Study on Free Vibration Analysis of a Rotating Fibre-Graphene-Reinforced Hybrid Polymer Composites Pre-Twist Shell

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Abstract: *The present work aims to develop a computational procedure for investigating the vibration behaviour of pre-twisted laminated composite shell containing graphene inclusions in their matrix. According to nanoscopic empirical equations, graphene's mechanical properties are determined by its size dependence. It has been demonstrated that the orthotropic mechanical properties of composite laminates made from carbon fibres and hybrid matrix can be evaluated. Based on pre-twist and geometric configurations, finite element methods have been used to model hybrid materials shells that include carbon fibre, graphene, and graphene-fibre reinforcement. As part of the validation process, the proposed method is compared with other methods when possible. Finally, the vibrational behaviour of the composite shell is extracted by imposing a twisted angle on a cantilever boundary condition. An analysis of vibrations for each configuration is presented in this paper, as well as the effects of graphene inclusions on natural frequencies. As graphene volume fractions in the matrix increase, the natural frequencies of every mode also increase. When the hub radius and rotational speed are increased, the frequency parameter increases with an increase in graphene volume in the hybrid polymer composite pre-twisted shell.*

Key Words: *Pre-twisted shell, graphene, hybrid composite, vibrations, rotational speed, finite element method*

1. INTRODUCTION

By combining two or more constituent materials, a material that is physically and chemically distinct from its constituents can be obtained [1]. This is the result of the combination of constituent materials. During the past decade, composite structures have made significant progress. Several industries have already adopted fiber-reinforced composites, including marine [2-3], civil [4] and other industries [5–7]. Due to advances in manufacturing technology, composite materials have become stronger, stiffer, denser, and more economical.

Graphene is the thinnest known material and possesses incredible strength, being about 200 times stronger than steel. It is also made up of a single carbon atom, making it one of the most versatile materials in existence [8-10]. In addition, it has favourable properties in terms of optical, electrical, and thermal characteristics [11-13]. Because graphene is a two-dimensional (2D) structure with a nanometre-scale thickness, its in-plane tensile behaviour is the most appropriate for a wide range of research applications. [14-15]. Combining graphene with different materials results in a strong reinforcement material in nanocomposites. Many studies have demonstrated that graphene nanocomposites are capable of advanced research applications [16-23]. Research in materials science and engineering has focused on graphene-reinforced composites (GRCs) with advanced mechanical, electrical, and thermal properties. The idea is developing that GRCs [24] will serve as factors in a variety of fields, including electrical devices, energy storage, and sensors.

New materials created for potential applications, polymer nanocomposites [25] have seen significant improvements in research and development. For structural applications, fibre-reinforced polymers (FRP) are widely used due to their outstanding properties of strength and weight [26]. Some advantages are associated with fibre-reinforced polymers, which motivate researchers to apply them to various structural contours. An increase in matrix-dominant characteristics is achieved by mixing nanoparticles with a polymer matrix. Graphene is a filling agent that provides both fibre-dominant and matrix-dominant properties in the polymer matrix. As such, graphene enhances the technical capabilities of a composite.

Using pre-twisted laminated [27-28] composite shells containing graphene inclusions in their matrix, the present work aims to develop a computational method for investigating vibration behaviour. Simulation of physical phenomena using FEM (Finite Element Method) is among the most effective methods for obtaining accurate results. Therefore, FEM can be applied both for academic and research purposes [29-30]. Its effectiveness in terms of precision and design effort is a major advantage of applying the finite element method using four node, quadrilateral, stress/displacement shell finite elements; it proposes a finite element approach for the analysis and design of composite laminated structures using a simplified integration formulation based on four nodes. The Halpin-Tsai equations and the outcomes of the nanomechanical analysis are required for describing the elastic properties of the carbon nanostructure/polymer matrix and subsequently their counterparts for the composite laminate by calculating the elastic constants of the hybrid composite. It is shown that the mechanical properties of the composite material are analysed under different loading conditions depending on the configuration of the plates.

Nanoparticles have been used to reinforce composite structures in a few publications [31-32], and they have been primarily focused on carbon fiber-based laminates with graphene inclusions. The modelling of CFGR (carbon fiber graphene reinforced) hybrid materials shells using traditional analysis method (FEM) is explored in this paper. By imposing a twisted angle on a cantilever boundary condition, it can be obtaining the vibrational behaviour of the composite shell. For each configuration, vibrations are calculated along with the effects of graphene inclusions on natural frequencies.

