Temperature-dependent 2-dimensional porous FG plates free vibration analysis using finite element approach

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Abstract: *This work aims to address the free vibration analysis of two-dimensional porous 2-D FG plates with temperature-dependent properties that are subjected to thermomechanical loading. The revised element finite element model has seven degrees of freedom and has been developed based on higher-order shear deformation theory. The material constitutions in two directions along the length and thickness are varied according to the revised index law of power. The current proposed numerical approach has been evaluated with existing literature results to find good amiability between them. With all side-supported boundary conditions, the effect of thermomechanical loads and porosity allocation on free vibration analysis of temperature-dependent material FG plates is explored. Also, the consequence of consistent and erratic porous portions of free vibration characteristics on 2D FG plates is studied. 2D properties are disbanded, which affects the natural frequency of the FG plate. In this case, the rise in environmental temperature and index values reduces the frequencies of the porous FG plates. Changeable porous distribution on FG plates obtained exemplary frequency characteristics compared with the consistent porous distribution.*

Key Words: *FG material plates, porosity, thermal field, finite element approach, free vibration*

1. INTRODUCTION

Advanced composites based on functionally graded materials consist of two or more components, the direction of which varies along the component direction to produce new materials. These integrated combinations of material percentages are transformed in single or multiple directions, correspondingly. In 1980 the Japanese started the perception of FGM to reduce the thickness of nuclear plants efficiently by Koizumi [1]. Increasing usage of technology and applications may raise complicated mathematical modelling of 2-dimensional FG plates. By changing the ratio between constituents in FG modelling, the thickness and length directions of the model are continually shifted in two-dimensional directions.

Several investigators enlightened the soliciting intensive research inducements in this area with considerable industrial applications of FGM, significantly focused on free vibration, static and dynamic characteristics of FG material modelling with one-dimensional direction. Materials such as FGM are used extensively in high-temperature environments such as engine combustion chambers and nuclear reactors. The structural properties may degenerate due to the operating conditions and manufacturing defects. The source of structural failure may be due to imperfections like porosities degrading the materials during manufacturing.

Accordingly, the study of free vibration analysis of porous 2-D FGM plates is considered consequential in this study.

In this study, some researchers developed different methods to analyze 1D FGM plates' vibration characteristics. Three-dimensional exact analytical solutions were developed by Senthil and Batra [2] for the frequencies of FG rectangular plates. Ferreira et al. [3] established a meshless technique for the study of natural frequency analysis of plates that incorporated a global collection of theories for first-order and third-order shear deformation. According to Using an analytical approach, Hasani Baferani et al. [4] calculated the free vibration of FG thin film plates. As a result of their methodology, they can distinguish between the three coupled partial differential equations of motion, which are reformulated into two decoupled equations by the Navier method. By employing the element-free Ritz method, Zhao et al. [5] analysed the free vibration of FG plates. Hosseini et al. [6] used a first-order shear deformation theory to analyse the free vibration of a rectangular plate with FG boundary conditions. Using the physical neutral surface principle, Zhang and Zhou [7] investigated FG thin plates for their deflection, buckling, and free vibration behavior.

Several studies have been conducted on FGM plates' vibration analysis in recent years. In one of the studies, Reddy and Chin [8] considered the thermomechanical dynamic response of functionally graded plates and cylinders. The mathematical model used advanced finite element methods with thermomechanical coupling. As part of their study of vibration characteristics in a thermal environment, Yang and Shen [9] used a Galerkin approach and modal superposition method. Naghdabadi and Hosseini [10] provided a finite element model of the plate in accordance with the Rayleigh-Ritz method combined with a non-linear heat-transfer equation. According to Young-Wann [11], a vibration analysis of functionally graded rectangular plates with temperature-dependent material properties is based on Rayleigh-Ritz calculations with non-linear heat-transfer equations. Rayleigh-Ritz equations were used to obtain frequency equations. By applying second-order shear deformation theory to functionally graded plates subjected to thermal loads, Shahrjerdi et al [12] performed free vibration analysis of solar panels. A thermomechanical induced vibration of ceramic-metal plates was investigated by Talha and Singh [13] by using finite elements. During the isoperimetric Lagrangian element C0, each node has 13 degrees of freedom.

FGM beams are currently being studied to determine the effect of porosity on vibration characteristics. By Farzad Ebrahimi and Ali Jafari [15], a thermomechanical vibration analysis was performed on FGM beams with porosities. By combining higher-order shear deformation theory with Hamilton's principle, the authors developed the equation of motion. Based on a Navier-type solution, they solved the equations.

