A theory on understanding aerodynamic phenomena of airfoils and the significance of airfoil's thickness on lift and drag

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Abstract: *This paper proposes a new theory explaining aerodynamic phenomena of airfoils. The theory is based on the application of Newton's classical mechanics rather than differential equations of fluid dynamics. The approach in developing the equations contains both geometrical and fluid dynamics perspectives of motion of body in a fluid. Based on the theory, it is shown that new airfoil designs could be generated through the application of "contraction" and subsequent "expansion" in the geometry for lift generation. The effect of thickness of airfoil is important in the context of lift and drag and further investigation on its effect on airfoil aerodynamics is conducted. The obtained results are reported and discussed.*

Key Words: *airfoil theory, airfoil design, aerodynamics, airfoil thickness, lift, drag*

1. INTRODUCTION

One of the most important aerodynamic theories in the last century was proposed by Professor Ludwig Prandtl of Germany in 1904 [1]. Prof. Prandtl published his paper on "Motion of fluids with very little viscosity" at the International Mathematical Congress, Heidelberg which later became known as the Boundary Layer theory. According to this theory, the physical quantity of interest in the (boundary) layer between fluid and the body is the velocity. Prandtl observed that there are high velocity differences in the thin transition layer and this is significant even in the case of low viscous fluids. In addition, flow separation around bodies is reasoned as due to the pressure increase along the surface in the direction of flow. The transition layer imparts a characteristic "impress" or pressure to the free flow through the "emission" of turbulence. Prandtl along with his students Albert Betz and Max Munk further contributed to theoretical aerodynamics through the introduction of lifting line theory which is based on the concept of circulation and Kutta-Joukowski theorem. The theory predicts the lift distribution over a 3D wing. According to this theory, a bound vortex along the wing span loses strength and it is shed as a vortex sheet from the trailing edge. The thin airfoil theory was proposed by Max Munk and was developed by Hermann Glauert. According to this theory, the airfoil is described numerically by setting the vortices along the mean camber line of an airfoil section. This arrangement forms a vortex sheet which is placed along the chord line. In addition, the strength of the vortex sheet is balanced that when the uniform stream is superimposed on this vortex sheet, the camber line turns into a streamline. The Kutta condition is fulfilled by this aforementioned flow configuration.

All the above three theories were introduced at least nearly a century ago. There are not many aerodynamic theories published in open literature. In this paper, a simple fluid flow theory is proposed which aims to understand the aerodynamic phenomena of lift and drag generation airfoils. Through a combination of approaching from both geometrical and fluid dynamics perspective, equations for flow along with lift and drag equations are proposed. By applying this theory, new aerodynamic geometries could be generated which is also described in the forthcoming Section 2.

2. THEORY

This section proposes a new theory for streamlined bodies (airfoil) travelling in fluids of low viscosity.

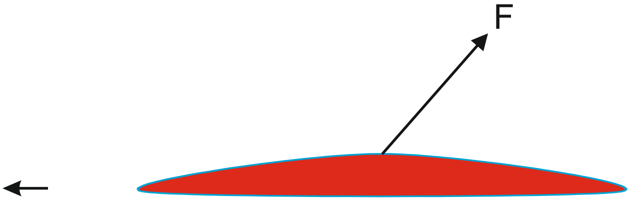

Fig. 1 – Body (airfoil) moving in fluid with velocity *V* generates a force vector, *F* with magnitude and direction

Let's imagine a body travelling at a velocity V in a fluid of small viscosity, μ. The body due to its particular shape generates a force that acts in a certain direction. This is illustrated in Fig. 1. An assumption that the fluid flow is incompressible is made.