2. MATHEMATICAL FORMULATION

2.1 Twisted plate characteristics

Pre-Twisted composite shell is designed using the design software below. Figure 1 represents the pre-twist shell of graphene fiber-reinforced polymer hybrid composites.

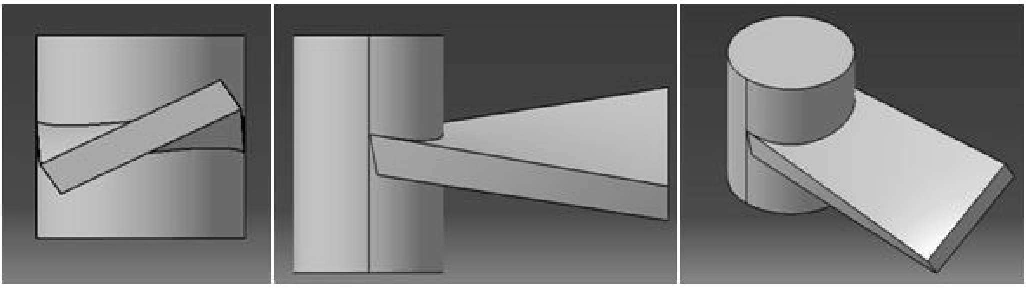


Fig. 1 Laminated Pre-Twist Shell

The pre-twisted doubly curved shell panel differential equations of equilibrium are given as

$$\frac{\partial Fx}{\partial x}+\frac{\partial Fxy}{\partial y}-\frac{1}{2}\left(\frac{1}{C_y}-\frac{1}{C_x}\right)\frac{\partial B_{xy}}{\partial y}+\frac{P_x}{C_x}+\frac{P_y}{C_{xy}}=Q_1\frac{\partial^2 u}{\partial t^2}+Q_2\ \frac{\partial^2\theta_x}{\partial t^2} \tag{1}$$

$$\frac{\partial Fxy}{\partial x}+\frac{\partial Fy}{\partial y}+\frac{1}{2}\left(\frac{1}{C_y}-\frac{1}{C_x}\right)\frac{\partial B_{xy}}{\partial x}+\frac{P_y}{C_y}+\frac{P_x}{C_{xy}}=Q_1\frac{\partial^2 v}{\partial t^2}+Q_2\ \frac{\partial^2\theta_y}{\partial t^2} \tag{2}$$

$$\frac{\partial P_x}{\partial x}+\frac{\partial P_y}{\partial y}-\frac{F_x}{C_x}-\frac{F_y}{C_y}-2\frac{F_{xy}}{C_{xy}}+F_{x^0}+F_{y^0}\frac{\partial^2 w}{\partial y^2}=Q_1\frac{\partial^2 w}{\partial t^2} \tag{3}$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=Q_3\frac{\partial^2\theta_x}{\partial t^2}+Q2\frac{\partial^2 u}{\partial t^2} \tag{4}$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=Q_3\frac{\partial^2\theta_y}{\partial t^2}+Q2\frac{\partial^2 v}{\partial t^2} \tag{5}$$

where, F_{x^0} and F_{y^0} are external in plane force in '*x*' and '*y*' respective directions,
C_x and C_y are the curvature radius in the *x* and *y* respective directions,
C_{xy} is the radius of twist,
Consider, pre-twisted element,
Fx, Fy, and *Fxy* are plane forces,
Px, and *Py* are shearing forces,
Bx, By and *Bxy* are bending moment components.

$$(Q_1, Q_2, Q_3)=\sum_{k=1}^{n}\int_{zk-1}^{zk}(\rho)_k\,(1,z,z^2)\partial z \tag{6}$$

Here, n = number of layers. $\rho_k = k^{th}$ layer density.

