A mathematical model for describing the operation of airborne gun-laying radars in conditions of active counteraction to enemy interference and military-economic assessment of the feasibility of its implementation

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Abstract: *The combat capabilities of modern means of warfare in the air largely depend on the reliable operation of on-board electronic means (OEM) and weapon control systems of fighter aircraft. Therefore, in the course of military operations, each of the warring parties seeks to disorganise the operation of the enemy's radio-electronic systems and weapon controls as much as possible and ensure the stable operation of their own OEM by all means. This task is assigned to the means of electronic warfare (EW). During air combat, this task is assigned to the aircraft jamming station, whose place and role in modern air combat is constantly growing. The efficiency of the operation of this system is directly related to the aircraft survivability during a combat mission. This study considers the developed approach to the output of initial data on modelling the operation of airborne radars in conditions of interference and simultaneous active counteraction to enemy interference stations. The authors in this study have suggested a mathematical description of the air combat situation, which occurs when using* *methods of active countermeasures to enemy airborne interference systems by changing the parameters of the operation of the airborne gun-laying radar (AGLR). Indicators and criteria that characterise the effectiveness of the operation of airborne radars, depending on the method of counteraction used, were proposed, including a sequence of possible applications of the known four methods of counteraction, taking into account the features of their application.*

Key Words: *mathematical modelling, algorithm, model, active electronic countermeasures system, methods of active counteraction to interference stations.*

1. INTRODUCTION

Based on the proposed sequence, an algorithm was developed to substantiate the choice of methods for countering enemy interference stations, taking into account the range between aircraft for continuous tracking of the target. The calculations were made regarding the probability of tracking the target, taking into account the use of various combinations of counteraction methods. Since the situation of air combat has features of uncertainty, which are caused by the influence of a large number of factors of air combat, it is proposed to use fuzzy sets and game theory for a qualitative and complete assessment of the effectiveness of the operation of airborne radars [1].

The study also considers examples of the military-economic feasibility of implementing the proposed model in existing algorithms for the operation of the AGLR of fighter aircraft of the armed forces of Ukraine.

The experience of local wars and conflicts shows that aviation without electronic warfare has very low survivability. The use of active interference systems for individual protection of aircraft brings the probability of failure of the aircraft in an air battle to 0.45-0.75, and the integrated use of electronic warfare (electronic warfare) systems allows bringing this probability to 0.8-0.95. Thus, due to the importance of tasks performed by on-board EW system, the issues of counteraction to the electronic warfare equipment itself, the so-called electronic countermeasure system (ECM), or active counteraction to airborne interference by acting on their reconnaissance devices, have recently been increasingly studied [2].

Today, there are a number of ways to counteract the jamming stations, the use of which is random [3], namely: 1) a method to counteract the electronic countermeasures of the enemy by changing the plane of polarisation of the AGLR signal; 2) a method to counteract the electronic countermeasures of the enemy by changing the power of the radiating signal of the AGLR; 3) a method to counteract the electronic countermeasures of the enemy by violating the regularity of radiation; 4) a method to counteract the electronic countermeasures of the enemy by changing the parameters of the radiating signal of the AGLR.

However, the analysis of its use suggests that most methods of active counteraction have different conditions for their use, which is associated with changes in the operation of the AGLR, which imposes restrictions on the range and time of their use during air combat [4]. The automatic direction finder (ADF) of a fighter aircraft during air combat is one of the main AGLRs, since in the event of a failure of the automatic range tracking and automatic speed tracking systems, but with the normal operation of the ADF system, missile launch remains possible.

This determines the need for the development and practical implementation of the existing methods of active counteraction or electronic countermeasure systems (ECM) to enemy interference stations in air combat, as well as a rational combination of the use of existing methods of counteraction and the search for ways to implement them on board of fighter aircraft to increase the efficiency of the AGLR [5].

2. MATERIALS AND METHODS

A number of studies are devoted to the study of the possibility of countering jamming stations, as evidenced by the analysis of research efforts (R&D), dissertation papers, and scientific publications of leading scientists. However, existing methods and mathematical models have a number of disadvantages.

In particular, they do not address the issues of their joint use during air combat (four methods simultaneously).

The developed methods of countering enemy jamming stations are described by separate mathematical models that are suitable only for a specific method. Existing mathematical models practically do not take into account the various conditions of their use during air combat, which are associated with changes in the operation of the airborne gun-laying radars and restrictions on the range of their use.

Today, full-scale modelling of the use of methods of countering jamming stations during air combat to check the effectiveness of their application and develop a decision on their practical implementation has a high cost [6].

Therefore, in order to make a decision on the feasibility of practical implementation of the proposed methods in fighter aircraft, it is necessary to first check their effectiveness by mathematical modelling [7].

For this purpose, it is necessary to create adequate mathematical models that take into account different types of air combat situations.

Thus, the relevance of the subject matter is conditioned by the need to eliminate the discrepancy between the effective use of existing methods of active counteraction to enemy air interference and the limited capabilities of existing mathematical models that are used for this purpose.

In accordance with this, the purpose of the study is to improve the mathematical model of the operation of the AGLRs in conditions of active counteraction to enemy air interference [8], [9], [10], [11], [12].

