Bending of an elastoplastic circular sandwich plate on an elastic foundation in a temperature field

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Abstract: *Today, the development of the general theory of quasi-static deformation of three-layer structural elements, including plates, is not yet complete and is being intensively studied. Mathematical models of deformation under complex thermo-force and thermo-irradiation loads are created. The problems of strength, stability, and dynamic behaviour are considered. In strength calculations of three-layer structural elements, it is necessary to take kinematic hypotheses for each layer separately, which complicates the mathematical side of the problem but leads to significant refinement of the stress-strain state. The reaction of an elastic foundation is described by the Winkler model. The use of variational methods allows one to obtain a refined system of three differential equations of equilibrium in internal forces. The thermo-force bending of an elastoplastic circular sandwich plate with a light core connected to an elastic foundation is considered. The polyline normal hypotheses are used to describe the kinematics of a plate package that is not symmetric in thickness. In thin base layers, the Kirchhoff-Love hypotheses are accepted. In a light relatively thick core, the Timoshenko hypothesis is true, while the normal remains rectilinear, but rotates at some additional angle, the radial displacements change linearly in thickness. The differential equations of equilibrium are obtained using the Lagrange variation method. The statement of the boundary value problem in displacements is given in a cylindrical coordinate system. Numerical results for circular metal-polymer sandwich plates are presented.*

Key Words: *deformation, stress, design, oscillation, coefficient*

1. INTRODUCTION

For the first time, three-layer structures were used in construction in the 19th century. In the 1940s, the first aircraft with sandwich power hull elements began to appear. Nowadays, such structures have found their application in aerospace and transport engineering, construction, production and transportation of hydrocarbons. All this led to the demand for layered, including three-layer, structural elements, which necessitated the development of mathematical models and methods for calculating layered structural elements for various types of loads. Layered rods, plates, and shells are usually composed of materials with substantially different physical and mechanical properties. The load-bearing layers made of materials of high strength and rigidity are designed to absorb the main part of the mechanical load. The binding layers that serve to form a monolithic structure are designed to redistribute forces between the load-bearing layers. This combination of layers facilitates the reliable operation of systems in adverse environmental conditions (temperature, radiation), helps to create structures that combine high strength and rigidity with a relatively low weight.

Numerous studies have been devoted to the dynamics and vibrations of sandwich structural elements, including [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. Free oscillations of sandwich cylindrical shells are considered in [1]. The solution is written out in the form of an expansion into a double trigonometric series, the frequencies of free oscillations are analysed. Nonstationary dynamic effects on cylindrical shells and parabolic cylinders are considered in [2], [3], [4], [5]. The acoustic effect on layered plates and aerodynamic dumping in the layers are investigated in [6], [7]. In papers [8], [9], the dynamic behaviour of three-layer aircraft structural elements is considered. Natural and forced oscillations under the influence of harmonic, pulsed, and resonant loads are analysed. The kinematics of the deformation is assumed to correspond to the kinematic hypotheses of the polyline, the solutions are constructed as a series expansion in a system of proper orthonormal functions. The frequencies of natural oscillations under various boundary conditions are studied.

In [10], [11], [12], [13], [14], numerical modelling and software for determining the static and kinematic parameters of growing isotropic and anisotropic bodies in the process of nonstationary additive heat and mass transfer are presented. A method is proposed for modelling the effects of the loss of stability of thin-walled parts manufactured using selective laser melting (SLM) technology. The application of composite heat shields in intensive energy flows with diffusion is considered. A mathematical model of the energy efficiency of mechatronic modules and power supplies for promising mobile objects is being developed. Papers [15], [16], [17] are devoted to the study of the fluidity of reinforced plates made of unique rigid-plastic materials, taking into account the two-dimensional stress state in the fibres. The modelling of flexural deformation of layered plates with a regular structure made of nonlinear memory materials is elaborated. The theory of moderately large deflections of multilayer shells with a transversely soft core and reinforcement along the contour is proposed.

Various quasi-static problems are considered in the papers [18], [19], [20]. This is the deformation of inhomogeneous wire structures. The mechanical properties and microstructure of stainless steel made by laser sintering are investigated, and the assumption of the continuity of the energy of interfacial deformation of an orthotropic sandwich plate is analysed using a refined layer-by-layer theory. Papers [21], [22], [23] are devoted to the analysis of the assumption of the energy continuity of interfacial deformation of an orthotropic sandwich plate using the refined layer-by-layer theory, the approximate solution of the problem of plastic indentation of circular sandwich panels, and the study of the influence of heat flow on the stress state of a sandwich rod. Here the formulation and solution of a boundary value problem are presented, including a system of differential equilibrium equations and boundary conditions on the deformation of a circular sandwich plate with a hole connected to the Winkler foundation. The effect of the temperature field is taken into account.

