure 4(c); Figure 5(a), Figure 5(b) & Figure 5(c)), it is noticed that the TS increased with an increase in WS from 30 to 40 mm/min by varying the tool TA from 1° to 2° and then reduced with further increase of the speed. Due to this reason, we kept TRS is constant at 900 rpm. According to the results, an increase of the TRS up to 60 mm/min results in temperature increase and provides better stirring and it also directs efficient plasticized material flow. WS should be maintained at a minimum to produce a successful joint while producing the required plastic flow. Similarly, the TA of a tool creates the forging pressure.





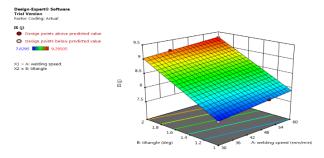


Figure 4. (b) Interaction effect of WS & TA on IS of weldment by STC profile.

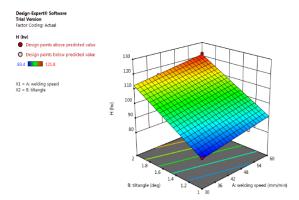


Figure 4. (c) Interaction effect of WS & TA on H of weldments by STC profile.

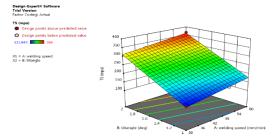


Figure 5. (a) Interaction effect of WS & TA on TS of weldment by TCT profile.

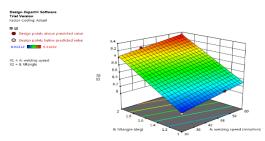


Figure 5. (b) Interaction effect of WS & TA on IS of Weldment by TCT profile.

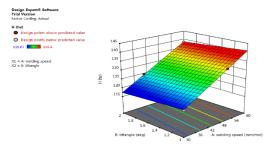


Figure 5. (c) Interaction effect of WS & TA on H of weldment by TCT profile.

5.1 Developing Mathematical Model

The relation between control factor values and response values of weldments as a response is developed using multiple regression analysis. From the regression analysis, it is found that second-order polynomial hypothesis holds true. An equation between control factor variables and response variables was developed to predict the error. The coefficient of determination (R^2) is calculated for the TS, IS &H variables based on which the decision could be taken to suggest the equation [28-29]. Empirical models were developed for the prediction of mechanical properties.

5.1.1 Regression analysis

Multiple regressions predictive equations generated for three factors such as TS, IS, and H, and the equations of two different tool pin profiles are shown as follows.

SCT profile

Tensile strength = -192.4+4.721*a+198.8*b-0.03937**a*aImpact strength = 5.780+0.01801*a+1.402*b-0.000102*a*aHardness = 54.45-0.1850*a+29.78*b+0.005250*a*a

TCT profile

Tensile strength = -20.37 - .408 * a + 183.7 * b + 0.04541 * a * aImpact strength = 6.413 + 0.02517 * a + 1.012 * b - 0.000175 * a * aHardness = 97.47 + 0.8855 * a - 2.638 * b - 0.001925 * a * a

Table 2. Analysis of variance of weldments made by SCT/TCT profiles

Source	TS (Mpa)		IS (J)		H (HV)		Remarks	
Source	SCT	ТСТ	SCT	ТСТ	SCT	ТСТ	Keinai KS	
P-value	0.03	0.04	0.01	0.03	0.01	0.01	<0.05 (confidence level 95%)	
Predicted R ² value	0.97	0.98	0.99	0.98	0.98	0.99	The difference is less than 0.2	

Adjusted R ²	0.99	0.99	0.99	0.99	0.99	0.99	
Adeq Precision	34.33	29.75	93.93	42.56	42.2	74.96	It measures the signal to noise ratio. A ratio is greater than 4 is desirable.

An evaluation of experimental values and predicted values of mechanical properties for both tool pin profiles appears to be in good agreement and the same are tabulated in Table 3. The models are validated by R-Squared, Adjusted R-Squared and Prediction R-Squared and standard error of the regression (S).