3. RESULTS AND DISCUSSIONS

The proposed mathematical model of the operation of the AGLRs in conditions of active counteraction to enemy air interference, in contrast to existing approaches, has improved system of indicators and criteria for describing the operation of airborne gun-laying radars in interference conditions, taking into account the features and restrictions on the use of methods of active counteraction to enemy interference stations during air combat [13].

Firstly, construction of mathematical model describing the operation of AGLR in conditions of active counteraction to enemy interference starts with generation of source data on modelling of airborne gun-laying radar operation in the conditions of interference and simultaneous active counteraction to enemy interference stations [14].

There are four methods for countering enemy jamming stations:

Method 1: Polarisation interference – by changing the polarisation of the signal at the reception of the AGLR; Method 2: Complex of interference – by changing the parameters of the irradiating signal of the AGLR (frequency, pulse parameters); Method 3: Complex of interference – by changing the power of the irradiating signal of the AGLR; Method 4: Failure of automatic activation of the enemy interference by violating the irradiation discreteness.

To visualize possible implementations of this methods, diagram could be built (Figure 1).

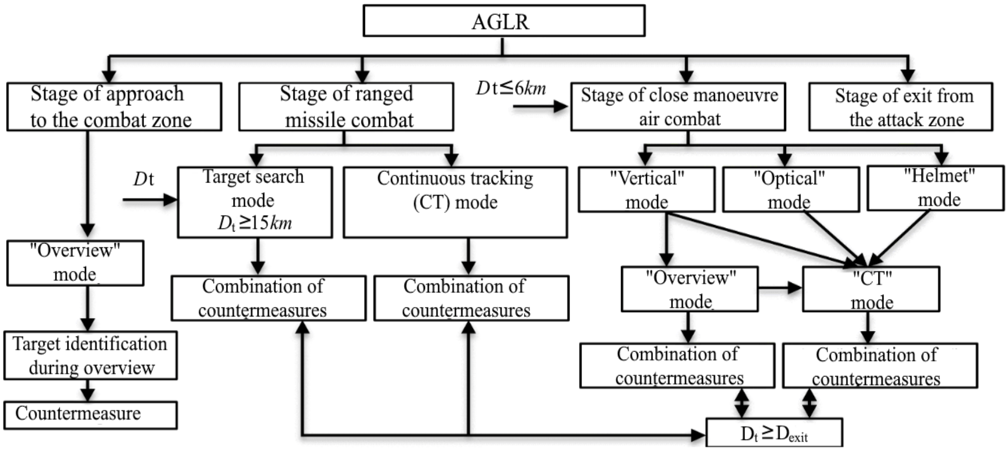

Fig. 1 – Diagram of possible application

After, the initial data is obtained, which include: the power of transmitters of AGLRs and jamming stations; the normalised radiation pattern of the antenna of the AGLR; the maximum gain of the AGLRs and jamming stations; the range between the AGLR and jamming station receiver; the radiation band of the interference signal; the coefficient that takes into account the discrepancy between the polarisation of jammer antenna and the radar, which is suppressed; the AGLR suppression coefficient; the interference power and signal input of the radar antenna, which is suppressed; the gain of the radar antenna in conditions of interference and without it; the effective surface target scattering; operating wavelength; noise power at the input of the radar antenna, which is suppressed; angle of change in the polarisation plane of the obstacle when received from the orthogonal position [15], selection of indicators and criteria that characterize the AGLR operation in the conditions of active counteraction to enemy interference could be described.

Performance indicators of AGLR in the conditions of interference taking into account the countermeasures to enemy jamming stations:

1. The range of target detection using the first and second methods of counteraction:

$$D = 4\sqrt{\frac{N_s G_s \lambda^2 G_r \sigma_t}{64\pi^3 N_{r.in}} \cdot cos^2 \Delta\beta} \tag{1}$$

where N_r, N_s – power of AGLR and jammer transmitters; G_s, G_r - maximum gain of AGRL and jammer antennas; λ – working wavelength; $N_{r.in}$, $N_{s.in}$ – interference power and signal at the input of the suppressed radar antenna; $\Delta\beta$ – angle of the plane of obstacle polarization when receiving from the orthogonal position; D – distance between jammer and AGLR; σ_t – target RCS; N_{no} – noise power at the input of the suppressed radar antenna; and in terms of application of the 3rd method:

$$D = 4\sqrt{\frac{1}{2}\frac{N_S G_S G_r \sigma_t \lambda^2}{64\pi^3 N_{r.in}}} = 4\sqrt{\frac{1}{2}} \cdot D_{max} \tag{2}$$

where G^k, G – gain of the radar antenna, in conditions of interference and without them;

2. The range of continuous tracking mode D_{ct} is not changed.

$$D_{Ct} = \beta \cdot D_{Max}, \beta = f(W_{fa} = 0{,}5[1 - \Phi(q_0)], W_d = [1 + \Phi(q - q_0)]) \quad (3)$$

where W_d – detection probability; W_{fa} – false alarm probability

3. The signal-to-noise ratio at the input of the AGLR receiver:

$$\frac{N_{s.in}}{N_{r.in}} = \frac{N_{s.in} \cdot G + N_r \cdot G\, sin^2 \Delta\beta}{(N_{r.in} + N_{no}) \cdot G^k\, cos^2\, \Delta\beta} \quad (4)$$