2. MATERIALS AND METHODS

Statement of the boundary value problem. This study considers a sandwich plate with a central hole (Figure 1) in the cylindrical coordinate system r, φ, z. The middle plane of the core is taken as the coordinate plane, the z-axis is directed perpendicular to it up to the first layer. For isotropic base layers with a thickness of h_1, h_2, the Kirchhoff-Love hypothesis is accepted. The incompressible core (h_3 = 2c) is light, i.e., it ignores the work of shear stresses σ_{rz} in the tangential direction. The deformed normal of the core remains rectilineal but rotates by some additional angle ψ. At the boundaries of the layers, the movements are continuous. On the outer and inner contours of the plate, it is assumed that there are rigid diaphragms that prevent the relative shift of the layers [24], [25], [26].

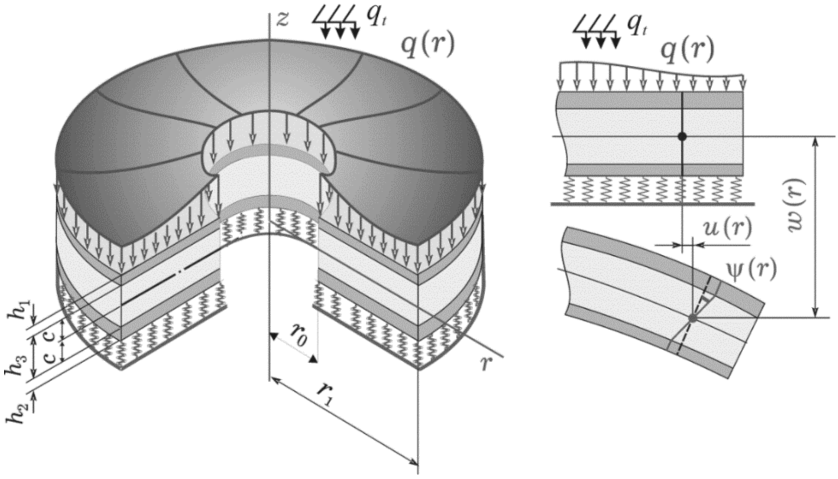

Fig. 1 – Design scheme of a sandwich plate

Let at the initial moment of time on a flat outer surface:

$$z = c + h1 \tag{1}$$

of a circular sandwich plate with a total relative thickness:

$$H = h1 + h2 + 2c \tag{2}$$

Connected with the elastic foundation, asymmetric vertical load q_0 (r) and a heat flux of intensity q_t, directed perpendicular to the bearing layer 1, begin to act. Surface

$$z = -c - h2, \tag{3}$$

of the outer and inner contours of the plate are considered to be heat-insulated. This allows the inhomogeneous temperature field $T(z)$, calculated from a certain initial temperature T_0, to be calculated with sufficient accuracy according to the equation given in [23]. Based on the hypothesis of the core normal rectilinearity

$$2\varepsilon_{rz}^{(3)} = u_r^{(3)},_z + w,_r = \psi \tag{4}$$

after integration, expressions for radial displacements in the layers $u_r^{(k)}$ through the desired functions are obtained:

$$u_r^{(1)} = u + c\psi - zw,_r\,, c \le z \le c + h_1 \tag{5}$$

$$u_r^{(3)} = u + z\psi - zw,_r\,, -c \le z \le c \tag{6}$$

$$u_r^{(2)} = u - c\psi - zw,_r\,, -c - h_2 \le z \le -c \tag{7}$$

where z is the coordinate of the fibre in question, a comma in the lower index denotes the differentiation operation by the coordinate following it, and the upper index – the number of a layer. Using the components of the stress tensor $\sigma_{\alpha}^{(k)}$ ($\alpha = r, \varphi$), $\sigma_{rz}^{(3)}$ generalised internal forces and moments are introduced:

$$T_{\alpha} \equiv \sum_{k=1}^{3} T_{\alpha}^{(k)} = \sum_{k=1}^{3} \int_{h_k} \sigma_{\alpha}^{(k)}\, d\, z,\ M_{\alpha} \equiv \sum_{k=1}^{3} M_{\alpha}^{(k)} = \sum_{k=1}^{3} \int_{h_k} \sigma_{\alpha}^{(k)} z\, d\, z,\ H_{\alpha} = M_{\alpha}^{(3)} + c\left(T_{\alpha}^{(1)} - T_{\alpha}^{(2)}\right),\ Q = \int_{-c}^{c} \sigma_{rz}^{(3)}\, d\, z. \quad (8)$$