According to Farzad Ebrahimi et al [14], compositionally graded Euler beams with porosities were used to study vibration effects of temperature. Further, compositionally graded beams appear to respond differently to temperature variations depending on their porosity volume fraction.

The authors have not encountered any research on vibration analysis of 2D FG material plates from the literature that they have reviewed. FGM plates with porosity were studied with finite element method.

The aim was to achieve an approximate solution that could be compared to free vibration characteristics. In this study, the kinematics of the plate element have been developed based on the theory of higher order shear deformations. In order to solve the governing equation of motion of the plate, Hamilton's principle is applied. By examining a 2D FG plate in a high temperature environment, the formulated solution investigates the effect of porous distribution.

2. FORMULATION OF THE PROBLEM

A nonlinear equation is considered to analyze the temperature-dependent properties, which can be represented as

$$Q = Q_0(Q_{-1}Q^{-1} + Q_1T + Q_2T^2 + Q_3T^3 + 1) \tag{1}$$

where, T indicates temperature, Q_0, Q_{-1}, Q_1, Q_2, and Q_3 are the coefficients of temperature-dependent material properties.

2.1 Mathematical formulation of 2-Dimensional FGM plates

As shown in Figures 1(a, b) and 1(c), the plate with FG porous distribution and changing metal and ceramic distribution is considered for this analysis. A Voigt model determines the changing material properties (P) for 2D FGM plates as the young's modulus, density, etc.

$$P(x, z, Q) = P_m(Q)\left(m_v(x, z) - \frac{\varsigma}{2}\right) + P_c(Q)\left(c_v(x, z) - \frac{\varsigma}{2}\right) \tag{2}$$

As in Shafiei et al. [16] ceramic quantity fraction can be obtained by simple power law.

$$c_v(x, z) = \left(\frac{1}{2} + \frac{z}{t}\right)^{xl}\left(\frac{x}{L}\right)^{zl} \tag{3}$$

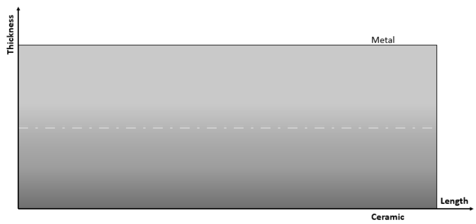

Fig. 1 (a) Thickness direction distribution of material

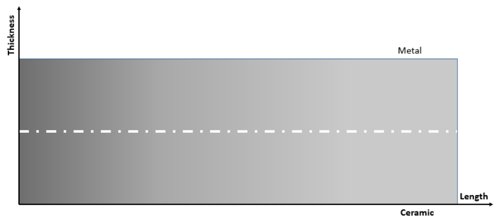

Fig. 1(b) Length direction material distribution

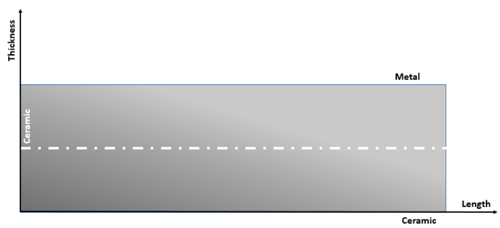

Fig. 1 (c) Thinness and length distributions of material properties.

A porous FGM-1 is characterized by an even distribution of porosity, whereas porous FGM-2 is characterized by an irregular distribution, as shown in figures 2a and 2b.

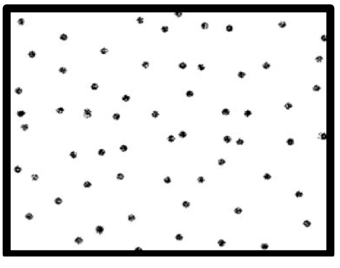

Fig. 2 (a) A 2D FG plate with even porosity distribution

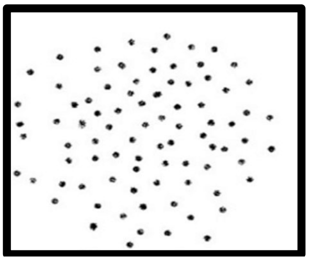

Fig. 2 (b) A 2D FG plate with uneven porosity distribution

Accordingly, the above equation young's modulus 'E', Poisson's ratio 'ϑ', the thermal conductivity 'α', and mass density 'ρ' of porous 2D FGM-I plate is applied as

$$\alpha(x,z)=\alpha_m(Q)+\big(\alpha_c(Q)-\alpha_m(Q)\big)v_c(x,z)-\frac{\varsigma}{2}\big(\alpha_c(Q)+\alpha_m(Q)\big) \tag{4}$$