Power of the useful signal at the input of AGLR receiver, taking into account the use of the first and second method of enemy jammer counteraction:

$$N_{s.in} = \frac{N_s G_s}{4\pi\, D^2} \cdot \frac{\lambda^2 G_r}{4\pi} \cdot \frac{\sigma_t}{4\pi\, D^2} \cdot \cos\, \Delta\beta \pm \frac{N_r G_s}{4\pi\, D^2} \cdot \frac{\lambda^2 G_r}{4\pi} \cdot \frac{\Delta\, f_{bnd}}{\Delta\, F_n} \cdot \alpha \cdot sin\, \Delta\beta \quad (5)$$

where: Δf_{bnd} – bandwidth of the radar receiver; ΔF_n – interference signal radiation band; α – coefficient that takes into account the discrepancies between the polarisation of the jammer antenna and the suppressed AGRL;

4. Coefficient of AGLR suppression using countermeasures:

$$K_s \geq \frac{N_{r.in} G^k\, cos\, \Delta\beta}{N_{s.in} G\, cos\, \Delta\beta + \alpha\, N_{r.in}\, G\;\; sin\, \Delta\beta} \quad (6)$$

where K_s – suppression factor of AGLR.

Following the performance indicators definition, the probability of target tracking in the conditions of interference, taking into account the counteraction of the enemy jamming station could be calculated. For jamming station P'_{fail} – the probability of failure of target tracking (complex of four obstacles):

$$P'_{fail} = 1 - \prod_{i=1}^{4} (1 - k_i^{co} P_i) \quad (7)$$

where k^{co}_i – coefficient of effectiveness in response to the application of the i-th method (determined by experts).

For AGLR: $P'_{track} = 1 - P'_{fail}$ – probability of tracking the target in the interference conditions.

The system of indicators and criteria for describing the operation of the AGLR in conditions of interference was improved, taking into account the features and restrictions on the use of methods of active counteraction to enemy interference during air combat. The main indicator of restriction on the use of selected methods of active counteraction is the target detection range in the conditions of air combat, taking into account the specifics of the practical implementation of each of the methods defined above:

$$D = \sqrt[4]{\frac{N_s G_s \lambda^2 G_r \sigma_t}{64\pi^3 N_{s.in}} \cdot cos^2\;\; \Delta\beta} \quad (8)$$

$$D' = \sqrt[4]{\frac{1}{2} \frac{N_s G_s G_r \sigma_t \lambda^{\,2}}{64\pi^3 N_{s.in}}} = \sqrt[4]{\frac{1}{2}} \cdot D_{max} \quad (9)$$

where: N_s ($N_{s.in}$) – transmitter signal strength and antenna input signal, G_s (G_r) – maximum gain of radar and jamming stations, λ – operating wavelength, σ_t – effective target scattering plane, $\Delta\beta$ – angle of change of the polarisation plane. Calculations show that when using methods 1 and 4, the maximum range decreases by 4%; when using methods 2 or 3, the maximum range decreases by 16%, and when using all four methods – by 20 %. The criterion for the effectiveness of the radar operation in conditions of interference and simultaneous counteraction to them is the probability of tracking the target, thus, in equation 7 the probabilities of tracking the target in conditions of interference with and without counteraction are determined, where the coefficient of counteraction efficiency is additionally applied [16].

Then, mathematical model substantiating the choice of methods of active counteraction to enemy interference during air combat should be defined by obtaining data for the choice of a combination of active counteraction methods. Selection of elements of the mathematical model presented below:

$| a_{ij} |$ – matrix evaluating the AGLR efficiency during counteraction to the enemy jamming station;

a_{ij} – AGLR efficiency for each solution A_i, $i = \overline{(1, n)}$ and each state of the medium B_j j = 1, of the interference method;

A_1, A_2 – strategies of the enemy party A (creation of appropriate interference);

B_1, B_2 – pure strategies of the party B (AGLR) (considered counteraction methods to enemy jammers);

a_{ij}, $i = \overline{(1,2,)}$ $j = \overline{1,2}$ – fuzzy values representing the probabilities of tracking the AGLR target in the conditions of creating interference while ECM is active, which are indicators of the effectiveness of counteraction by the proposed methods.

Output data for the choice of a combination of active counteraction methods substantiation

1) If $\frac{\sin^2 \Delta\beta}{(\cos^2 \Delta\beta)}$, then by determining the range of target detection using the 1st method of counteraction by using (Equation 1):

$$D = 4\sqrt{\frac{N_s G_s \lambda^2 G_r \sigma_t}{64\pi^3 N_{r.in}} \cdot \cos^2 \Delta\,\beta} \tag{10}$$

Results in to $D_{avble} = 0.96\ D_{max}$ with P_{fail} by the 1st countermeasure method. With this data, matrixes elements of the payoff (probability of target tracking by AGLR) in the conditions of interference and without them, not taking into account the counteraction methods could be constructed.

Coefficient of effectiveness in response to the application of the 1-th method ($k^{co}{}_1$) could be determined, based on the results of construction of matrixes (Figures 2, 3).