The deformations in the layers follow from (5) and the Cauchy relations. The physical equations of state of the theory of small elastoplastic deformations of Ilyushin are used to relate stresses and deformations:

$$s_{\alpha}^{(k)} = 2G_k(1 - \omega_k(\varepsilon_u^{(k)}, T_k))э_{\alpha}^{(k)},\ \sigma^{(k)} = 3K_k(\varepsilon^{(k)} - \alpha_{0k}T_k),$$
$$s_{rz}^{(3)} = 2G_3(1 - \omega_k(\varepsilon_u^{(3)}, T_k))э_{rz}^{(3)}\ (k = 1, 2, 3;\ \alpha = r, \varphi), \quad (9)$$

where: $s_{\alpha}^{(k)}$, $э_{\alpha}^{(k)}$ – deviatoric, $\sigma^{(k)}$, $\varepsilon^{(k)}$ – spherical parts of stress and strain tensors; $G_k(T)$, $K_k(T)$ – temperature-dependent shear and volume deformation modules; a_{0k} – coefficient of linear temperature expansion; $\omega_k(\varepsilon_u^{(k)}, T_k)$ – functions of plasticity of materials of bearing layers and physical nonlinearity of the core, $s_{rz}^{(3)}$, $э_{rz}^{(3)}$ – tangential stress and shear deformation in the core; $\varepsilon_u^{(k)}$ – strain intensity [26], [27].

In the components of the stress tensor $\sigma_{\alpha}^{(k)}$, $\sigma_{rz}^{(3)}$ using the equations (7), the linear and nonlinear components are distinguished:

$$\sigma_{\alpha}^{(k)} = \sigma_{\alpha e}^{(k)} - \sigma_{\alpha\omega}^{(k)},\ \sigma_{rz}^{(3)} = \sigma_{rze}^{(3)} - \sigma_{rz\omega}^{(3)},$$
$$\sigma_{\alpha e}^{(k)} = 2G_k э_{\alpha}^{(k)} + 3K_k(\varepsilon^{(k)} - \alpha_k T),\ \sigma_{\alpha\omega}^{(k)} = 2G_k\omega_k(\varepsilon_u^{(k)}, T)э_{\alpha}^{(k)}, \quad (10)$$
$$\sigma_{rze}^{(3)} = 2G_3 э_{rz}^{(3)},\ \sigma_{rz\omega}^{(3)} = 2G_3\omega_3(\varepsilon_u^{(3)}, T)э_{rz}^{(3)},\ (\alpha = r, \varphi)$$

The generalised internal forces and moments (6) are also represented as the difference between the linear and nonlinear parts:

$$T_{\alpha} = T_{\alpha e} - T_{\alpha\omega} = \sum_{k=1}^{3} T_{\alpha e}^{(k)} - \sum_{k=1}^{3} T_{\alpha\omega}^{(k)},\ M_{\alpha} = M_{\alpha e} - M_{\alpha\omega} = \sum_{k=1}^{3} M_{\alpha e}^{(k)} - \sum_{k=1}^{3} M_{\alpha\omega}^{(k)},$$
$$H_{\alpha e} = M_{\alpha e}^{(3)} + c\left(T_{\alpha e}^{(1)} - T_{\alpha e}^{(2)}\right),\ H_{\alpha\omega} = M_{\alpha\omega}^{(3)} + c\left(T_{\alpha\omega}^{(1)} - T_{\alpha\omega}^{(2)}\right) \quad (11)$$

The equations of equilibrium in forces for an elastic circular sandwich plate connected to a deformable foundation are obtained in [8] without using the physical relations between stresses and deformations, therefore, they will be valid here as well [28], [29], [30].

Substituting in them the equations for internal forces and moments (7), obtain a system of equations of equilibrium in forces describing the physically nonlinear deformation in the temperature field of a circular sandwich plate with a light core resting on an elastic foundation:

$$T_{r,r} + \frac{1}{r}(T_r - T_{\varphi}) = p_{\omega},\ H_{r,r} + \frac{1}{r}(H_r - H_{\varphi}) = h_{\omega},$$
$$M_{r,rr} + \frac{1}{r}(2M_{r,r} - M_{\varphi,r}) = q_0 - q_R + q_{\omega}. \quad (12)$$

where q_0 – the intensity of the external distributed load; q_R – the reaction of the base; the lower index "*e*" in the left parts of the equations is omitted for simplicity.

The nonlinear additions are carried to the right-hand sides of equations (8) and have the following form:

$$p_\omega = T_{r\omega},_r + \frac{1}{r}(T_{r\omega} - T_{\varphi\omega}),\ h_\omega = H_{r\omega},_r + \frac{1}{r}(H_{r\omega} - H_{\varphi\omega}),$$
$$q_\omega = M_{r\omega},_{rr} + \frac{1}{r}(2M_{r\omega},_r - M_{\varphi\omega},_r). \quad (13)$$

Corresponding force boundary conditions on the contours ($r = r_0$, $l = 0$; $r = r_1$, $l = 1$):

$$T_r = T_r^l + T_\omega,\ H_r = H_r^l + H_\omega,\quad M_r = M_r^l + M_\omega,$$
$$M_r,_r + \frac{1}{r}(M_r - M_\varphi) = Q^l + M_{r\omega},_r + \frac{1}{r}(M_{r\omega} - M_{\varphi\omega}). \quad (14)$$

It is assumed that the relationship between the reaction of the base and the deflection of the plate w is described by the Winkler model:

$$q_R = -\kappa_0 w. \quad (15)$$

where κ_0 is the stiffness coefficient of the elastic foundation (modulus of foundation).