From the results it is noticed that the TCT profile gives better results over a SCT profile.

6. OPTIMIZATION OF PROCESS PARAMETERS BY USING GENETIC ALGORITHM (GA)

Several traditional methods like dynamic programming have been used in the past to optimize the process parameters in FSW.

Traditional techniques have some drawbacks such as the possibility for the solution to get confined into local minima, and hence there is a need for a multipurpose algorithm to solve different optimization problems.

In the current study, a non-traditional optimization technique, namely GA, was used to solve the problems and which gives better results compare to existing techniques.

From the experimental results, it is found that the TCT profiles exhibit better results compare to SCT.

The optimization process was carried out for TCT.

The procedural program was written and executed in MATLAB GA TOOLBOX using regression equations. from these equations, the functions were developed to find the optimal value.

6.1 Fitness.m.file

Function y =venu_fitness(x)

 $\begin{array}{l} Y(1) = -20.37 - 2.408 * x(1) + 183.7 * x(2) + 0.04541 * x(1) * x(1); \\ Y(2) = 6.413 + 0.02517 * x(1) + 1.012 * x(2) - 0.000175 * x(1) * x(1); \\ Y(3) = 97.47 + 0.8855 * x(1) - 2.638 * x(2) - 0.001925 * x(1) * x(1); \end{array}$

6.2 Constraint.m.file

 $\begin{aligned} & Function[c, ceq] = venu_constraint(x) \\ & c(1) = [468 - (-20.37 - 2.408 * x(1) + 183.7 * x(2) + 0.04541 * x(1) * x(1))]; \\ & c(2) = [12 - (6.413 + 0.02517 * x(1) + 1.012 * x(2) - 0.000175 * x(1) * x(1))]; \\ & c(3) = [155 - (97.47 + 0.8855 * x(1) - 2.638 * x(2) - 0.001925 * x(1) * x(1))]; \\ & ceq = []; \end{aligned}$

6.3 Optimization of tool box:

Solver = ga_genetic algorithm Fitness function =@fitness No. of variables = 2 Bounds = [30 1], [60 2]

TOOLS	WS (mm/min)	TA (°)	TS (Mpa)	IS (J)	H (HV)	TS (Mpa)	IS (J)	H (HV)
			Experi	mental V	alues	Predicted values		
	30	1	131.94	8.02	119.67	127.40	8.03	119.86
	40	2	323.33	9.16	124.54	327.87	9.14	124.34
TCT	50	1	156.45	8.24	134.3	160.99	8.22	134.10
101	60	2	366	9.31	138.4	361.45	9.33	138.59
	30	1	112.56	7.62	83.4	116.50	7.63	82.87
	40	2	330.97	9.13	115	327.03	9.12	115.52
SCT	50	1	144	7.82	88.1	140.06	7.81	88.62
201	60	2	346.66	9.29	121.8	350.59	9.30	121.27

Table 3. Analyzed results of multiple regression

7. GENETIC ALGORITHM RESULTS

To measure the performance of MATLAB genetic algorithm approach, constrained optimization of input parameters of FSW. From this, the improved results are obtained in 102 iterations and these results were compared with the experimentation results which shown in Table 4. From the GA results it is observed that the WS of 58 mm/min and TA angle of 1.95° are found to be the better combination for carrying out the experiments using TCT profile.

Table 4. Comparison of experimental results with the results of GA approach

S. No	Technique	WS (mm/min)	TA (Degree)	TS (Mpa)	IS (J)	Hardness (HV)
1	Experimentation	60	2	366	9.31	138.4
2	Genetic Algorithm	58	1.95	366	9.31	138.4

8. CONCLUSIONS

The effect of TA and WS at a constant TRS of 900 rpm on mechanical properties of AA2014 weldments comparing the tool pin profiles of SCT and TCT are investigated using ANOVA and regression analysis.