2) By using power of the useful signal at the input of AGLR receiver (Equation 5), with 2nd method of enemy jammer counteraction:

$$N_{s.in} = \frac{N_s G_s}{4\pi\, D^2} \cdot \frac{\lambda^2 G_r}{4\pi} \cdot \frac{\sigma_t}{4\pi\, D^2} \cdot \cos\ \Delta\beta \pm \frac{N_r G_s}{4\pi\, D^2} \cdot \frac{\lambda^2 G_r}{4\pi} \cdot \frac{\Delta\, f_{bnd}}{\Delta\, F_n} \cdot \alpha \cdot sin\, \Delta\beta \tag{11}$$

Results in to $D_{avble} = 0.86\ D_{max}$ with P_{fail} by the complex of methods. With this data, matrixes elements of the payoff in the conditions of interference (probability of target tracking by AGLR) and without them with the application of counteraction methods could be constructed. Coefficient of effectiveness in response to the application of the 2-nd and i-th method ($k^{co}{}_2$, $k^{co}{}_i$) could be determined, based on the results of construction of matrixes (Figures 4, 5).

Output data for the mathematical model of fuzzy matrix game for estimating the efficiency of AGLR in the conditions of enemy counteraction to select the optimal method

1) If $\frac{\sin^2 \Delta\beta}{(\cos^2 \Delta\beta)}$, then by determining the range of target detection using the 3[rd] method of counteraction by using (Equation 2):

$$D = 4\sqrt{\frac{1}{2}\frac{N_s G_s G_r \sigma_t \lambda^2}{64\pi^3 N_{r.in}}} = 4\sqrt{\frac{1}{2}} \cdot D_{max}, \tag{12}$$

Results in to $D_{avble} = 0.86\ D_{max}$ with P_{fail} by the complex of methods. Therefore, the mathematical model of fuzzy matrix game for evaluating the effectiveness of the AGLR in the conditions of counteraction of the enemy's jamming station for the selection of the optimal method has the form (Equation 7).

$$A = \|a \quad ij\| = \begin{matrix} & B_1 & B_2 \\ A_1 & a_{11} & a_{12} \\ A_2 & a_{21} & a_{22} \end{matrix} \tag{13}$$

2) The signal-to-noise ratio at the input of the AGLR receiver using the 4[th] method of counteraction by using (Equation 4):

$$\frac{N_{s.in}}{N_{r.in}} = \frac{N_{s.in} \cdot G + N_r \cdot G \sin^2 \Delta\beta}{(N_{r.in} + N_{no}) \cdot G^k \cos^2 \Delta\beta} \tag{14}$$

Results in to $D_{avble} = D_{max}$ with P_{fail} by the failure of automatic activation of the enemy jamming interference. Therefore, the solution by approximate (iterative) method will be:

$$S_A \approx |p_1; p_2|, p_1 = \frac{m_i^*}{n_\Sigma}; p_2 = \frac{m_i^*}{n_\Sigma}, S_B \approx |q_1; q_2|, q_1 = \frac{n_i^*}{n_\Sigma}; q_2 = \frac{n_i^*}{n_\Sigma} \tag{15}$$

$$v_A = \frac{\alpha + \beta}{2} \approx \frac{\alpha\ \Sigma_{min} + \beta\ \Sigma_{max}}{2 \cdot k} \tag{16}$$

where: $\alpha\ \Sigma_{min}$ – worth of game for player A (AGLR*), $\beta\ \Sigma_{max}$ – worth of game for player B (*jamming station*)*. Coefficient of effectiveness in response to the application of the 3-rd and 4-th method ($k^{co}{}_1$, $k^{co}{}_i$) could be determined, based on the results of mathematical model (Equations 7, 8, 9). A mathematical model is presented to substantiate the choice of a combination of methods of active counteraction to enemy interference during air combat (Figure 1). Further, in Figures 2, 3, fuzzy estimates of the probability of tracking the airborne gun-laying radars in conditions of interference and countering enemy jamming stations in the case of using all methods of counteraction are developed. The optimal combination of counteraction methods is calculated using a fuzzy matrix game for evaluating the AGLR efficiency in the conditions of countering jamming stations. Using the results obtained, it is proposed to divide a combat mission performed by a fighter aircraft into 5 stages (Figure 6), depending on the range to the target, as opposed to the two existing ones.

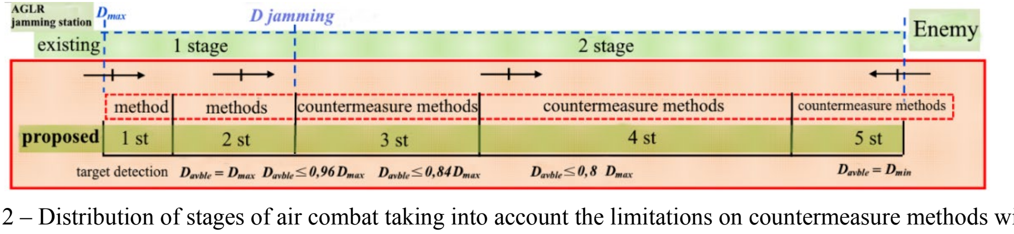

Fig. 2 – Distribution of stages of air combat taking into account the limitations on countermeasure methods with regard to the distance to target