The linear generalised internal forces in equations (10) and boundary conditions (12) can be expressed in terms of the desired displacements using the physical relations (7).

As a result, the system of nonlinear differential equations of equilibrium (8), taking into account (13) in displacements, takes the form:

$$L_2(a_1 u + a_2\psi - a_3 w,_r) = p_\omega,\ L_2(a_2 u + a_4\psi - a_5 w,_r) = h_\omega,$$
$$L_3(a_3 u + a_5\psi - a_6 w,_r) - \kappa_0 w = q_0 + q_\omega. \quad (16)$$

where L_2, L_3 – second-and third-order differential operators.

$$L_3(g) \equiv g,_{rrr} + \frac{2g,_{rr}}{r} - \frac{g,_r}{r^2} + \frac{g}{r^3},\ L_2(g) \equiv g,_{rr} + \frac{g,_r}{r} - \frac{g}{r^2} \quad (17)$$

the coefficients a_i are determined by integral relations, since the elastic modulus of the materials in the layers changes in thickness along with the temperature

$$a_1 = \sum_{k=1}^{3} K_{k0},\ a_2 = c(K_{10} - K_{20}),\ a_3 = \sum_{k=1}^{3} K_{k1},\ a_4 = K_{32} + c^2(K_{10} + K_{20}),$$
$$a_5 = K_{32} + c(K_{11} - K_{21}),\ a_6 = \sum_{k=1}^{3} K_{k2},\ K_{km} = \int_{h_k}\left[K_k(T_k) + \frac{4}{3}G_k(T_k)\right] z^m dz, \quad (18)$$
$$(m = 0, 1, 2)$$

The problem of finding the functions u®, ψ®, w® is closed by adding force (12) or kinematic boundary conditions to equations (14).

In the latter case, when the contour of the plate is firmly sealed, the requirements must be met on it:

$$u = \psi = w = w,_r = 0 \quad (19)$$

With a hinged support:

$$u = \psi = w = M_r = 0. \quad (20)$$

In the case of a free contour of the plate:

$$\psi = 0, Tr = Mr = Mr,r = 0. \quad (21)$$

3. RESULTS AND DISCUSSIONS

The formulated boundary value problem is nonlinear, so it is not necessary to find its exact solution. Next, the study considers the procedure for applying the Ilyushin method of a linear approximation to the problem under consideration. To do this, the system (10) is written in iterative form:

$$L_2(a_1u^n + a_2\psi^n - a_3w,_r^n) = p_\omega^{n-1}, L_2(a_2u^n + a_4\psi^n - a_5w,_r^n) = h_\omega^{n-1},$$
$$L_3(a_3u^n + a_5\psi^n - a_6w,_r^n) - \kappa_0w^n = q_0 + q_\omega^{n-1} \quad (22)$$

here n – the number of the approximation, the values p_ω^{n-1}, h_ω^{n-1}, q_ω^{n-1} are called "additional" external loads and are assumed to be zero at the first step, and then calculated based on the results of the previous approximation. In this case, equations of the type (11) are used, in which all the terms have the index "n -1" at the top:

$$p_\omega^{n-1} = T_{r\omega}^{n-1},_r + \frac{1}{r}(T_{r\omega}^{n-1} - T_{\varphi\omega}^{n-1}), h_\omega^{n-1} = H_{r\omega}^{n-1},_r + \frac{1}{r}(H_{r\omega}^{n-1} - H_{\varphi\omega}^{n-1}),$$
$$q_\omega^{n-1} = M_{r\omega}^{n-1},_{rr} + \frac{1}{r}(2M_{r\omega}^{n-1},_r - M_{\varphi\omega}^{n-1},_r), \quad (23)$$

where:

$$T_{\alpha\omega}^{n-1} \equiv \sum_{k=1}^{3}\int_{h_k}\sigma_{\alpha\omega}^{(k)n-1}dz = \sum_{k=1}^{3}\int_{h_k}2G_k\omega_k(\varepsilon_\alpha^{(k)n-1})Э_\alpha^{(k)n-1}dz,$$

$$M_{\alpha\omega}^{n-1} \equiv \sum_{k=1}^{3}\int_{h_k}\sigma_{\alpha\omega}^{(k)n-1}zdz = \sum_{k=1}^{3}\int_{h_k}2G_k\omega_k(\varepsilon_\alpha^{(k)n-1})Э_\alpha^{(k)n-1}zdz, \quad (24)$$

$$H_{\alpha\omega}^{n-1} = M_{\alpha\omega}^{(3)n-1} + c\left(T_{\alpha\omega}^{(1)n-1} - T_{\alpha\omega}^{(2)n-1}\right)(\alpha = r, \varphi),$$