Consequently, Young's modulus 'E', Poisson's ratio 'ϑ', the thermal conductivity 'α', mass density 'ρ' can be expressed for porous FGM-II as

$$\alpha(x,z)=\alpha_m(P)+\big(\alpha_c(P)-\alpha_m(P)\big)v_c(x,z)-\frac{\varsigma}{2}\big(\alpha_c(P)+\alpha_m(P)\big)\left[1-\frac{2|z|}{t}\right] \tag{5}$$

3. MATHEMATICAL FORMULATION

3.1 Basic kinematics

Based on Reddy's equations, the third-order shear deformation theory is used to develop basic kinematics of plate structures.

$$\begin{gathered} u=u_n+z\theta_x-c_1z^3\big(\theta_x+w_{n,x}\big),\\ v=v_n+z\theta_y-c_1z^3\big(\theta_y+w_{n,y}\big)\\ w=w_n \end{gathered} \tag{6}$$

Strain-displacement constitutive law relations on the neutral axis can be expressed as follows:

$$\{\varepsilon^{bd}\}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^{(nx)}\\ \varepsilon_y^{(n)}\\ \gamma_{xy}^{(nx)}\end{Bmatrix}+z\begin{Bmatrix}\varepsilon_x^{(1)}\\ \varepsilon_y^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix}-z^3\begin{Bmatrix}\varepsilon_x^{(3)}\\ \varepsilon_y^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix} \tag{7}$$

$$\{\gamma^{sh}\}=\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\gamma_{yz}^{(nx)}\\ \gamma_{xz}^{(nx)}\end{Bmatrix}+z^2\begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix} \tag{8}$$

here $c_1=\frac{4}{3\square^2}$ and $c_2=3c_1$.
Taking the stress-strain relationship of the functionally graded plate as a 2D plane, one can describe it as follows:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}q_{11}&q_{12}&0&0&0\\ q_{21}&q_{22}&0&0&0\\ 0&0&q_{44}&0&0\\ 0&0&0&q_{55}&0\\ 0&0&0&0&q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{9}$$

where, $q_{11}=q_{22}=\frac{E(x,z)}{(1-v^2(x,z))}$, $q_{12}=q_{21}=\frac{E(x,z)v(x,z)}{(1-v^2(x,z))}$, $q_{44}=q_{55}=q_{66}=\frac{E(x,z)}{2(1+v(z,x))}$

3.2 Finite Element Approach

In this study, a four-nodded quadrilateral with one node at each corner is considered to have five degrees of freedom.

It is possible to observe two in plane displacements along x and y axes, as well as one transverse displacement in the thickness direction and rotations about x and y axes. The displacement vector of an element $\delta^{(n)}$ is written as

$$\left\{\delta^{(n)}\right\} = \sum_{i=1}^{n} N_i\, \delta_i, \tag{10}$$

where $\left\{\delta^{(n)}\right\} = \left\{u, v, w, \theta_x, \theta_y\right\}$, nodal displacement vector of an element

$$\left\{\delta^{(e)}\right\} = \left\{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}\right\}_{i=1,2,3,4} \tag{11}$$

The matrix of shape functions is follows as

$$[N] = \left\{\left[N_{u_n}\right]\left[N_{v_n}\right]\left[N_{w_n}\right]\left[N_{\theta_x}\right]\left[N_{\theta_y}\right]\right\}^T \tag{12}$$

Nodal displacement vector can be used to express strain vectors $\left\{\delta^{(e)}\right\}$ as follows

$$\{\varepsilon^{bd}\} = [B_{bd}]\left\{\delta^{(e)}\right\} \tag{13}$$

$$\{\gamma^{ss}\} = [B_{ss}]\left\{\delta^{(e)}\right\} \tag{14}$$

As an example of strain energy, consider the following:

$$U_{PE}^{(e)} = \frac{1}{2}\int_0^l \int_0^b \left[\left\{\delta^{(e)}\right\}^T \left(\left[K_b^{(e)}\right] + \left[K_s^{(e)}\right]\right)\left\{\delta^{(e)}\right\}\right] dxdy \tag{15}$$