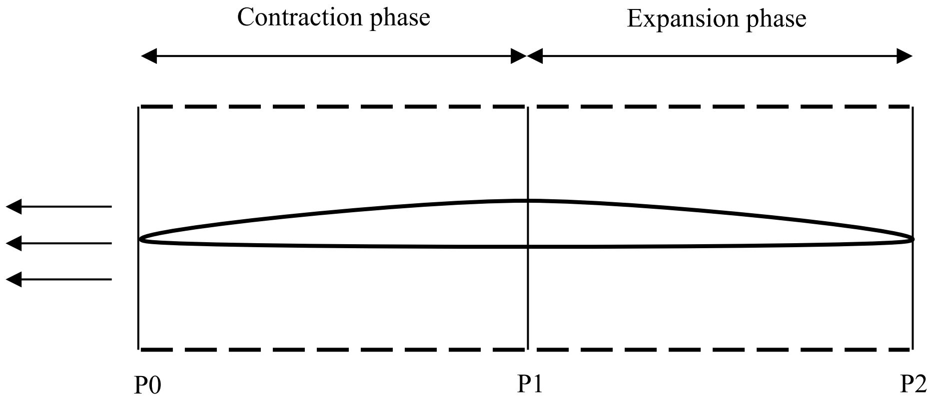

Fig. 2 – Theoretical basis of working of an airfoil [2]. A contraction phase exists between Plane 0 (P0) and Plane 1 (P1) and an expansion phase between Plane 1 and Plane 2 (P2). Assuming incompressible flow around the airfoil, Equation 1 is satisfied

Now, consider Fig. 2. If the velocity of the body is steady, then the volume of fluid starting at Plane 0 and exiting at Plane 2 is constant. This can be expressed as per Equation 1 [2]:

$$v = \frac{m}{\rho} = constant \tag{1}$$

where v is the volume, m is the fluid mass and ρ is the density. Equation 1 applies to bodies of finite length and not too large.

Assuming negligible viscous forces due to friction, the change in velocity and pressure can be related by Bernoulli's equation as in Equation 2:

$$p + \frac{1}{2}\rho V^2 = constant \tag{2}$$

The fluid at reference Plane 0 has the following physical properties: density (ρ), velocity (V), pressure (p), viscosity (μ) and temperature (T) maintained unmodified. Assuming equilibrium at the top surface when the motion of body in the contraction phase reaches Plane 1 after traversing infinitesimal distance l, ρ, T, μ of fluid will remain unchanged while the pressure and velocity are varied. The velocity of the fluid particle along a streamline passing through plane 1 will have higher velocity than at Plane 0. Similarly, the pressure acting on a fluid particle is less at Plane 1 than at Plane 0. This can be expressed as per the notation provided in Equation 3.

$$\begin{aligned} &A_1<A_0 \\ &V_1>V_0 \\ &p_1<p_0 \end{aligned} \tag{3}$$

In the expansion phase, between Plane 1 and 2, the inverse phenomenon takes place. The velocity of the fluid particle decreases whereas pressure on the fluid particle increases at Plane 2 than at Plane 1. This can be expressed as per the notation provided in Equation 4.

$$\begin{aligned} &A_2>A_1 \\ &V_2<V_1 \\ &p_2>p_1 \end{aligned} \tag{4}$$

A similar phenomenon can be observed in the bottom surface of the airfoil. However, the fluid pressure at Plane 1, velocity is reduced lesser than at the top. This causes the fluid to move at slightly higher speeds than at the top surface and at Plane 2, the fluid exits at slightly higher speeds due to subsequently reduced expansion. The amount of expansion is slightly lower at the lower surface than at the top surface.

The pressure difference or gradient, p_l–p_u, contributes to the force F. This can be expressed as per Equation 5:

$$F = \Delta p.S \tag{5}$$

where S is the area of the surface body. Resolving F in the normal and tangential direction provides the lift (L) and drag (D). L can only be calculated through mathematical integration as per Eqn. 6.

$$L = \frac{1}{(x_{TE} - x_{LE})} \int_{x_{LE}}^{x_{TE}} (p_l - p_u).S.dl \tag{6}$$

where l represents the chord length, (x_{TE} - x_{LE}) represents the chord length in the co-ordinate system and p_l–p_u, is the pressure difference or gradient contributing to lift which is shown in Eqn.5.