With the boundary conditions, it is necessary to do the same. Then, at each step of the approximation, a linear problem of the theory of elasticity with known additional "external" loads is obtained, the loads are calculated by equations (21), (22) [31], [32], [33]. Using the first two in the third equation of the system (20), the coefficients are zeroed before the desired functions u^n and ψ^n. After two-fold integration of these equations, the system is reduced to the form

$$u^n = b_1w,_r^n - \frac{1}{a_1a_4 - a_2^2}\frac{1}{r}\int r\int(a_2h_\omega^{n-1} - a_4p_\omega^{n-1})drdr + C_1^nr + \frac{C_2^n}{r},$$

$$\psi^n = b_2w,_r^n + \frac{1}{a_1a_4 - a_2^2}\frac{1}{r}\int r\int(a_1h_\omega^{n-1} - a_2p_\omega^{n-1})drdr + C_3^nr + \frac{C_4^n}{r}, \quad (25)$$

$$L_3(w,_r^n) + \kappa^4w_r^n = -q + f_\omega^{n-1},$$

where $C_1^n,\ C_2^n,\ C_3^n,\ C_4^n$ – the integration constants at the n-th step,

$$\kappa^4 = \kappa_0D, q = q_0D, b_1 = \frac{a_3a_4 - a_2a_5}{a_1a_4 - a_2^2}, b_2 = \frac{a_1a_5 - a_2a_3}{a_1a_4 - a_2^2},$$

$$f_\omega^{n-1} = -Dq_\omega^{n-1} + D_1\frac{1}{r}(rp_\omega^{n-1}),_r + D_2\frac{1}{r}(rh_\omega^{n-1}),_r,$$

$$D = \frac{a_1(a_1a_4 - a_2^2)}{(a_1a_6 - a_3^2)(a_1a_4 - a_2^2) - (a_1a_5 - a_2a_3)^2}, \quad (26)$$

$$D_1 = \frac{a_1(a_3a_4 - a_2a_5)}{(a_1a_6 - a_3^2)(a_1a_4 - a_2^2) - (a_1a_5 - a_2a_3)^2},$$

$$D_2 = \frac{a_1(a_1a_5 - a_2a_3)}{(a_1a_6 - a_3^2)(a_1a_4 - a_2^2) - (a_1a_5 - a_2a_3)^2},$$

The third equation in (23) in the expanded form is the following:
the index "n -1" at the top:

$$w_{,rrrr}^{n}+\frac{2}{r}w_{,rrr}^{n}-\frac{1}{r^2}w_{,rr}^{n}+\frac{1}{r^3}w_{,r}^{n}+\kappa^4 w^n=-q+f_{\omega}^{n-1}, \tag{27}$$

Its general solution can be written as:

$$w^n=C_5^n\,ber(\kappa r)+C_6^n\,bei(\kappa r)+C_7^n\,ker(\kappa r)+C_8^n\,kei(\kappa r)+w_0^n(r), \tag{28}$$

where: $ber(\kappa r)$, $bei(\kappa r)$, $ker(\kappa r)$, $kei(\kappa r)$ – zero-order Kelvin functions; $w_0^n(r)$ – a particular solution of equation (25), to find which, in the general case, the Cauchy kernel is used.

The recurrent solution of the problem of thermo-force bending of a physically nonlinear circular sandwich plate on an elastic foundation takes the form

$$\begin{aligned}
u^n&=b_1w_{,r}^{n}-\frac{1}{a_1a_4-a_2^2}\frac{1}{r}\int r\int(a_2h_{\omega}^{n-1}-a_4p_{\omega}^{n-1})drdr+C_1^n r+\frac{C_2^n}{r},\\
\psi^n&=b_2w_{,r}^{n}+\frac{1}{a_1a_4-a_2^2}\frac{1}{r}\int r\int(a_1h_{\omega}^{n-1}-a_2p_{\omega}^{n-1})drdr+C_3^n r+\frac{C_4^n}{r},\\
w^n&=C_5^n\,ber(\kappa r)+C_6^n\,bei(\kappa r)+C_7^n\,ker(\kappa r)+C_8^n\,kei(\kappa r)+w_0^n(r),
\end{aligned} \tag{29}$$

where the integration constants $C_1^n,\ C_2^n,\ \dots,\ C_8^n$ at each iteration step are determined from the boundary conditions.