As a kinetic energy of an element is

$$V_{KE}^{(e)} = \frac{1}{2}\int_v \rho(z)(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)dxdydz \tag{16}$$

The velocities $\dot{u}, \dot{v} and \dot{w}$ as a shape function and nodal velocity vector, are represented as follows

$$\dot{u} = \left[\left[N_{u_n}\right] + z\left[N_{\theta_x}\right] - c_1 z^3\left(\left[N_{\theta_x}\right] + \left[N_{\frac{\partial w}{\partial x}}\right]\right)\right]\{\delta^{(e)}\}^T,$$

$$\dot{v} = \left[\left[N_{v_n}\right] + z\left[N_{\theta_y}\right] - c_1 z^3\left(\left[N_{\theta_y}\right] + \left[N_{\frac{\partial w}{\partial y}}\right]\right)\right]\{\delta^{(e)}\}^T \tag{17}$$

$$\dot{w} = \left[N_{w_n}\right]\{\delta^{(e)}\}^T$$

As a result of the element kinetic energy ($T^{(e)}$)

$$V_{KE}^{(e)} = \frac{1}{2}\int_0^l \int_0^b \left[\left\{\delta^{(e)}\right\}^T \left[M^{(e)}\right]\left\{\delta^{(e)}\right\}\right] dxdy \tag{18}$$

3.3 Thermal environment condition

To conduct an analysis, the temperature gradient should vary nonlinearly in the thickness direction.

A non-linear temperature gradient z-direction is presumed to be associated with the simple power function.

$$T = T_a + \Delta T \left(\frac{1}{2} + \frac{z}{t}\right)^{\iota\alpha} \tag{19}$$

When using FGM, it is necessary to consider nonlinear variations of material properties and nonlinear heat distribution conditions, especially in high temperature applications.

3.4 Numerical Equation

Calculating the fundamental calculation of equation entails applying Hamilton's principle.

$$\delta \int_{t_1}^{t_2} \left(U_{PE}^{(e)} - V_{KE}^{(e)}\right) dt = 0 \tag{20}$$

Using the nodal displacement vector for a particular element, the energy of kinetic and potential of the plate can be articulated below:

$$U_{PE}{}^{(e)} = \frac{1}{2}\{\delta^{(e)}\}^T [K^{(e)}]\{\delta^{(e)}\} \tag{21}$$

$$V_{KE}{}^{(e)} = \frac{1}{2}\{\dot{\delta}^{(e)}\}^T [M^{(e)}]\{\dot{\delta}^{(e)}\} \tag{22}$$

According to the temperature-dependent porous FGM plate element in matrix form moves as follows:

$$[M^{(e)}]\{\ddot{\delta}^{(e)}\} + \left[K^{(e)}\right]\{\delta^{(e)}\} = 0 \tag{23}$$

We could substitute the displacement vector, stiffness matrix, and mass matrix of plate in the equation above to determine the following equation:

$$[[K] - \omega_n{}^2[M]]\{\delta\} = 0 \tag{24}$$

4. RESULTS AND DISCUSSIONS

4.1 Numerical validation

Temperature dependent material properties are considered from the Reddy and Chin [8] for this numerical study.

To ensure that the numerical technique works, a natural frequencies analysis of a temperature-dependent FG material plate is solved.

Before calculating the vibration characteristics of the temperature dependent FGM plate in a thermal environment, it is necessary to validate the derived formulation.

In Table 1, the comparison of the frequencies of one-dimensional FG plates found from this study and that obtained by Huang and Shen [17], is performed based on the law of power variation for material properties disbandment as studied in this study.

In an effort to authenticate the numerical correctness of their method, the authors used the published consequences of Huang and Shen [17] to determine the vibration characteristics of the temperature-dependent materials plates.

According to the authors, the numerical results in table 1 are comparable to those reported in the literature.