During air combat, there may be situations in which possible strategies for conducting suppression by the enemy are known, but it is not known which of the known strategies is currently taking place [17], [18]. This situation can occur in an air battle, when there is known data on the types of fighter aircraft and countermeasures that are installed on these aircraft, as well as the general combat capabilities of enemy aircraft equipment and options for their use during the operation and the possibility of implementing counteraction methods, but it is not known at present what types of obstacles it now forms or whether the jamming station is enabled at all with radiation and further options for continuing the battle [19]. As defined above, the mathematical model in the form of a matrix game is the most adequate to describe this situation of the struggle of EW systems with the AGLR of a fighter aircraft in the conditions of ECM. This mathematical model for evaluating the efficiency of AGLRs has the form of a fuzzy efficiency assessment matrix $|a_{ij}|$, where a_{ij} – efficiency of the ADF of AGLR system for each solution A_i , $i = \overline{1,n}$ and each state of the environment I_j, $j = 1,m$ – a method of interference [16].

To make a decision in air combat on the use of a particular action and counteraction, fuzzy game models can be used in which performance estimates are set by fuzzy values [20]. Fuzzy models are models that are described in the framework of fuzzy sets, fuzzy relations, fuzzy dependencies, fuzzy processes and fuzzy quantities and allow one to make a fuzzy (i.e., in the form of fuzzy quantities) forecast of the behaviour, current and final state of systems and processes modelled for any term, the quality of the forecast worsens with increasing forecast terms [21]. Obvious difficulties when using a model in the form of a matrix game to evaluate the effectiveness of counteraction arise when determining the elements of win matrices (a_{ij} – efficiency of the ADF of AGLR for each decision of all parties, respectively), because the amount of information about the effectiveness of the ADF of AGLR in the proposed ECM methods is limited, which is not enough to objectively determine these elements by existing methods. The correct determination of the situation and effectiveness of conducting a particular method of ECM in air combat is associated with the accumulation and analysis of a priori information about the effectiveness of interference on the AGLR and the impact of the corresponding jamming methods on self-defence EW systems to reduce their effectiveness. There are two fundamentally different ways to get a priori information. The first of them consists in a direct study of the actual situations that develop in air combat based on the analysis of the experience of combat operations of fighter aircraft. The second method is heuristic construction of a priori information based on the methods of engineering psychology [22]. When searching for the optimal solution for using a particular ECM method in the case when it consists of mixed strategies, it was noted above that the resulting solution becomes sensitive to setting model elements a_{ij}.

The processing of the results of this survey should be presented in the form of fuzzy numbers with corresponding membership functions, which would depend on the specific situation developing in air combat in conditions of limited information about the use of a particular ECM method.

The departure from probability theory is conditioned by the fact that the model based on probability theory is adapted to processing accurate information distributed across implementations.

As soon as there is an inaccuracy in a separate implementation, the model becomes unable to use it [23].

Thus, the estimates a_{ij} of situations (A_i, B_j) of matrix games would be unclear due to incomplete information about the performance characteristics of the enemy jamming stations, unknown conditions of air combat, and the lack of practical assessments of the noise immunity of the radar during ECM at a certain stage of the battle.

Improved mathematical model of AGLR operation in the conditions of active counteraction to enemy interference stations:

1) If k_4^{co}, then probability of target tracking when using the 4th counteraction method will be:

$$k_4^{co} = 0 \div 0{,}45 \rightarrow (D_{avble} = D_{max}) \tag{17}$$

$$\mathrm{P}_{\mathrm{fail}}^{(4)} = 1-(1-k_4^{co}P_4)\cdot(1-P_1)\cdot(1-P_2)\cdot(1-P_3) \tag{18}$$

$$P_{trot}^{(4)} = 1-\mathrm{P}_{\mathrm{fail}}^{(4)} \tag{19}$$

Evaluation of the efficiency of AGLR operation with (Equation 12)

$$E_{trot}^{wECM} = \sum_{1,j=1}^{2} P_{trot.1,2}\cdot P\big(z(q)_{1,2}\big) = P_{trot.1}\cdot P(z_1) + P_{trot.2}\cdot P(z_2) \tag{20}$$

As the result a combination of countermeasures has been determined – (*4th method*).

2) If $k_1^{co};\ k_4$, then probability of target tracking when using the 1st and 4th counteraction methods will be:

$$k_1^{co} = 0 \tag{21}$$

$$k_4^{co} = 0 \div 0{,}37 \rightarrow (D_{avble} = 0{,}96\, D_{max}) \tag{22}$$

$$\mathrm{P}_{\mathrm{fail}}^{(1,4)} = 1-(1-k_1^{co}P_1)\cdot(1-k_4^{co}P_4)\cdot(1-P_2)\cdot(1-P_3) \tag{23}$$

$$P_{trot}^{(1,4)} = 1-\mathrm{P}_{\mathrm{fail}}^{(1,4)} \tag{24}$$

Evaluation of the efficiency of AGLR operation with (Equation 12):

$$E_{trot}^{wECM} = \sum_{1,j=1}^{2} P_{trot.1,2}\cdot P\big(z(q)_{1,2}\big) = P_{trot.1}\cdot P(z_1) + P_{trot.2}\cdot P(z_2) \tag{25}$$

As the result a combination of countermeasures has been determined – (*1st and 4th methods*).