When the boundary contours of the annular plate are tightly sealed, the solution (27) must be substituted for the boundary conditions (17). As a result, at each step of the approximation, a linear system of eight algebraic equations for determining the integration constants is obtained $C_1^n,\ C_2^n,\ \dots,\ C_8^n$. The first four equations that meet the requirements $u = 0$, $\psi = 0$ for $r = r_0$ and $r = r_1$ will be

$$C_1^n r_1+\frac{C_2^n}{r_1}=q_1^{n-1},\ C_1^n r_0+\frac{C_2^n}{r_0}=q_2^{n-1},\ C_3^n r_1+\frac{C_4^n}{r_1}=q_3^{n-1},\ C_3^n r_0+\frac{C_4^n}{r_0}=q_4^{n-1}, \tag{30}$$

where:

$$\begin{aligned}
&q_1^{n-1}=\frac{1}{a_1a_4-a_2^2}\int r\int(a_2h_{\omega}^{n-1}-a_4p_{\omega}^{n-1})drdr\Big|_{r=r_1},\ q_2^{n-1}=\\
&\frac{1}{a_1a_4-a_2^2}\frac{1}{r_0}\int r\int(a_2h_{\omega}^{n-1}-a_4p_{\omega}^{n-1})drdr\Big|_{r=r_0},q_3^{n-1}=-\frac{1}{a_1a_4-a_2^2}\int r\int(a_1h_{\omega}^{n-1}-\\
&a_2p_{\omega}^{n-1})drdr\Big|_{r=r_1},q_4^{n-1}=-\frac{1}{a_1a_4-a_2^2}\frac{1}{r_0}\int r\int(a_1h_{\omega}^{n-1}-a_2p_{\omega}^{n-1})drdr\Big|_{r=r_0},
\end{aligned} \tag{31}$$

They allow to obtain constants $C_1^n,\ C_2^n,\ C_3^n, C_4^n$ explicitly:

$$\begin{aligned}
C_1^n&=\frac{q_1^{n-1}-r_0q_2^{n-1}}{1-r_0^2},\ C_2^n=\frac{r_0(q_2^{n-1}-r_0q_1^{n-1})}{1-r_0^2},\\
C_3^n&=\frac{q_3^{n-1}-r_0q_4^{n-1}}{1-r_0^2},\ C_4^n=\frac{r_0(q_4^{n-1}-r_0q_3^{n-1})}{1-r_0^2}
\end{aligned} \tag{32}$$

From the conditions $w = w,_r = 0$ for $r = r_0$ and $r = r_1$, it follows:

$$\begin{aligned}
&C_5^n\,ber\,\kappa+C_6^n\,bei\,\kappa+C_7^n\,ker\,\kappa+C_8^n\,kei\,\kappa=-w_0^n(r_1),\\
&C_5^n\,ber(\kappa r_0)+C_6^n\,bei(\kappa r_0)+C_7^n\,ker(\kappa r_0)+C_8^n\,kei(\kappa r_0)=0,\\
&b_3C_5^n+b_4C_6^n+b_{30}C_7^n+b_{40}C_8^n=-w_{0}^{n},_r(r_1),\ b_{31}C_5^n+b_{41}C_6^n+b_{32}C_7^n+b_{42}C_8^n=\\
&0,
\end{aligned} \tag{33}$$

where:

$$b_{30} = \frac{\kappa\sqrt{2}}{2}[ker_1(\kappa r_1) + kei_1(\kappa r_1)],\ b_{40} = \frac{\kappa\sqrt{2}}{2}[-ker_1(\kappa r_1) + kei_1(\kappa r_1)],$$

$$b_{31} = \frac{\kappa\sqrt{2}}{2}[ber_1(\kappa r_0) + bei_1(\kappa r_0)],\ b_{41} = \frac{\kappa\sqrt{2}}{2}[-ber_1(\kappa r_0) + bei_1(\kappa r_0)], \quad (34)$$

$$b_{32} = \frac{\kappa\sqrt{2}}{2}[ker_1(\kappa r_0) + kei_1(\kappa r_0)],\ b_{42} = \frac{\kappa\sqrt{2}}{2}[-ker_1(\kappa r_0) + kei_1(\kappa r_0)],$$

Here the relations are used:

$$w,_r^n = \frac{\kappa\sqrt{2}}{2}\{C_5^n[ber_1(\kappa r) + bei_1(\kappa r)] + C_5^n[-ber_1(\kappa r) + bei_1(\kappa r)] +$$
$$+C_7^n[ker_1(\kappa r) + kei_1(\kappa r)] + C_8^n[-ker_1(\kappa r) + kei_1(\kappa r)]\} + w_0^n,_r(r), \quad (35)$$

$$w,_r^n(r_1) = b_3C_5^n + b_4C_6^n + b_{30}C_7^n + b_{40}C_8^n + w_0^n,_r(r_1),\ w_0^n(r_0) = 0,\ w_0^n,_r(r_0) = 0$$

As a result, the solution of the system of linear algebraic equations (20) can be written in the determinants:

$$C_5^n = \frac{\Delta_5^n}{\Delta},\ C_6^n = \frac{\Delta_6^n}{\Delta},\ C_7^n = \frac{\Delta_7^n}{\Delta},\ C_8^n = \frac{\Delta_8^n}{\Delta} \quad (36)$$

where: Δ – determinant of the system (31); the remaining determinants are obtained from it by replacing the column with the number (n-4), n – the lower index of the determinant, with the column of free members of the system (30) [34], [35].