3. FINITE ELEMENT FORMULATION

Whereas analytical methods are more difficult to use, the finite element method can be utilized to solve problems with complicated shapes and boundary conditions. The finite element approach for the structural analysis of functional graded materials pre-twisted shell elements was created using the First order shear deformation Theory. The assumption is that the plate is manufactured by a number of layers, each of which is supposed to be isotropic and homogeneous. The approach makes use of an eight nodded isoparametric quadratic shell element. There are nodes in the middle of the shell element. The five degrees of freedom for each node are u, v, w, ϕ_x and ϕ_y. The following is the shape function for the shell element with 8 nodes.

$$u(\ni,\vartheta) = \alpha_1 + \alpha_2\ni + \alpha_3\vartheta + \alpha_4\ni^2 + \alpha_5\ni\vartheta + \alpha_6\vartheta^2 + \alpha_7\ni^2\vartheta + \alpha_8\ni\vartheta^2 \tag{7}$$

Displacement between nodes of shape function is represented as follows:

$$\mathrm{N_i} = (1+\ni\ni_\mathrm{i})(1+\vartheta\vartheta_\mathrm{i})\frac{(\ni\ni_\mathrm{i}+\vartheta\vartheta_\mathrm{i}-1)}{4} \quad \mathrm{i=1\ to\ 4} \tag{8}$$

$$N_i = (1-\ni^2)\frac{1+\vartheta\vartheta_\mathrm{i}}{2}\mathrm{i=5,7} \tag{9}$$

$$N_i = (1+\ni\ni_\mathbf{i})\frac{1-\vartheta^2}{2} \quad \mathrm{i=6,8} \tag{10}$$

where:
$\ni$ and ϑ are the element natural coordinates,
$\ni_i$ is ith node values,
ϑ_i is values at ith node.
In this case, FSDT is used to provide the displacement field

$$\mathrm{u}(x,y,z) = u_0(x,y) + z\phi_y(x,y) \tag{11}$$

$$v(x,y,z) = v_0(x,y) + z\phi_x(x,y) \tag{12}$$

$$w(x,y,z) = w_0(x,y) \tag{13}$$

where: u is displacement in the x-axis, v is displacement in the y-axis; w is displacement in the x-axis; u_0 is displacement at the midplane in the x-axis; v_0 is displacement at the midplane in the y-axis; w_0 is displacement at the midplane in the z-axis; ϕ_x is the midplane rotations normal to the x-direction; ϕ_y is the midplane rotations normal to the y-direction.
Shape function displacements are represented by the following equations:

$$x = \sum N_i x_i \ , y = \sum N_i y_i \tag{14}$$

$$u_0 = \sum N_i u_i \,, v_0 = \sum N_i v_i \ , w_0 = \sum N_i w_i \tag{15}$$

$$\phi_x = \sum N_i \phi_{xi} \,, \phi_y = \sum N_i \phi_{\mathrm{yi}} \tag{16}$$

3.1 Element elastic stiffness matrix derivation

Linear strain components are represented by displacements as follows:

$$\{\varepsilon\} = \{B\}\{v_e\} \tag{17}$$

where $\{v_e\} = \{u_1 v_1 w_1 \phi_{x1}\phi_{y1} \ldots . u_8 v_8 w_8 \phi_{x8}\phi_{y8}\}$

$$[B] = [[B_1][B_2] \ldots [B_8]] \tag{18}$$

An element's elastic stiffness can be expressed as a matrix,

$$[k_e] = \int_{-1}^{1}\int_{-1}^{1}[B]^T\,[D][B][J]d\ni d\vartheta \tag{19}$$

3.2 Element initial stress stiffness matrix derivation

The matrix form of the nonlinear strain components is as follows:

$$\varepsilon_{nl} = \left\{\varepsilon_{xnl}\ \varepsilon_{ynl}\ \tau_{xynl}\right\}^T = \frac{\{C\}\{v\}}{2} \tag{20}$$

$$\text{where, } \{v\} = \left\{u_x\ u_y\ v_x\ v_y\ w_x\ w_y\ \phi_{x,y}\ \phi_{y,x}\ \phi_x\ \phi_y\right\}^T \tag{21}$$

Expression for displacement (v) is given by

$$\{v\} = \{G\}\{v_E\} \tag{22}$$

Thermal stress of element initial stress stiffness matrix is as follows:

$$\left[k_g\right]_e = \int_{-1}^{1}\int_{-1}^{1}[G]^T\,[S][G][J]d\ni d\vartheta \tag{23}$$