3) If $k_2^{co};\ k_3$, then probability of target tracking when using the 2nd or 3rd counteraction methods will be:

$$k_2^{co} = 0 \div 0{,}15 \tag{26}$$

$$k_3^{co} = 0 \div 0{,}25 \rightarrow (D_{avble} = 0{,}84\, D_{max}) \tag{27}$$

$$\mathrm{P}_{\mathrm{fail}}^{(2)} = 1-(1-k_2^{co}P_2)\cdot(1-P_1)\cdot(1-P_3)\cdot(1-P_4) \tag{28}$$

$$P_{trot}^{(2)} = 1-\mathrm{P}_{\mathrm{fail}}^{(2)} \tag{29}$$

$$\mathrm{P}_{\mathrm{fail}}^{(3)} = 1-(1-k_3^{co}P_3)\cdot(1-P_1)\cdot(1-P_2)\cdot(1-P_4) \tag{30}$$

$$P_{trot}^{(3)} = 1-\mathrm{P}_{\mathrm{fail}}^{(3)} \tag{31}$$

After evaluation of efficiency a combination of countermeasures has been determined (*2nd or 3rd method*)

4) If k_1^{co}; k_2^{co}; k_3^{co}; k_4^{co}, then probability of target tracking using all 4 counteraction methods will be:

$$k_1^{co} = 0; k_2^{co} = 0 \div 0{,}15 \tag{32}$$

$$k_3^{co} = 0 \div 0{,}25 \tag{33}$$

$$k_4^{co} = 0 \div 0{,}37 \rightarrow (D_{avble} = 0{,}8\cdot D_{max}) \tag{34}$$

$$\mathrm{P}_{\mathrm{fail}}^{(1,2,3,4)} = 1-(1-k_1^{co}P_1)\cdot(1-k_2^{co}P_2)\cdot(1-k_3^{co}P_3)\cdot(1-k_4^{co}P_4) \tag{35}$$

$$P_{trot}^{(1,2,3,4)} = 1-\mathrm{P}_{\mathrm{fail}}^{(1,2,3,4)} \tag{36}$$

After evaluation of efficiency *a* ^{combination} of countermeasures has been determined (*all 4 methods*)

$$k_1^{co} = 0; k_2^{co} = 0 \div 0{,}15 \tag{37}$$

$$k_3^{co} = 0 \div 0{,}25 \rightarrow (D_{avble} = 0{,}8\cdot D_{max}) \tag{38}$$

$$\mathrm{P}_{\mathrm{fail}}^{(1,2,3)} = 1-(1-k_1^{co}P_1)\cdot(1-k_2^{co}P_2)\cdot(1-k_3^{co}P_3)\cdot(1-k_4^{co}P_4) \tag{39}$$

$$P_{trot}^{(1,2,3)} = 1-\mathrm{P}_{\mathrm{fail}}^{(1,2,3)} \tag{40}$$

After evaluation of efficiency a combination of countermeasures has been determined (*three methods*)

Evaluation of the efficiency of AGLR in conditions of interference and without them for the entire battle:

$$E_{trot}^{wECM} = \sum_{1,j=1}^{2} P_{trot.1,2}\cdot P\big(z(q)_{1,2}\big) = P_{trot.1}\cdot P(z_1) + P_{trot.2}\cdot P(z_2) \tag{41}$$

Evaluation of the efficiency of AGLR in the conditions of enemy interference:

$$E_{trot}^{ECM} = P(a_{11})\cdot P(q_1) + P(a_{22})\cdot P\ (q_2) \tag{42}$$

Further, according to the calculations performed, for example, the dependence of the signal power at the input on a change in the polarisation plane on the receiving side during the operation of the enemy jamming stations and the dependence of the reconnaissance range of the AGLR on a change in the angle of the polarisation plane when countering interference stations are graphically given (Figures 2, 3).

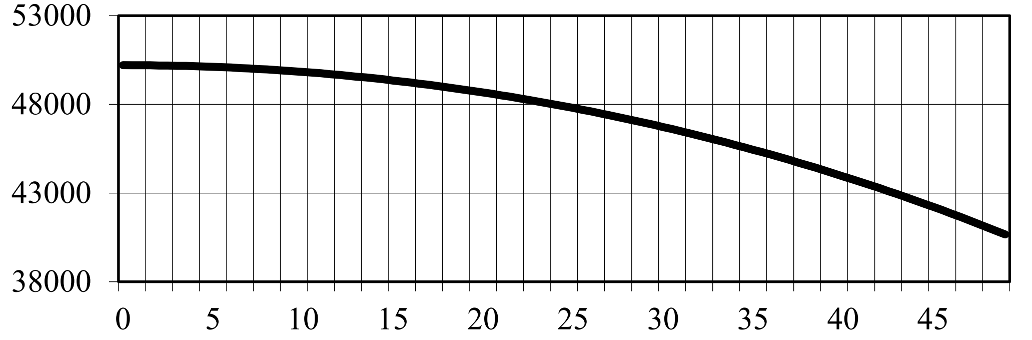

Fig. 3 – Calculation of changes in the radar reconnaissance range from changes in the polarisation plane during implementation of ECM

The calculations performed show that when performing a mismatch on $\Delta\gamma=30^0$ the reconnaissance range will change by 6.8% and will be 93.4 kilometres if the maximum value of the reconnaissance range without ECM ($\Delta\gamma=0$) is 100 kilometres.