Thus, the solution (27) with the integration constants (30), (34) describes the elastoplastic displacements of circular sandwich plates with a light core and sealed boundary contours, bent on an elastic foundation by an arbitrary symmetric load $q(r)$ and a heat flow q_t. Numerical studies were carried out for plates fixed along the contours, the layers of which were composed from D16T-fluoroplastic-D16T materials. Relative layer thicknesses $h_1 = h_2 = 0.04$, $h_3 = 0.4$, inner radius $r_0 = 0.2$, outer radius $r_1 = 1$. The intensity of the thermal load $q_t = 5000$ J/(m^2*s). Time of its action $t_0 = 60$ min. For the plates under consideration, the heat spent on heating the outer metal layer is neglected (due to the low heat capacity). Its temperature is assumed to be equal to the temperature of the aggregate at the bonding site: $T^{(1)} = T^{(3)}(c, t)$. At a heat flow of $q_t = 5000$ J/(m^2*s), the temperature in the outer layer reaches a value of $T_1 = 597$ K (at $t_0 =$ 60 min.), which corresponds to sufficient heating of duralumin, but less than the melting point of the filler – fluoroplastic. In the second layer in contact with the base, the temperature is constant. To describe the dependence of the elastic modulus of materials on temperature, the equation proposed by Bell is used:

$$\{G(T),\ K(T),\ E(T)\} = \{G(0),\ K(0),\ E(0)\}\phi(T),$$
$$\phi(T) = \begin{cases} 1, & 0 < T/T_{pl} \le 0{,}06, \\ 1{,}03(1 - T/(2T_{\text{pl}})), & 0{,}06 < \dfrac{T}{T_{pl}} \le 0{,}57, \end{cases} \quad (37)$$

where: T_{pl} – the melting point of the material; G (0), $K(0)$, $E(0)$ are the module values at the so-called zero temperature. For example, knowing the magnitude of the shear modulus G_0 at a certain temperature T_0, obtain $G(0) = \frac{G_0}{\varphi(T_0)}$. At higher temperatures $T/T_{pl} > 0.57$, a small deviation of the material behaviour from the linear law (22) is possible. The functions of the plasticity of the materials of the bearing layers and the physical nonlinearity of the aggregate, depending on the strain intensity $e_u^{(k)}$, the temperature T_k and the hydrostatic stress $\sigma^{(3)}$, are taken as

$$\omega_k(\varepsilon_u^k, T_k) = \begin{cases} 0, & \varepsilon_u^k \le \varepsilon_{T0}^k, \\ A_{1k}\left(1 - \frac{\varepsilon_{T0}^k}{\varepsilon_u^k + \varepsilon_{T0}^k - \varepsilon_T^k}\right)^{\alpha_{1k}}, & \varepsilon_u^k > \varepsilon_{T0}^k. \end{cases} \quad \varepsilon_T^k(T) = \frac{\sigma_T^k(T_k)}{E_k(T_k)},$$
$$\sigma_y^k = \sigma_{T0}^k \, exp\left\{\kappa_k\left(\frac{1}{T_k} - \frac{1}{T_{k0}}\right)\right\}, \tag{38}$$

where A_{1k}, a_{1k}, E_k, k_k, A_2, a_2, A_3, a_3 – the constants of the layer materials obtained experimentally; $e_T^{(k)}$ – the yield strength of the material under deformations at the temperature T_k, $e_{T0}^{(k)}$ – the yield strength at the initial temperature; p_0 – the minimum pressure covering all internal defects in the core material.

The displacements (27) for an annular plate with integration constants (34) at a base with stiffness (κ_0 = 1000 MPa/m) are shown in Figure 2 (a-c): 1 – isothermal bending of the elastic plate; 2 – thermoelastic bending; 3 – thermoplastic. The intensity of the power load q_0=-40 MPa. The plasticity of the layer materials significantly affects the radial displacements in the plate. Here, the weak influence of the physical nonlinearity of the materials on the deflection and relative shear is conditioned by the sufficiently high rigidity of the foundation and plate.