3.3 Matrix derivation for the element mass

As a result, the mass matrix of the elements looks like this:

$$[m_e] = \int_{-1}^{1}\int_{-1}^{1}[N]^T\,[P][N][J]d\ni d\vartheta \tag{24}$$

$$(P_1P_2P_3) = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}(\rho)_k\,(1, z, z^2)dz \tag{25}$$

3.4 Centrifugal stiffness matrix of a rotating shell element

As a result of the rotation of the plate, centrifugal forces are generated, thus, rotational speed results in work being done by the shell.

$$W_c^e = \frac{1}{2}F_c^e\dot{w}^2dx = \frac{1}{2}\{v^e\}^T[k_c]^e\{v^e\} \tag{26}$$

On an element of a shell, centrifugal force can be expressed as follows:

$$F_c^e = \int_{x_i}^{x_i+l}\int_{-t/2}^{t/2}b\rho\,(t)\Omega^2(R+x)dt\,dx \tag{27}$$

where x_i is the distance of i^{th} node from axis of rotation, Ω *(rad/s)* is angular velocity of plate element and R is the radius of hub.
Shell elements have kinetic energy as follows:

$$T = \frac{1}{2}\iint_A \rho h\,\dot{w}^2dA \tag{28}$$

An element's centrifugal stiffness matrix can be found below:

$$[k_c] = F_c\iint[N]^T\,[N]dxdy \tag{29}$$

3.5 Analysis of free vibration

The analysis of free vibration is represented by:

$$[M_m]\{\ddot{v}\} + \left[K_{ef}\right]\{v\} = 0 \tag{30}$$

$$[K_{ef}] - f_n{}^2[M_m] = 0 \tag{31}$$

where, $[M_m]$= mass matrix, f_n= natural frequencies, $[K_{ef}]$ =stiffness matrix

$$[K_{ef}] = [k_e] + [k_c] + [K_{m1}]$$

$\{v\}$ = vector of degrees of freedom.

4. RESULTS AND DISCUSSIONS

An analysis of free vibrations of a pre-twisted cantilever shell is presented in this work. Calculations are performed using a MATLAB based on FEM. By using eight-nodded shell element the analysis was done. To validate the model, the results are compared with those of previous studies using the present approach.

According to Tables 1 and 2, a comparison has been made between the first five natural frequencies characteristics of rotating cantilever plates of different speeds. Increasing the aspect ratio (δ), hub radius ratio (σ), and dimensionless rotation speed parameter (**Ψ**) leads to an increase in shell rotation frequency.

As a result of these parameters, the speed at which the shell rotates and the frequency at which it rotates are directly influenced. In addition, the outcomes are also accompanied by the results of Yoo and Kim [33] and Dokainish and Rawtani [34], which show substantial agreement with our current approach.

Table 1. An analysis of the frequency distributions of the rotating cantilever plate based on the first five natural frequencies (δ= 1, σ=0, **Ψ**= 0.01).

	Dokainish and Rawtani [34]	Yoo and Kim [33]	Present	Dokainish and Rawtani [34]	Yoo and Kim [33]	Present
1	3.654	3.652	3.682	4.112	4.113	4.146
2	8.643	8.645	8.664	8.984	9.003	9.153
3	21.556	21.533	21.586	22.012	21.966	22.425
4	27.474	27.384	27.412	27.866	27.623	27.912
5	31.218	31.218	31.546	32.176	31.585	32.842

Table 2. An analysis of the frequency distributions of the rotating cantilever plate based on the first five natural frequencies (δ= 1, σ=1, **Ψ**= 0.01).

	Dokainish and Rawtani [34]	Yoo and Kim [33]	Present	Dokainish and Rawtani [34]	Yoo and Kim [33]	Present
1	3.808	3.861	3.901	4.639	4.813	5.085
2	8.728	8.735	8.856	9.305	9.343	10.642
3	21.741	21.719	21.613	22.726	22.679	23.174
4	27.626	27.425	27.784	28.459	27.790	29.321
5	31.776	31.362	31.821	33.683	32.149	32.942

Table 3. Comparison of physical properties of the composite material and Graphene inclusion