As opposed to lift, drag is a force that can be attributed to fluid viscosity. This would imply that the drag force will be zero, if the motion of an airfoil is assumed to be in a perfect fluid. In real fluids, the viscosity causes resistance which is drag. When the velocity increases, the fluid particles exert different drag force. As per Edme Mariotte and Newton's law [3], the magnitude of aerodynamic forces generated by the airfoil is proportional to the square of the fluid velocity. The profile shape has caused a pressure gradient to develop due to the incurrence of fluid velocity differences due to geometric variation between the upper and lower surfaces.

The flow field in the immediate vicinity of the moving body can be discretized into small areas distributed with connected points. Considering points on either surface (upper and lower) of the body, the velocity of the particle at the suction side is higher than at the pressure side in Plane 1. The pressure acting on the particle at the suction side is less than at pressure side in Plane 1. This is expressed in Equations 6 and 7:

$$\begin{gathered} A_{1,s}<A_{1,p} \\ V_{1,s}>V_{1,p} \\ p_{1,s}<p_{1,p} \end{gathered} \tag{7}$$

Similarly, the velocity of the particle at the suction side is less than at pressure side in Plane 2, whereas the pressure acting on the particle at the suction side is higher than at pressure side in Plane 1. At Plane 2,

$$\begin{gathered} A_{2,s}<A_{2,p} \\ V_{2,s}>V_{2,p} \\ p_{2,s}<p_{2,p} \end{gathered} \tag{8}$$

The aforementioned discussions in a way will satisfy the following relation:

$$A_1[\rho_1 V_1^2 + p_1] = [\rho_2 V_2^2 + p_2]A_2 \tag{9}$$

For incompressible flow, Equation 9 can be re-written as,

$$A[\rho V^2 + p] = constant \tag{10}$$

The underlying concept in the aforementioned discussions is based on the assumption that the motion of the body is steady. The force generated by an airfoil in real world is transient in nature. To explain unsteadiness, I would like to resort to the following method. According to Newton [4], an external applied force induces acceleration of the body in the direction of motion.

$$F_{ext} = ma \tag{11}$$

The acceleration of the body can be further described in terms of velocity change over time as,

$$a = \frac{dV}{dt} \tag{12}$$

The velocity change induces an unsteady *F* that is described in Equation 5. Many other scientists over the years have proposed different theories to explain an unsteady airfoil. Here, it is assumed that the variation of velocity provides an explanation of the cause of unsteadiness. It thus provides an explanation for motion of oscillating bodies if the velocity vector is considered perpendicular to the direction of motion. Thus, should the fluid is maintained at zero velocity, the following condition will be satisfied:

$$\frac{dF_{ext}}{dt} = \frac{d\left(m\frac{dV}{dt}\right)}{dt} = m\frac{d}{dt}\left(\frac{dV}{dt}\right) \tag{13}$$

where $\frac{dV}{dt}$ is small in magnitude. It will be stressed that although the above equation provides the desired physical reasoning, it may not provide the model which addresses real flight.

A summary of the commonalities and differences existing between Prandtl's theory and the proposed theory is provided in Table 1.

Table 1 – Commonalities and differences between Prandtl's theory and the proposed theory

	Element	**Prandtl's theory**	**Proposed theory**
Commonalities	Fluid property	Incompressible	Incompressible
Differences	Fluid	Viscous	Includes motion in inviscid and viscous fluids
	Assumption	Fluid dynamics differential equations	Newton's mechanics
	Novelty	Continuity assumption (existent)	Own formulation of constant volume
	Type	Experimental	Theoretical
	Basis	Mathematical	Physical

The application of the proposed theory will be subsequently discussed. Different lift-generating geometries could be generated based on contraction and subsequent expansion conceptualization. Examples of such geometries are provided in Fig. 3(a)-(d). The derived concepts from this method can be combined with existing methods like gradient based or heuristic optimization algorithms [5], [6] or inverse design method as mentioned in [7] to generate optimized designs.