During the ECM, in a certain way (mismatch in the range $\Delta\gamma=10\div150$), calculations show that the maximum reconnaissance range will decrease to about 5 kilometres from the maximum value.

The fighter aircraft would cover this distance in too short a time (units of seconds) at modern flight speed values [24].

The criterion for the effectiveness of the AGLR in conditions of interference and simultaneous counteraction is the probability of tracking the target, which additionally takes into account the coefficient of counteraction effectiveness:

$$P_{fail} = 1 - \prod_{\mathrm{i}=1}^{4} (1 - k_{\mathrm{i}}^{ca} P_{\mathrm{i}}) \tag{43}$$

where: k_i^{ca} – coefficient of counteraction effectiveness, taking into account the application of i-th method (the value is determined by an expert method); $P_{track} = 1 - \mathrm{P_{fail}}$ – the probability of tracking the target in conditions of obstacles for the airborne radar.

A mathematical model of the operation of the AGLR under interference conditions is presented, taking into account the features and limitations of the use of active counteraction methods [25].

Using the results obtained in the mathematical model and the counteraction efficiency coefficient, the probability of tracking the target is calculated in the same way for each combination of counteraction methods, then the efficiency of the AGLR operation is evaluated.

Then the calculated optimal combination of counteraction methods falls into the algorithm for conducting ECM to active enemy air interference stations, which takes into account 5 stages of air combat [26].

Further, in Equations 41 and 42, using the results obtained, an assessment of the effectiveness of the operation of the AGLR with or without interference and taking into account the implementation of the enemy jamming stations for the entire air battle was carried out. Taking into account the peculiarities of using counteraction methods and calculations made regarding the probability of tracking the target, it is proposed to determine the following combination of methods given in Table 1.

Table 1 – A combination of counteraction methods proposed depending on the distance to the enemy

Distance to the target	$D_{avble}=D_{max}$	$D_{avble}\leq 0.96D_{max}$	$D_{avble}\leq 0.84D_{max}$	$D_{avble}\leq 0.8D_{max}$	$D_{avble}=D_{min}$
Combination of methods	“4”	“1” and “4”	“2” or “3”	“1”, “2”, “3” and “4”	“1”, “2” and “3”
P_{track} of target with the use of a combination of countermeasures	0.71	0.74	0.81	0.88	0.85
P_{track} of target without the use of a combination of countermeasures	0.35	0.33	0.28	0.22	0.19
P_{track} of target in unobstructed conditions	0.91	0.91	0.93	0.94	0.95

To confirm the calculations performed, Figure 4 graphically shows the dependence of the probability of continuous tracking of a target in an air battle on the range to the target for cases: jamming stations do not form obstacles; in conditions of interference and active counteraction to them (the use of methods according to an improved mathematical model); in conditions of interference without the use of counteraction methods.

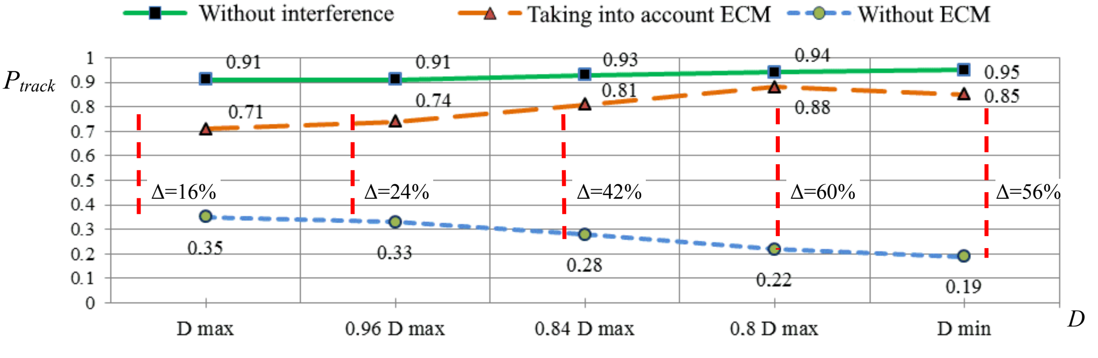

Fig. 4 – Probability of continuous tracking of the target in an air battle

Using the obtained estimates of the effectiveness of the radar operation, an assessment of the military-economic feasibility of implementing an improved mathematical model was carried out according to the criterion of prevented damage. The results of the military-economic assessment of the proposed mathematical model indicate the economic feasibility of its implementation.

The economic effect of implementing these recommendations would amount to: 277.9-344.6 million UAH for duelling air combat; 1.3-1.55 billion UAH during group air combat (5 aircraft) (Table 2).

With this data model of evaluation of the military-economic feasibility of implementing an improved mathematical model could be defined.