	Vgr	E_1(GPa) [1]	E_{g1}(Gpa) [Present]	E_2(GPa) [1]	E_{g2}(Gpa) [Present]	ρ_c (g/cm3) [1]	ρ_g (g/cm3) [Present]
1	0.00	139.20	139.41	7.799	7.898	1.560	1.719
2	0.10	145.47	145.56	16.673	16.784	1.602	1.766
3	0.20	152.95	153.12	23.073	23.121	1.645	1.798
4	0.30	162.00	162.21	30.305	30.413	1.687	1.823
5	0.40	173.20	173.39	39.064	39.196	1.730	1.861
6	0.50	187.42	187.62	50.094	50.226	1.772	1.901

where E_1 & E_2 are Young's modulus of composite material in the longitudinal direction & in the transverse direction, E_{g1} & E_{g2} are Young's modulus of graphene inclusion material in the longitudinal direction & in the transverse direction, ρ_c & ρ_g are the equivalent mass density of the composite structure and graphene inclusion.

The proposed method is used to compare the physical properties of the composite material from literature K.G. Stelios' [1] work with that of the graphene inclusion in the present study. Table 3 shows the results of physical properties of components compared with K.G. Stelios' work [1] and the proposed approach. The present results are in good agreement with those presented by K G Stelios in his work [1] at a different volume fraction of graphene.

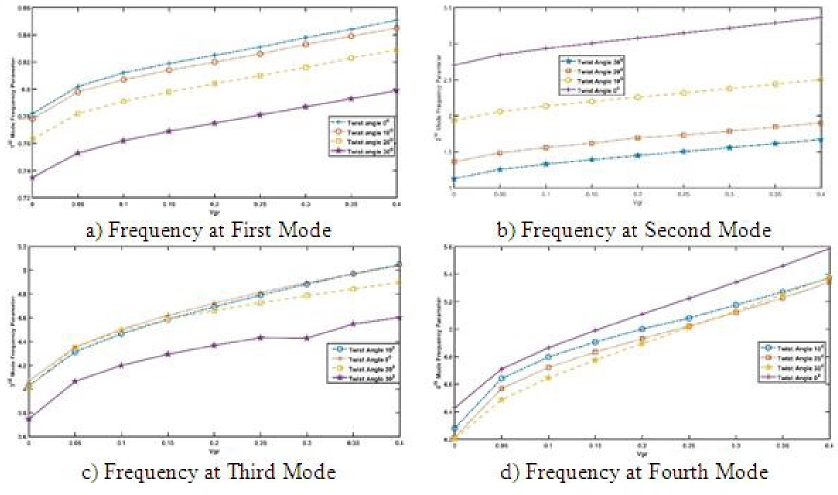


Fig. 2 Variation of four modes of frequency parameter of twisted shell with graphene volume fraction

Graphene with varying volume percentage and cantilever shell with twist angle is shown in Figure 2 to illustrate the first four natural frequencies. For this analysis, twist angles of 0, 10, 20 and 30 degrees were considered. In general, the frequency parameter drops as the twist angle rises. The findings of this study indicate that with an increase in the graphene volume fraction index, there is also an increase in the frequency of the phenomenon observed. As the volume fraction varies from 0 to 0.4, it appears that there is a significant variation in the frequency parameters.

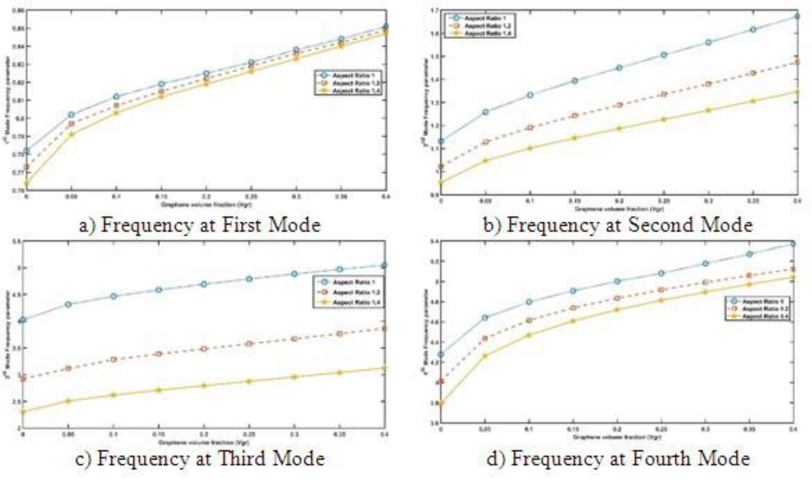


Fig. 3 Four modes of frequency parameter variation with aspect ratio

At different graphene volumes, Figure 3 (a) to (d) illustrates the first four natural frequency parameters that will have a different impact on the aspect ratio of the cantilever plate. With an increase in the graphene volume, there is an increase in the frequency. While increasing the volume of graphene, there is a corresponding increase in stiffness, which will result in a rise in frequency parameter.