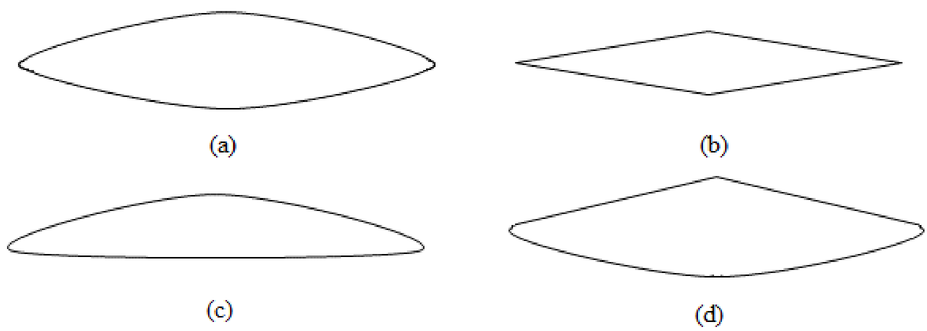

Fig. 3 – Generation of different lift-generating concepts from the proposed theory. Symmetric concept: (a) and (b); Asymmetric concept: (c) and (d)

Assume a symmetrical contraction-expansion airfoil that has a shape in the form as shown in Fig. 3(a). At 0° α, this shape does not produce lift. But at α higher than 0°, it will produce a lift due to velocity differences in the flow on the pressure and suction sides. At specific velocities, the lift can be increased if the shape allows the flow to be accelerated higher in the suction side with gradually improving thickness.

It can be added that the limitation to aerodynamic lift is the maximum thickness for a given velocity condition. The maximum thickness for a lift-generating geometry (airfoil) is illustrated in Fig. 4.

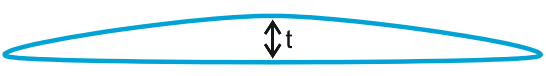

Fig. 4 – Maximum thickness of an airfoil

3. METHODOLOGY USED TO INVESTIGATE AIRFOILS

In order to study the effect of thickness on the aerodynamic performance of an airfoil, 5 NACA airfoils with incremental thickness are analyzed using XFOIL, a panel method code. The airfoils selected are NACA 0009, NACA 0010, NACA 0011, NACA 0012 and NACA 0013 designed with incremental thickness. The airfoils are symmetric with no camber. A viscous flow assumption is made. XFOIL helps in reducing the computational cost significantly when testing more number of airfoil for different *Re* conditions. The software is used here for analyzing angles of attack in increment of 0.1^0. The thickness corresponding to the selected airfoils and the maximum thickness location are listed in Table 2.

Table 2 – Selection of airfoils and their geometrical properties

Airfoil	Maximum thickness, *t*	Location of maximum thickness along chord
NACA 0009	8.95%c	29.7%c
NACA 0010	9.95%c	29.7%c
NACA 0011	10.93%c	29.7%c
NACA 0012	11.93%c	29.7%c
NACA 0013	12.92%c	29.7%c

c=1 unit chord length

4. EFFECT OF AIRFOIL THICKNESS ON LIFT AND DRAG

It is inherent that the normal component of force *F* described in Equation 5 concerns actual flight and particular importance is attached to lift generation rather than drag. Drag is only of secondary importance compared to lift since it reduces the aerodynamic efficiency. Airfoils are designed to produce high lift and low drag and the geometry chosen depends on the intended application and *Re* of flow. A detailed investigation on the effect of thickness on lift and drag has been performed which will be subsequently discussed.

Effect on lift

Fig. 5 shows the lift variation for five NACA airfoils designed with incremental thickness. The Reynolds number condition for the flow is 50000. At this *Re*, it is observed that there is no variation of C_L. On the contrary, the C_L is α dependent. At lower angles, between 0^0 and 2.2^0, it is observed that lower thickness contributes to higher lift. Beyond 2.2^0, until about 9.2^0, higher thickness contributes to higher lift and this is significant for the entire aforementioned range of α. Beyond 9.2^0, no correlation could be observed.