Recommendations for evaluation: Implementation of a combination of the choice of methods of countering the enemy jamming stations, suitable if they do not require large financial expenses $C_i^{ECM}\leq(0{,}005{:}0{,}01)\cdot C_a$. If $\frac{E_{trot}^{ECM}-E_{trot}^{wECM}}{E_{trot}^{ECM}}\leq 0{,}2$ – false, then:

Table 2 – Military-economic assessment of the application of the proposed mathematical model and algorithm for a combination of methods of active counteraction of enemy jamming stations in various situations of air combat

Aircraft type	Cost of aircraft, C_a, ths. UAH	Damage probability without ECM, $P_{dmg}^{no\ ECM}$	Damage probability with ECM, P_{dmg}^{ECM}	Damage probability to enemy aircraft, $P_{dmg.\ pr.}$	Cost of quipment, C_{eq}, ths. UAH	Cost of expected damage without using ECM in air combat, $\Delta C_1 = \frac{C_a P_{dmg}^{no\ ECM}}{P_{dmg.pr}}$ ths. UAH	Cost of military equipment, the loss of which will be prevented due to the improved method, $\Delta C_2 = \frac{C_a P_{dmg}^{ECM}}{P_{dmg.pr}}$ ths. UAH	Difference between the total cost of prevented damage and equipment costs, $\Delta C=(\Delta C_2-\Delta C_1)-C_{eq}$ ths. UAH
Duelling dogfight								
Su-27 (against the F-16)	864.000.0	0.32	0.71	0.64	2.172.0	432.000.0	778.816.901	344.644.901
MiG-29 (against the F-16)	648.000.0	0.29	0.69	0.62	1.213.0	303.096.774	582.260.870	277.951.095
Group dogfight (5 aircraft)								
Su-27 (against various opponents)	4.320.000	0.31	0.72	0.62	8.688.0	2.160.000.0	3.720.000.0	1.551.312.0
MiG-29 (against various opponents)	3.240.000	0.28	0.71	0.61	4.852.0	1.487.213.115	2.783.661.972	1.291.596.857

Recommendations for evaluation: Establishment of requirements for the implementation of the proposed methods of countering the enemy jamming stations and assessment of the military-economic feasibility of their implementation according to the criterion of prevented damage:

$$C_i^{ECM} \leq \sum_{j=1}^{10} P_j^{wECM} \cdot C_a \tag{44}$$

The scientific novelty of the proposed mathematical model for describing the operation of airborne gun-laying radars in conditions of active counteraction to enemy interference is due to the following:

– the system of indicators and criteria for describing the operation of the AGLRs in conditions of interference has been improved, taking into account the features and restrictions on the use of methods of active counteraction to enemy interference stations during air combat;

– a mathematical model for substantiating the choice of a combination of methods of active counteraction to enemy interference during air combat is developed based on a fuzzy matrix game, taking into account the coefficient of counteraction effectiveness, which allows a more reasonably determination of the probability of tracking a target in interference conditions;

– a mathematical model of the operation of the AGLR in conditions of interference is developed, taking into account the features and limitations of the use of methods of active counteraction to enemy air interference, which allows using active counteraction methods more efficiently and evaluating the effectiveness of the AGLR in conditions of uncertainty of the air situation [27].

The calculation of the improved mathematical model and developed recommendations showed that the application of the proposed combination of methods of countering enemy interference at certain stages of air combat according to the mathematical model has the following advantages: at the first stage, when only the 4^{th} method of counteraction is created, the probability of tracking the target of the AGLR can increase by 1.4÷2.7 times; at the second (simultaneous use of the 1st and 4th methods of counteraction) it increases by 2.2÷3.2 times; at the third stage (using the 2nd or 3rd method of counteraction) is determined by mixed strategies of their use, the probability of tracking the target increases by 2.7÷3.3 times; at the fourth stage, if it is possible to create all four methods simultaneously, it is equal to its maximum value and increases by 3.1÷3.6 times; at the fifth stage, when the use of the fourth method becomes impossible, it increases by 2.9÷3.3 times, given that without counteraction, the probability of tracking the target at each of the stages is within: ΔP_{track} without ECM ≈ {0.13;0.18;0.23;0.28} [28], [29].

The use of a fuzzy matrix game for each of the probable situations of applying appropriate strategies by the parties, provided that each method of counteraction is implemented simultaneously, indicates that all elements of the matrix (probabilities of tracking the goal) increase by $\Delta P_{track} \approx 0.1 \div 0.3$.

And the calculated mixed strategies (SA ≈ |0.53;0.47|, SB ≈ |0.3;0.7|) of using methods of active counteraction to enemy air interference confirm that the assessment of the effectiveness of the AGLR increases by 1.5 times [30], [31], [32], [33].

4. CONCLUSIONS

Thus, the proposed mathematical model for describing the operation of the AGLR in conditions of active counteraction to enemy interference would facilitate, according to preliminary calculations, an increase in the efficiency of the operation of the AGLR by 14-33%, as well as take into account the uncertainty of the situation that develops during air combat. The developed algorithm improves the existing approach to the operation of the AGLR in terms of the probability of tracking a target in the conditions of using the proposed methods of countering interference, taking into account the limitations of their use.

The results obtained would create the necessary conditions for improving the efficiency of the operation of the AGLR in the conditions of active counteraction to enemy air interference by 27 %.

Further studies should be directed to providing recommendations for the introduction of an improved method in the modernisation of fighters of the Ukrainian Air Force, taking into account various types of interference and the emergence of new methods of counteraction.

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