Frequency parameter $\Omega = \overline{\omega}(a^2/\square)\sqrt{(\rho_0(1-\nu^2)/E_0)}$

Table. 1 Fundamental frequency parameters of simply supported temperature-dependent ZrO2/Ti–6Al–4V plate

	ZrO2		0.5		1		2		Ti–6Al–4V	
Mode	Ref. [16]	Present	Ref. [16]	Present	Ref. [16]	Present	Ref. [16]	Present	Ref. [16]	Present
(1,1)	7.86	7.9	6.87	6.94	6.43	6.51	6.1	6.09	5.32	5.28
(1,2)	18.65	18.66	16.26	16.37	15.2	15.36	14.37	14.38	12.45	12.47
(2, 2)	28.2	28.16	24.57	24.7	22.95	23.16	21.65	21.68	18.76	18.81
(1, 3)	34.01	34.68	29.65	30.4	27.69	28.51	26.11	26.69	22.6	23.17
(2, 3)	42.04	41.56	36.66	36.34	34.23	34.01	32.23	31.82	27.92	27.76

4.2 The effect of equal and asymmetrical porosities distribution on natural frequencies

Two-dimensional FG plates have a much different distribution of thickness and axial dimensions than one-dimensional FG plates. For this study, a square plate index values of axial $a_x = 1$, thickness $a_z = 1$, porosity 0.2 and with a/h=10 is considered. Table 2 describes the frequency parameters of temperature-dependent material two-dimensional distribution of porous plate. The first four frequencies of 2D FG plates are compared with even and uneven porous distribution. By examining table 2 in further detail, it becomes clear that it is the frequency parameters of even porous distribution that are most sensitive to temperature variations at various temperatures.

Table. 2 Frequency parameter of even and uneven porosities (0.2) of a temperature-dependent material plate in a thermal environment with simply supported boundary condition.

Temperature change	Porosities	Frequency parameter			
		1	2	3	4
0	Even	3.14	6.57	9.56	12.40
	Uneven	3.16	6.63	9.64	12.50
200	Even	3.10	6.48	9.45	12.25
	Uneven	3.13	6.55	9.52	12.35
400	Even	3.07	6.43	9.35	12.13
	Uneven	3.09	6.48	9.43	12.23
600	Even	3.04	6.38	9.23	12.03
	Uneven	3.07	6.43	9.35	12.13
800	Even	3.02	6.33	9.21	11.95
	Uneven	3.03	6.34	9.22	11.96

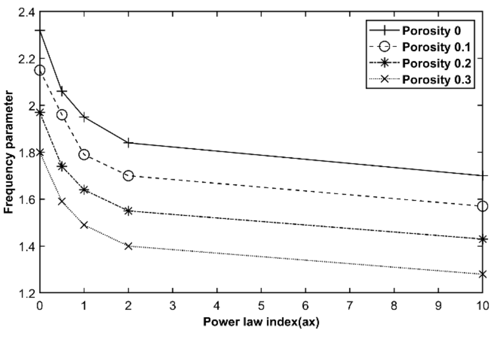

Fig. 3 (a) The first mode's variation with uniform porosity disbandment influences on first fundamental frequency

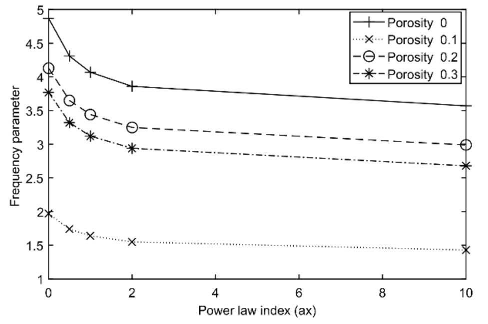

Fig. 3(b) The second mode's variation with uniform porosity disbandment influences on second fundamental frequency

This figure displays the disparity of the first two-mode fundamental frequencies concerning the axial grading indexes as shown in Figures 3(a) and 3(b). Different porosities are used in these figures to illustrate the variation of the frequency parameters. Despite constant index values, the consequence of the even dispersal of porosity on the frequency parameter slightly decreases with increasing porosity (0, 0.1, 0.2, 0.3). It is due to the increase of porosity volume fraction on the 2D FG plate that may cause a decrease in the volume fraction of FG content. Decreased FG content reduces the stiffness of the plate.

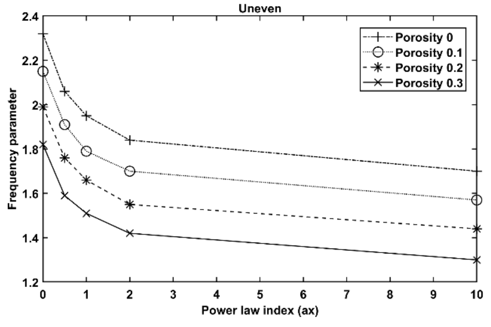

Fig. 4(a) The influence of unequal porosity disbandment on the first fundamental frequency.