The following conclusions can be drawn:

- The AA2014 friction stir weldments made by SCT have defects due to lack of contact area with the metal plates.
- From results of ANOVA, it is observed that the WS has more effect on the TS (0.037 of p-value), IS (0.0136 of p-value) and H (0.0137 of p-value) at the confidence level of 95% when the SCT is used.
- From the results of ANOVA, weld speed has added effect on the TS (0.0446 of p-value), IS (0.0312 of p-value) and H (0.0194 of p-value) at the confidence level of 95% when TCT is used.
- From the experimental results it is noticed that the TCT tool pin profile exhibits better mechanical properties such as 366 Mpa of TS, 9.319 J of IS, and 138.4 HV of H at 60 mm/min of WS and 2° of TA. This may be due to the best coupling action at stir zone of weldments compared to SCT profile.
- Optimization of process parameters was carried out by using GA. From the GA results it is observed that the WS of 58 mm/min and TA of 1.95° are found to be the better combination for carrying out the experiments using TCT profile.

ACKNOWLEDGMENT

The authors express their deepest gratitude to the authorities of SERB/F/11385/2018-2019 for granting fund to this project and Vignan's Foundation for Science, Technology & Research (Deemed to be University), for extending the facilities to complete the project work.

REFERENCES

- L. E. Murr, A review of FSW research on dissimilar aluminum alloys, J. Mater. Eng. Perform, 19:1071–1089, 2010.
- [2] W. M. Thomas, E. D. Nicholas, Friction stir welding for transportation industries, *Mater. Des.*, 18: 269–273, 1997.
- [3] V. Patel, W. Li, G. Wang, F. Wang, A. Vairis and P. Niu, Friction, Stir Welding of Dissimilar Aluminum Alloy Combinations: State-of-the-Art, *Metals*, 9, 270, 2019.
- [4] W. M. Thomas, E. D. Nicholas, J. C. Needham, M. G. Murch, P. Temple-Smith and C. J. Dawes, Friction stir butt welding, GB patent No.91259788.
- [5] R. Mishra, Z. Ma, R. Mishra, Friction stir welding and processing, *Mater. Sci. Eng. R*, 50, 1–78, 2005.
- [6] Z. Y. Ma, A. H. Feng, D. L. Chen, J. Shen, Recent Advances in Friction StirWelding/ Processing of Aluminum Alloys: Microstructural Evolution and Mechanical Properties, *Crit. Rev. Solid State Mater. Sci.*, 43, 269– 333, 2017.
- [7] P. L. Threadgill, A. J. Leonard, H. R. Shercliff and P. J. Withers, Friction Stir Welding of aluminum alloys, *Int Mater Rev*, 54:49-93, 2009.
- [8] B. Kaled, S. Ben Salem, J. Bessrour, Prediction model of uts and investigation on microstructure characterization of friction stir welded AA2024-T3, *Int J Adv Manuf Technolog*, 95:423-486, 2018.
- [9] B. Venu, L. S. Raju, Influence of Tool Geometry and Effects of Plunge Depth in Friction Stir Welding Of AA2014, International Journal of Innovative Technology and Exploring Engineering (IJITEE), 8(7), 2077-2079, 2019.
- [10] K. S. Arrora, S. Pandey, M. Schaper, R. Kumar, Effect of process parameters on FSW of AL alloy 2219-T87, Int J Adv Manuf Technology, 50(9):941-952, 2010.
- [11] G. Padhy, C. Wu, S. Gao, Friction stir based welding and processing technologies processes, parameters, microstructures and applications: A review, J. Mater. Sci. Technol., 34, 1–38, 2018.
- [12] P. H. Shah, V. J. Badheka, Friction stir welding of aluminium alloys: An overview of experimental findings— Process, variables, development and applications, *Proc. Inst. Mech. Eng. Part L: J. Mater. Des. Applic.*, 6, 1464420716689588, 2017.
- [13] Y. Zhou, S. Chen, J. Wang, P. Wang, J. Xia, Influences of Pin Shape on a High Rotation Speed Friction Stir Welding Joint of a 6061-T6 Aluminum Alloy Sheet, *Metals*, 8, 987, 2018.
- [14] R. S. S. Prasanth, K. H. Raj, Determination of Optimal Process Parameters of Friction Stir Welding to Join Dissimilar Aluminum Alloys Using Artificial Bee Colony Algorithm, *Trans. Indian Inst. Met.*, 71,453– 462, 2017.
- [15] S. Azeez, E. Akinlabi, Effect of processing parameters onmicrohardness and microstructure of a double-sided dissimilar friction stir welded aa6082-t6 and aa7075-t6 aluminum alloy, *Mater. Today: Proc.*, 5, 18315– 18324, 2018.
- [16] Y.-h. Zhao, S.-b. Lin, W. Lin, F.-X. Qu, The influence of pin geometry on bonding and mechanical properties in friction stir weld 2014 Al alloy, *MaterLett.*, 59:2948–52, 2005.
- [17] Q. Zehng, Jiakuizu, L. Zhang, G. Dai, Designing expert system with artificial neural networks for in situ toughene Si3N4, *Materials and Design*, 23:287-290, 2002.
- [18] S. R. Prasad, A. Kumar, Ch. S. Reddy, L. S. Raju, Influence of tool Shoulder geometry on microstructure and mechanical properties of friction stir welded 2014-T6 aluminium alloy, *materials today: proceeding*. 4:10207-10211, 2017.
- [19] T. Nagira, X. C. Liu, K. Ushioda, Y. Iwamoto, G. Ano & H. Fujii, Role of annealing twinning in microstructural evolution of high purity silver during friction stir welding" science technology of welding and joining, 24(7):1-8, 2019.
- [20] B. Venu, L. S. Raju, I. BhavyaSwathi, G. Santhanam, A review on Friction Stir Welding of various metals and its variables, *materials today:proceedings*, 18(1):298-302, 2019.
- [21] Y. Zhou, S. Chen, J. Wang, P. Wang, J. Xia, influences of pin shape on a high rotation speed friction stir welding joint of a 6061-T6 aluminium alloy sheet, *metals*, 8, 987, 2018.