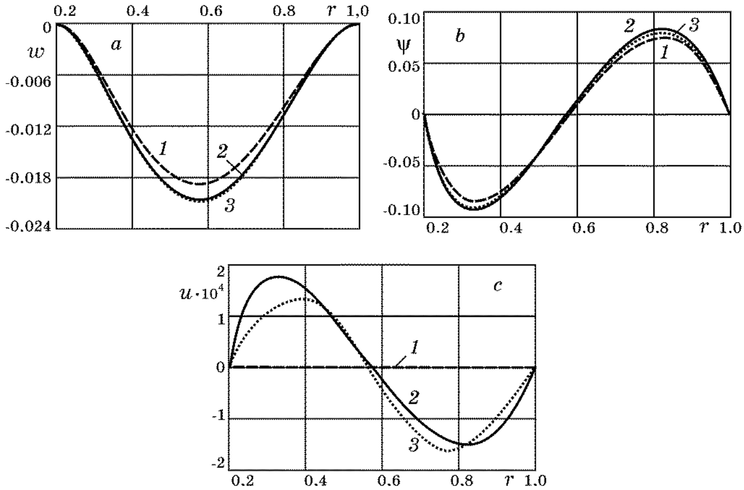

Fig. 2 – Change along the radius of the plate: a – deflection w, b – displacement in the core ψ, c – radial displacement u

The changes in the corresponding radial and tangential deformations on the outer surface of the first layer are shown in Figure 3. Plasticity causes an increase in the maximum radial deformations in the inner contour by 30%.

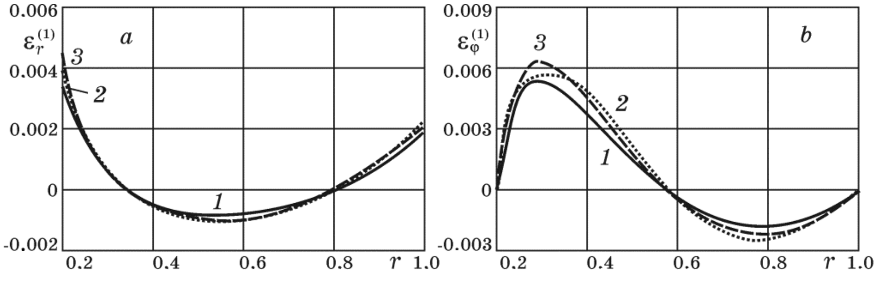

Fig. 3 – Variation of radial and angular deformations along the plate radius

Figure 4 shows the change in radial stresses along the thickness of the plate on its outer and inner contours. Due to heating, the first layer and part of the core expand and experience compression due to this.

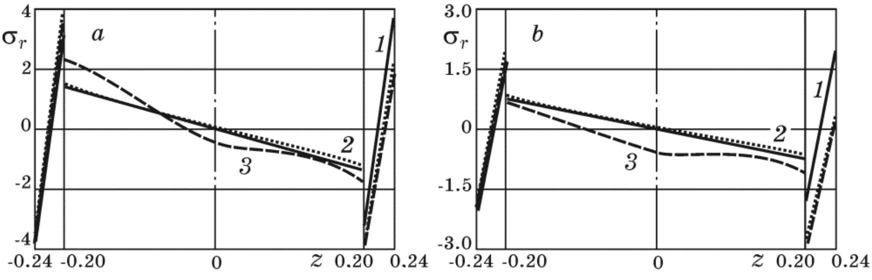

Fig. 4 – Variation of radial stresses along the plate thickness on: a – outer boundary, b – inner boundary

This causes them to shift the stress to the negative area. In the first layer, their increase in modulus is observed. In the second layer, the stress change is insignificant.

4. CONCLUSIONS

The description of the nonlinear deformation of layer materials is carried out using the relations of the theory of small elastoplastic deformations. The application of the method of linear approximation allows reducing the solution of a nonlinear system of equilibrium equations to the corresponding equations of the linear theory of elasticity with additional "external" forces determined by the results of the previous approximation. The resulting system of equilibrium equations is reduced to a single inhomogeneous fourth-order differential equation with respect to the deflection of the plate. The general solution of the corresponding inhomogeneous differential equation can be written out in Kelvin functions. The partial solution of an inhomogeneous equation with a uniformly distributed load is determined by a constant, in the case of a load of a more complex type – using the Cauchy kernel. The integration constants follow from the boundary conditions on the outer and inner plate contours at each step of the approximation.

Numerical studies were carried out for a plate with layers made of duralumin – fluoroplastic-4 – duralumin materials. It was found that with increasing base stiffness, the maximum deflection and relative shear significantly decrease. The stresses reach a maximum on the inner contour, where they are 1.5 times greater than the stresses on the outer contour. When the stiffness of the foundation increases to high, the maximum stresses decrease in modulus by 2.6 times. Thus, the proposed mathematical model allows studying the stress-strain state of physically nonlinear circular sandwich plates with a central hole connected to an elastic foundation of arbitrary rigidity, with different methods of fixing its external and internal contours under any axisymmetric loads. The resulting analytical solution can be used for conducting appropriate numerical experiments when performing calculations of composite structural elements in construction and mechanical engineering.

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