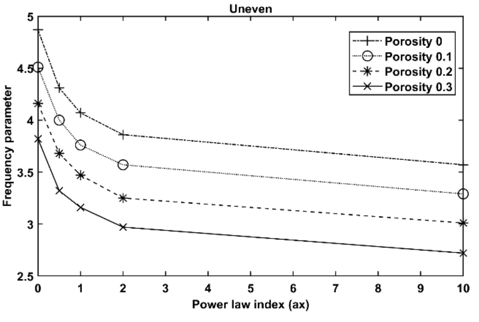

Fig. 4(b) The influence of unequal porosity disbandment on the second fundamental frequency.

The influence of porosity distribution on mode frequency parameters is depicted in Figures 4 (a) and (b). Increasing the axial index value of the 2D FG plate decreases both first and second mode frequency parameters. Thus, by raising the uneven porous volume fraction, we could decrease not only the frequency parameter but also the overall stiffness of a porous two-dimensional FG plate.

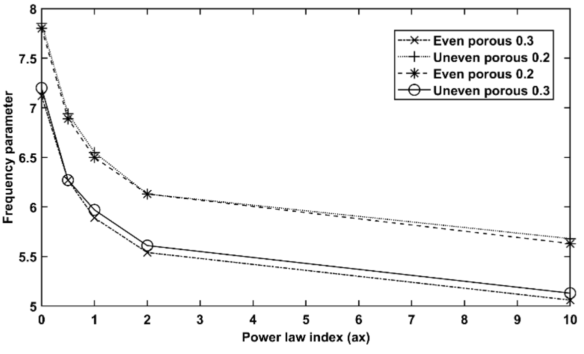

Fig. 5 The frequency parameter variation of a plate's distribution of uniform and uneven porosities.

The difference between the even and uneven porosity fractions of 0.2 and 0.3 is shown in figure 5 for a variety of axial index values. Temperature conditions and thickness direction indices are also maintained as constants in this case. The frequency parameter may be calculated by applying the identical thermal situation to the equal and uneven porous volume fractions. Frequency settings can be adjusted for uniform and uneven porous surfaces. As observed in Figure 5, the frequency of uneven porous distribution volume fraction of the two-dimensional FG plate is higher than that of the even volume fraction of the porous plate.

4.3 Natural frequencies of porous 2D FG plate in thermal fields

The influence of the thermal environment on the free vibrations of the 2D FG plate through various amalgamations of power-law axial and thickness index values are described in table 3. The increase in temperature reduces the frequency parameter of even and uneven porous

2D FG plate, sequentially. Furthermore, the even porous condition is more influenced by the frequency parameter compare with an uneven porous case.

Table. 3 Frequency parameters of simply supported temperature-dependent material porous plate with various combinations (x_a, z_a) of index values.

Temperature change	Porosities	Length and thickness index values (x_a, z_a)			
		(0,0)	(1,0)	(1,1)	(10,10)
0	Even	3.14	1.98	1.65	1.39
	Uneven	3.16	1.99	1.66	1.40
200	Even	3.10	1.96	1.63	1.38
	Uneven	3.13	1.97	1.64	1.38
400	Even	3.07	1.93	1.60	1.35
	Uneven	3.09	1.93	1.62	1.36
600	Even	3.04	1.90	1.57	1.31
	Uneven	3.07	1.91	1.57	1.32
800	Even	3.02	1.86	1.52	1.25
	Uneven	3.03	1.87	1.53	1.26

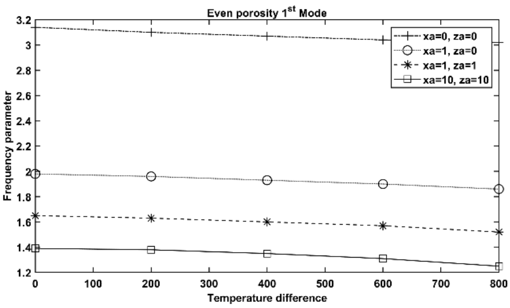

Fig. 6 (a) Change of even porosity on FGM plate, the first mode frequency characteristic variation with a temperature rise.