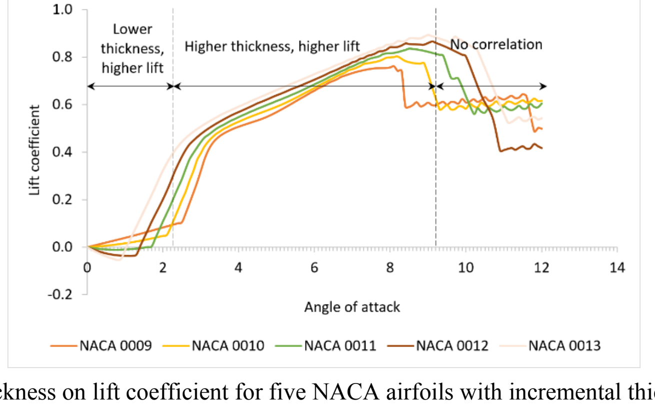

Fig. 5 – Effect of thickness on lift coefficient for five NACA airfoils with incremental thickness. *Re* of flow is 50000

In the next analysis, the *Re* of the flow is increased to 100000. It is observed (Fig. 6) that for α until 1.3^0, no correlation exists between thickness and lift.

Beyond the above angle, until 8^0, it is observed that higher airfoil thickness contributes to higher lift. Beyond 8^0, no correlation is observed.

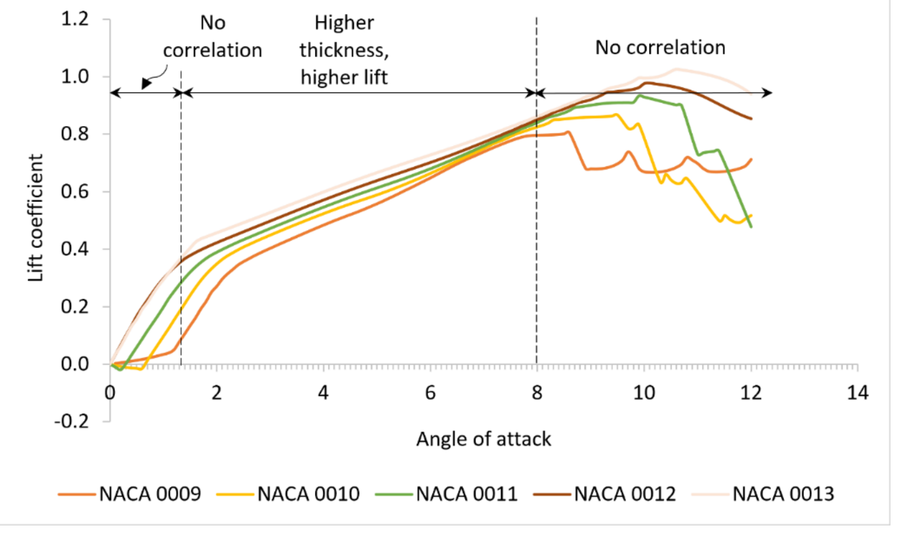

Fig. 6 – Effect of thickness on lift coefficient for flow *Re* of 100000

The aerodynamic characteristics of airfoils are now estimated for *Re* of 1000000. It is observed (Fig. 7) that for α of up to 1.9^0, uniform lift i.e. no considerable difference between the lift generated by airfoils is observed.

Between 1.9^0 and 5.6^0, airfoil with lower thickness produced higher lift at this *Re*. Between 5.6^0 and 8^0, a transition in lift production is observed in which airfoil with higher thickness continue to produce increased lift.

On the contrary, lift for airfoils with lower thickness subside in this α range. Beyond stall at 8^0, higher airfoil thickness contributes to higher lift.