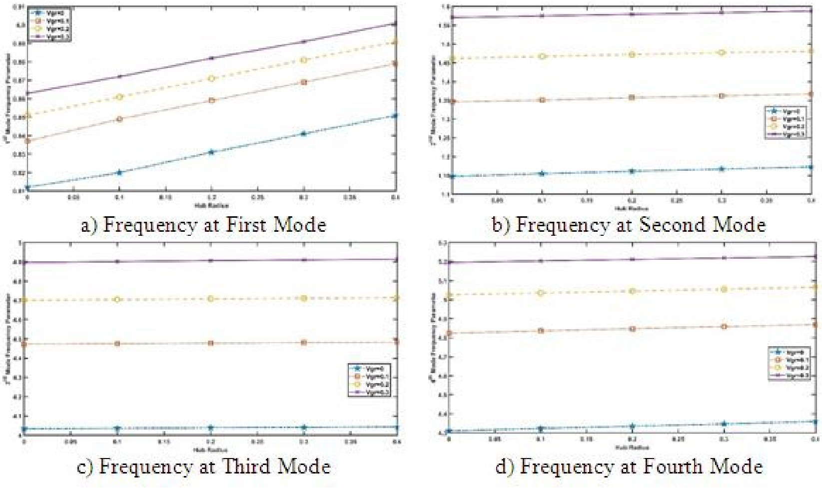


Fig. 4 Influence of graphene volume fraction on first four mode frequency parameter with hub radius

Figure 4 shows the effect of the diameter of the hub and the percentage of graphene on the first four frequencies of the rotating shell. It has an aspect ratio of 1 and a rotational speed of 200 revolutions per minute. There is a correlation between the frequency increase and the increase in the radius of the hub and also with the increase in graphene volume percentage. The increase of graphene volume in hybrid composite increases its stiffness of the pre-twisted shell; due to this the natural frequency parameter increases.

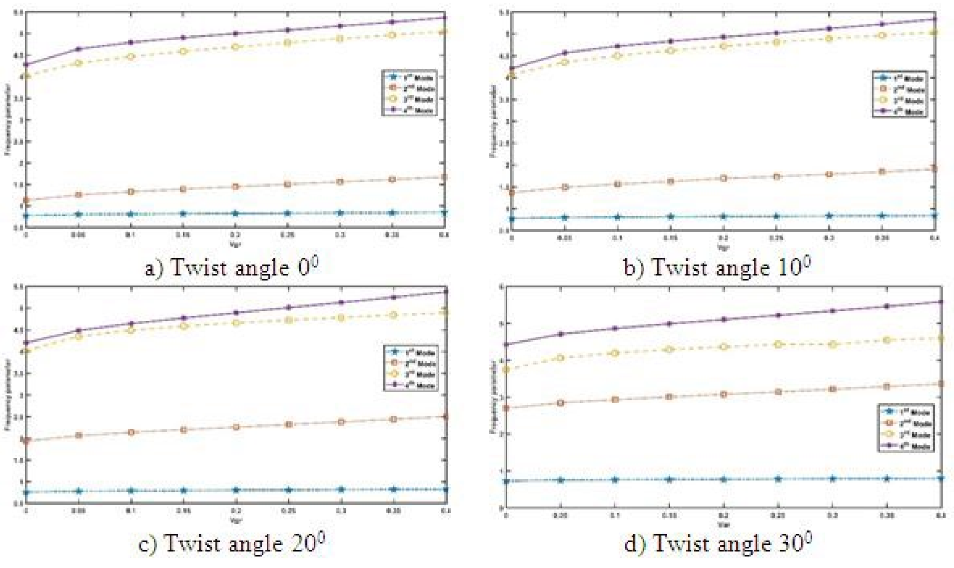
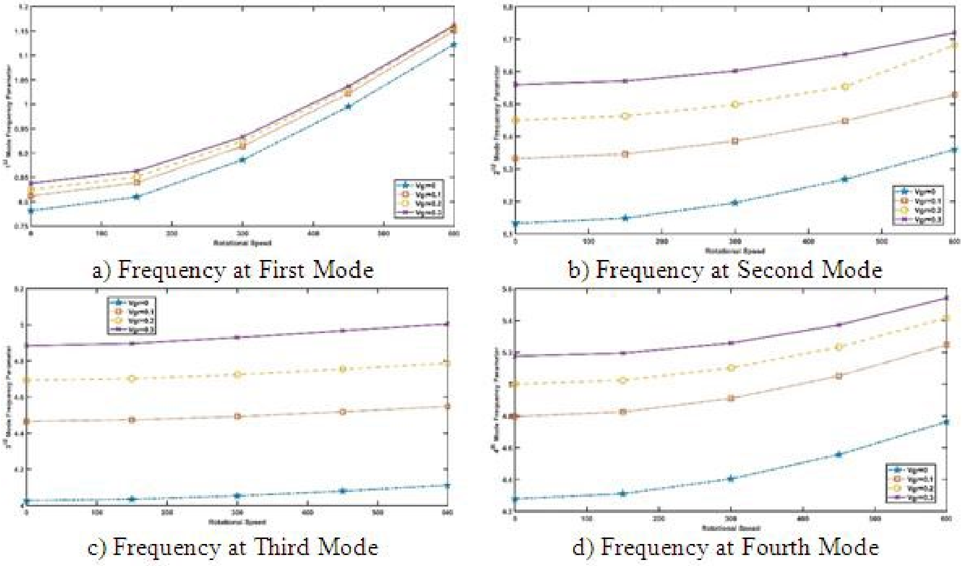