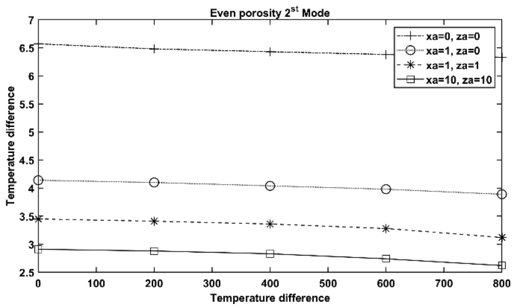

Fig. 6 (b) Change of even porosity on FGM plate, the second mode frequency characteristic variation with a temperature rise.

The variation of axial and thickness index values of the even porous plates are considered ceramic (0, 0), axial FG (1, 0), 2D FG (1, 1) & 2D FG (10, 10) plates. These even volume fraction (0.2) porous plates are influenced by the rise in temperature as revealed in figure 6. The rise of temperature reductions the first and second mode non-dimensional parameters of the even porous 2D FG plates are depicted in figure 6 (a) and (b), correspondingly. It explains that the upsurge in temperature difference reduces the even porous 2D FG plate frequency. The high thermal environment weakens the properties of material even porous plates and it begins to decrease the frequency parameter.

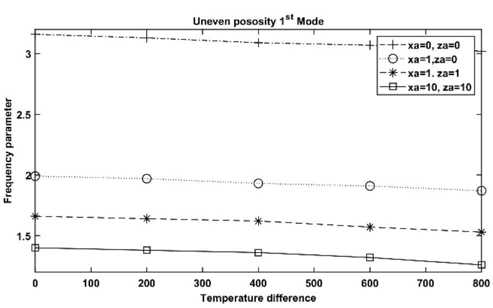

Fig. 7(a) Uneven distributed porosity of FG material plate with temperature for fundamental frequency

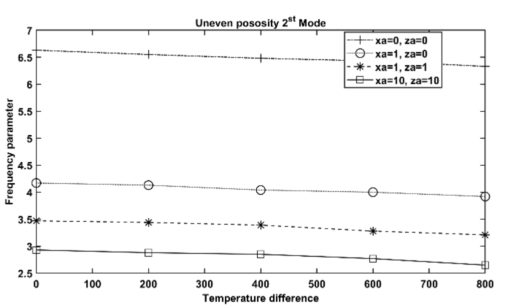

Fig. 7(b) Uneven distributed porosity of FG material plate with temperature for second fundamental frequency

The first and second parameter of frequency variation by the addition of temperature change of the uneven volume fraction porous plates are represented in Figures 7(a) and (b). In this case, also the temperature rise reduces the uneven porous plate frequencies of the first and second modes.

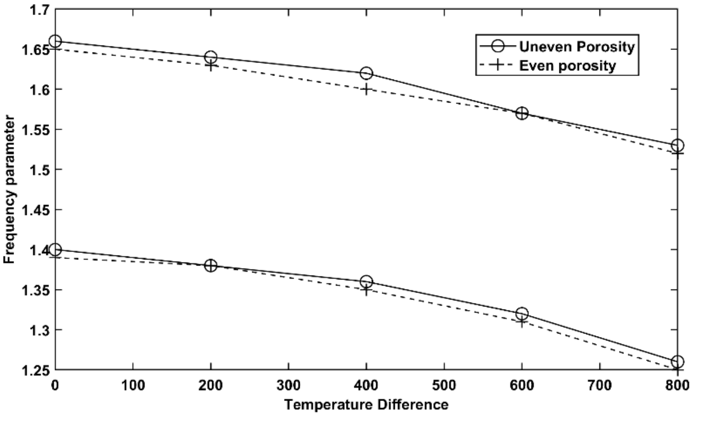

Fig. 8 The natural frequency of 2D FG plates by uniform and irregular porosities varies with temperature.

The effect of temperature change on a porous 2D FG plate with uniform and uneven conditions is shown in Figure 8. As the temperature rises, the frequency of non-dimensional zones decreases. Furthermore, it is found that even in porous conditions less natural frequencies exist by comparing uneven porous conditions. According to this, 2D FG plates can be studied in depth when the porous structure is uneven.

5. CONCLUSIONS

Using finite element analysis, a numerical solution for free vibration analysis of porous FG plates is developed using higher-order shear deformation theory. The present study analyzes the distribution of porous materials on FG plates in even and uneven conditions. The proposed finite element numerical approach implied that the parameters of frequencies are in good agreement with those published in research papers. The influences of even and uneven distributions of porosity, axial and thickness directions of the distribution of temperature-dependent properties of temperature-dependent materials, and the presence of high temperatures have been examined for 2D FG plates. As porosity concentration increases, the natural frequencies of 2D FG plates decrease, as has been observed in this study. Also, the frequency parameters slightly decrease with temperature. Furthermore, the uneven porous distribution influences less significantly the non-dimensional frequencies of the plate.