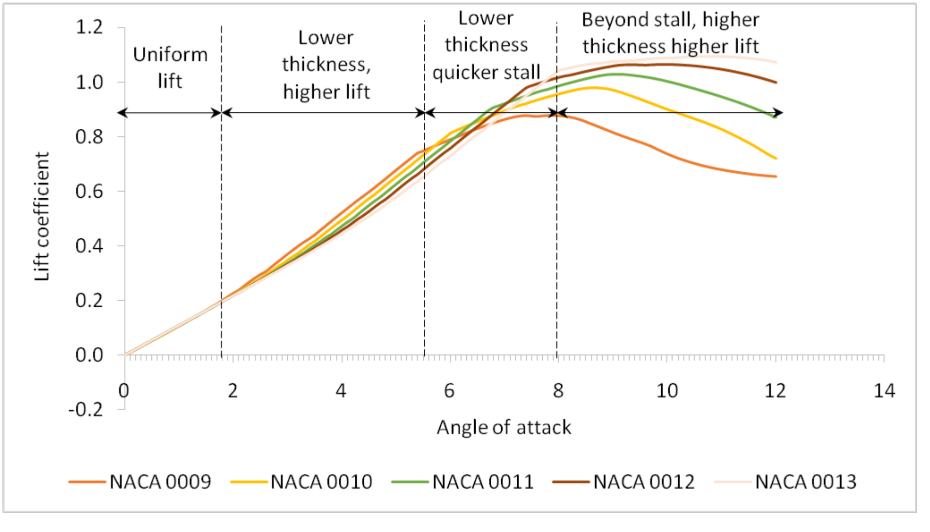

Fig. 7 – Effect of thickness on lift coefficient for flow *Re* of 1000000

Effect on drag

At lower *Re* (50000), the drag of airfoils show a periodic change. No particular trend (Fig. 8) in drag coefficient (based on airfoil thickness) could be observed. This shows that at this *Re* the drag is independent of airfoil thickness.

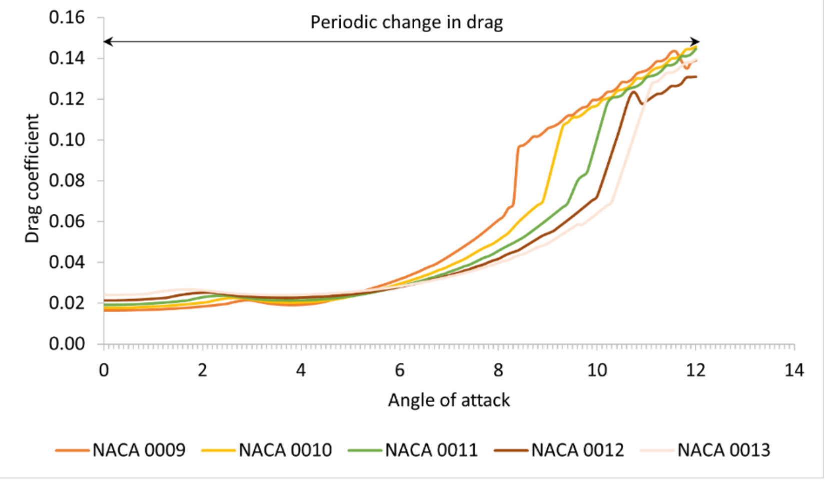

Fig. 8 – Effect of thickness on drag coefficient for flow *Re* of 50000

At *Re* of 100000, up to α of 5.7^0, drag variation based on thickness can be classified as non-periodic, non-uniform and transition with α could be observed.

Beyond 5.7^0, up to the measured α of 12^0, higher airfoil thickness leads to lower drag (Fig. 9).