Fig. 5 Effect of graphene on first four mode frequency parameter with twist angles
a) Twist angle 0^0, b) Twist angle 10^0, c) Twist angle 20^0, d) Twist angle 30^0

When considering the transverse vibration components in Figures 5a, 5b, 5c, and 5d in relation to their graphene volume fraction, Figure 5 shows that graphene volume fraction is the most significant factor for transverse vibration components. It has been demonstrated experimentally that the larger the volume of graphene, the higher the frequency of transverse vibrations of graphene for all distinct types of vibrations in graphene. It has been observed that graphene volumes of 0.1, 0.2, 0.3, and 0.4 increase in the frequency parameters of (1,1), (1,2), (2,1), and (2,2) modes with a twist angle of 0, 10, 20, and 30 degrees, respectively.



a) Frequency at First Mode

b) Frequency at Second Mode

c) Frequency at Third Mode

d) Frequency at Fourth Mode

Fig. 6 Graphene volume fraction influences on first four mode frequency parameter with rotational speed

Figure 6 shows that the graphene volume fraction and rotational speed are two factors influencing the transverse vibrations in Figure 6a, Figure 6b, Figure 6c, and Figure 6d. This natural frequency is correlated with graphene's volume fraction, in agreement with earlier descriptions of graphene. All the first four transverse modes have the same effect, and this observation holds true for all of them. As a result of this study, an increase in the frequency parameter of the pre-twisted shells was observed. Because graphene is present in the composite, the frequency of its normalized modes is higher than its basic modes. An increase in the graphene volume of the pre-twisted shell and its rotational speed increases the four frequency parameters.

5. CONCLUSIONS

A rotating twisted plate is analysed using the finite element method in order to determine the free vibration as a result of its rotation. Using the FSDT approach, the paper proposes a simple and straightforward approach to determine the transverse shear strain. With this approach, both the rotary inertia of the shell and the way the transverse shear strain is distributed throughout the thickness of the shell were considered. It was investigated how different parameters such as twist angle, hub radius, aspect ratio, graphene volume, and rotational speed affect the natural frequencies of the shell. When graphene is incorporated into a composite, it raises the natural frequencies significantly. This study shows that when the rotational speed parameter is increased, the natural frequencies of rotating twisted plates increase, which is in accordance with the present study. It is known that as the radius ratio of the hub increases, the frequency of vibration increases as well. It is also imperative to note that along with the aspect ratio, frequency parameters will also increase.

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