REFERENCES

[1] M. Koizumi, FGM activities in Japan, *Composites Part B: Engineering*, vol. **28**, pp. 1-4, 1997.
[2] Senthil S. Vel and R.C. Batra, Three-dimensional exact solution for the vibration of functionally graded rectangular plates, *Journal of Sound and Vibration,* vol. **272**, pp. 703–730, 2004.
[3] A. J. M. Ferreira, R. C. Batra, C. M. C. Roque, L. Q. Qian, R. M. N. Jorge, Natural frequencies of functionally graded plates by a meshless method, *Composite Structures,* vol. **75**, pp. 593–600, 2006.
[4] A. Hasani Baferani, A. R. Saidi, and E. Jomehzadeh, An exact solution for free vibration analysis of functionally graded rectangular plates, *Proc. IMechE, Part C: J. Mechanical Engineering Science*, vol. **225**, pp. 526-536, 2009.

[5] X. Zhao, Y. Y. Lee, K. M. Liew, Free vibration analysis of functionally graded plates using the Element-free kp-Ritz method, *Journal of Sound and Vibration,* vol. **319**, pp. 918–939, 2009.
[6] Sh. Hosseini-Hashemi, Rokni Damavandi Taher, H. Akhavan, M. Omidi, Free vibration of functionally graded rectangular plates using first-order shear deformation plate theory, *Applied Mathematical Modelling,* vol. **34**, pp. 1276–1291, 2010.
[7] D. G. Zhang, Y. H. Zhou, A theoretical analysis of FGM thin plates based on physical neutral surface, *Comp. Mater. Sci.,* vol. **44**, pp. 716–720, 2008.
[8] J. N. Reddy, and C. D. Chin, Thermomechanical analysis of functionally graded cylinders and plates, *Journal of Thermal Stresses*, vol. **21**, pp. 593-626, 1998.
[9] Y. Yang, and H. S. Shen, Vibration characteristics and transient response of shear-deformable functionally graded plates in thermal environments, *Journal of Sound and Vibration*, vol. **255**(3), pp. 579-602, 2002.
[10] R. Naghdabadi, S. A. Hosseini Kordkheili, A finite element formulation for analysis of functionally graded plates and shells, *Archive of Applied Mechanics*, vol. **74**, pp. 375-386, 2005.
[11] Y.-W. Kim, Temperature dependent vibration analysis of functionally graded rectangular plates, *Journal of Sound and Vibration*, vol. **284**, pp. 531-549, 2005.
[12] A. Shahrjerdi, F. Mustapha, M. Bayat, and D. L. A. Majid, Free vibration analysis of solar functionally graded plates with temperature dependent material properties using second order shear deformation theory, *Journal of Mechanical Science and Technology*, vol. **25**(9), pp. 2195-2209, 2011.
[13] M. Talha, and B. N. Singh, Thermo-mechanical induced vibration characteristics of shear deformable functionally graded ceramic-metal plates using the finite element method, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science,* vol. **225**, pp. 50-65, 2011.
[14] F. Ebrahimi, F. Ghasemi, E. Salari, Investigating thermal effects on vibration behavior of temperature-dependent compositionally graded Euler beams with porosities, *Meccanica,* vol. **51**, pp. 223–249, 2016.
[15] F. Ebrahimi and A. Jafari, A Higher-Order Thermomechanical Vibration Analysis of Temperature-Dependent FGM Beams with Porosities, *Journal of Engineering*, vol. **2016**, pp. 1-20, 2016.
[15] A. S. Rezaei, and A. R. Saidi, Exact solution for free vibration of thick rectangular plates made of porous materials, *Composite Structures,* vol. **134**, pp. 1051–1060, 2015.
[16] N. Shafiei, M. Kazemi, M. Safi, Nonlinear vibration of axially functionally graded non-uniform nanobeams, *International Journal of Engineering Science*, **106**(1), pp. 77–94, 2016.
[17] X.-L. Huang, H.-S. Shen, Nonlinear vibration and dynamic response of functionally graded plates in thermal environments, *International Journal of Solids and Structures*, vol. **41**, pp. 2403–2427, 2004.