- [22] B. Khaled, S. ben saren, J. Bessong, Prediction model of uts and investigation on microstructure characterization of friction stir welded AA2024-T3, *Int J Adv Manuf Technology*, 95:423-486, (2018.
- [23] K. S. Arrora, S. Pandey, M. Schaper, R. Kumar, Effect of process parameters on FSW of AL alloy 2219-T87, Int J Adv Manuf Technology, 50(9):941-952, 2010.
- [24] T. Sakthivel, G. S. Sengar, J. Mukhopadhyay, Effect of traverse speed on microstructure and mechanical properties of friction stir welded aluminum, *Int J Manuf Technology*, 43(5): 468-473, 2009.
- [25] A. Cxicxek, T. Kıvak and G. Samtasx, Application of Taguchi method for surface roughness and roundness error in drilling of AISI 316 stainless steel, J Mech Eng., 583: 165–174, 2012.
- [26] W. H. Yang and Y. S. Tarng, Design optimization of cutting parameters for turning operation based on Taguchi method, J Mater Process Tech., 84: 127–129, 1998.
- [27] B. M. Gopalsamy, B. Mondal and S. Ghosh, Taguchi method and ANOVA: an approach for process parameters optimization of hard machining while machining hardened steel, *J Sci Ind Res*, 68: 686–695, 2009.
- [28] C. X. Feng and X. Wang, Development of empirical models for surface roughness prediction in finish turning, Int J Adv Manuf Tech, 20: 348–356, 2002.
- [29] Dr. M. Nagaphani Sastry, B. Venu, Analysis & Optimization of Design Parameters of Mechanisms Using Ga, International Journal of Computational Engineering Research, 3(7):1-12, 2013.