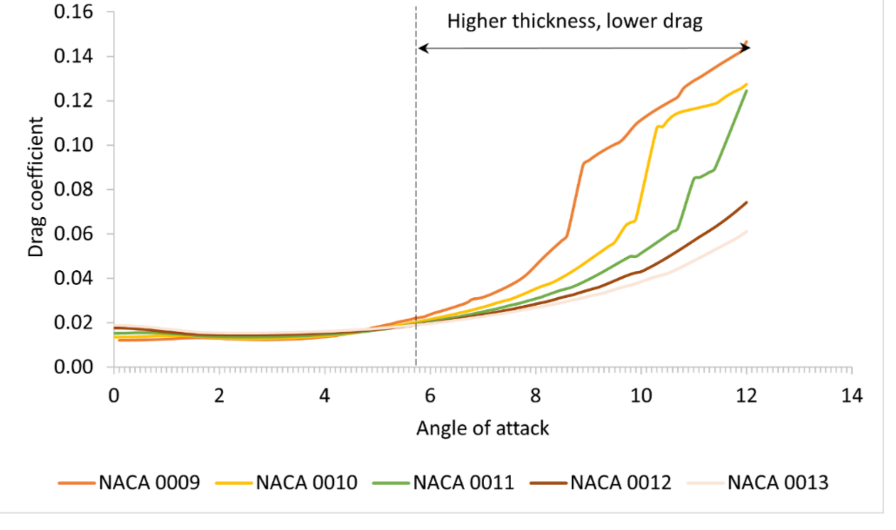

Fig. 9 – Effect of thickness on drag coefficient for flow *Re* of 100000

At high *Re* of 1,000,000 there is no significant variation in drag between airfoils for α up to 5.4^0. This means that at this *Re*, the drag is independent of thickness for 0^0 to 5.4^0 α range.

Beyond α of 5.4^0, higher thickness contributes to lower drag which means better aerodynamic performance and efficiency (*L/D*) (Fig. 10).

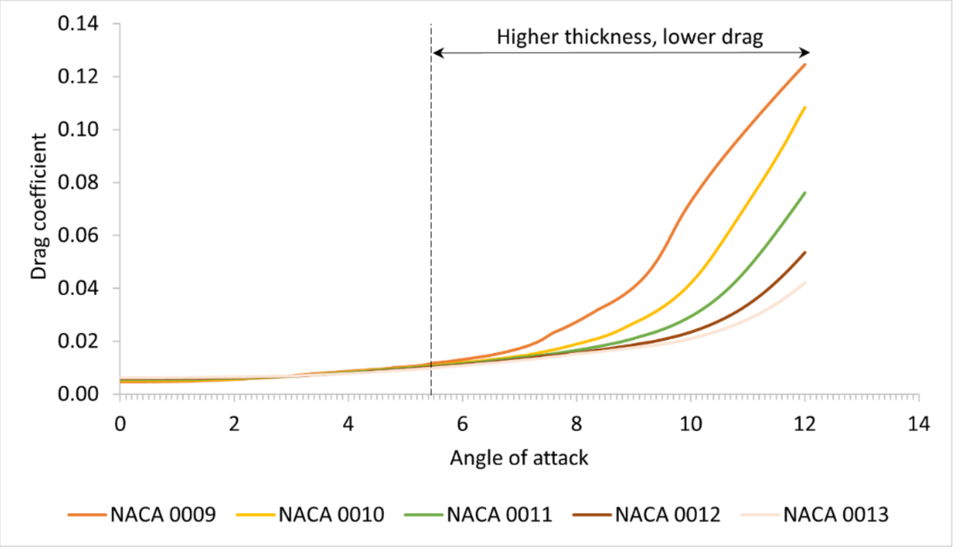

Fig. 10 – Effect of thickness on drag coefficient for flow *Re* of 1000000

5. IMPLICATION OF RESULTS

It can thus be assumed that the significance of lift and drag variation with airfoil thickness lies in the Reynolds number of flow.

Hence, it can be concluded that for any airfoil there is a non-linear relationship based on the following condition.

$$\begin{aligned} L &= f(Re, t) \\ D &= f(Re, t) \end{aligned} \tag{14}$$

The results imply that the selection of airfoil geometry should be based on the Reynolds number of flow and the aircraft designer should base his concept to get the derived lift and drag.

Furthermore, the airfoil thickness is a contributing factor in getting the desired airfoil performance. It is an important parameter that defines the airfoil geometry.

6. CONCLUSIONS

A theory on explaining aerodynamic phenomena of airfoils is proposed. The effect of airfoil thickness on lift and drag is investigated and discussed.

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