Longitudinal Absolute Stability of a BWB Aircraft-Pilot System with Saturated Actuator Model

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Abstract: *This paper deals with the analysis of the P(ilot) I(n-the-Loop) O(scillations) of the second category (with rate and position liming in the closed loop pilot-vehicle system), caused by the dynamic coupling between the human pilot and the aircraft. The analysis is made in the context of the longitudinal motion and the theoretical model of the airplane presented in this article is a (B)lended (W)ing (B)ody tailless configuration. In what concerns the human operator, this is expressed by the Synchronous Pilot Model, which is represented by a simple gain, without a specific delay. The Routh-Hurwitz criterion is used in order to analyze the longitudinal stability of the low-order pilot-airplane system without the influence of actuator nonlinearity (this means that the unsaturated actuator model is employed for the mentioned algebraic criterion). Most emphasis is put on the frequency Popov criterion, which is used to investigate the absolute stability property of the short-period model in the presence of the actuator rate saturation, in the condition of the Lurie problem. The transfer function of the longitudinal BWB model, obtained from open-loop analysis, has a double pole at the origin and, for the absolute stability feedback structure that contains the nonlinearity of the saturation type, the Popov frequency-domain inequalities are applied to the PIO II problem in this critical case.*

Keywords: *oscillations, Popov criterion, frequency domain inequality, Blended Wing Body, actuator saturation*

1. INTRODUCTION

The Pilot Induced Oscillations, Pilot Involved Oscillations or Pilot In-the-loop Oscillations, called PIOs for short, are dangerous phenomena known to have been the cause for several aircraft incidents and accidents (YF-22, JAS-39, X-31). In the scientific literature, a PIO phenomenon is generally defined as "sustained or uncontrollable oscillations resulting from the efforts of the pilot to control the aircraft" [1] or "inadvertent, sustained aircraft oscillation which is the consequence of an abnormal joint enterprise between the aircraft and the pilot".[2]

1.1 Classification of PIOs

According to common references (see [3]), PIOs phenomena can be divided into the following categories:

- **Category I**: Linear pilot-vehicle system oscillations, resulted from linear phenomena such as excessive lags introduced by filters, actuators, feel system and digital system time delays.

- **Category II**: Quasi-linear pilot-vehicle models, but with some nonlinear contribution, such as rate or position limiting.
- **Category III**: Severe life-threatening PIOs which are caused by nonlinearities and transitions in pilot or effective airplane dynamics.
- **Category IV**: Highly non-linear oscillations of the pilot-vehicle system. These PIOs are theoretically considered and less studied.

1.2 The BWB concept

Developed by the Boeing Company in support of NASA's Environmentally Responsible Aviation (ERA) programme, the flying wing "Blended Wing Body" (BWB) concept offers advantages in structural, aerodynamic and operating efficiencies over today's more conventional fuselage-and-wing designs. Among the advantages of the BWB configuration one can mention: high fuel efficiency, less noise and a greater lift-to-drag ratio than traditional aircraft (see Figure 1 for a comparision between the BWB configuration and wide-body aircraft).

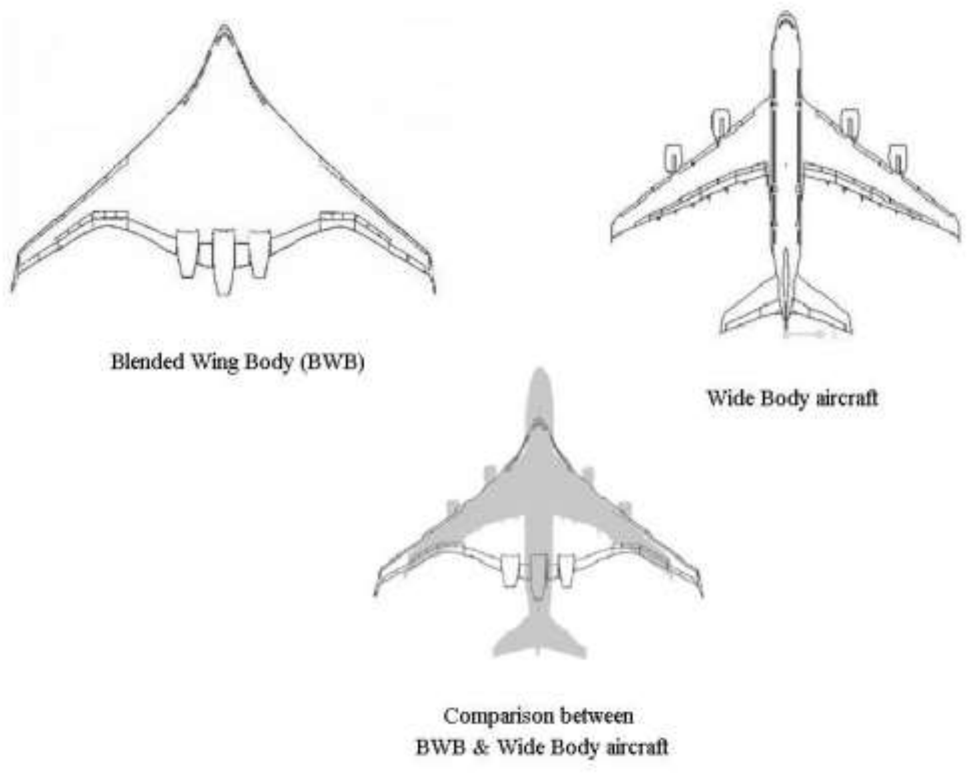

Figure 1 – Comparison between the BWB configuration and wide-body aircraft (adapted from [4])

Blended Wing Body aircraft, which can also be more simply called Blended Wing aircraft, or Hybrid Wing/Body aircraft, has an airframe design which incorporates design features of futuristic fuselage with a flying wing.

The body form is composed of distinct and separate wing structures, though the wings are smoothly blended into the body.

The original BWB configuration was conceived by McDonnel Douglas team. This first rudimentary design was the embryonic beginning of the BWB configuration.

An intuitive presentation of the BWB concept can be found in [5] pp. 71-92. Also, several researches regarding the BWB are, for example, [6] and [7].

2. THEORETICAL BACKGROUND

The general stabilization of the pilot-aircraft coupled system with rate saturation of the flight control surface actuator is shown in Figure 2, in which the limiter is of "AIAA type", as in [10], p. 166, and χ represents the output of the linear system.

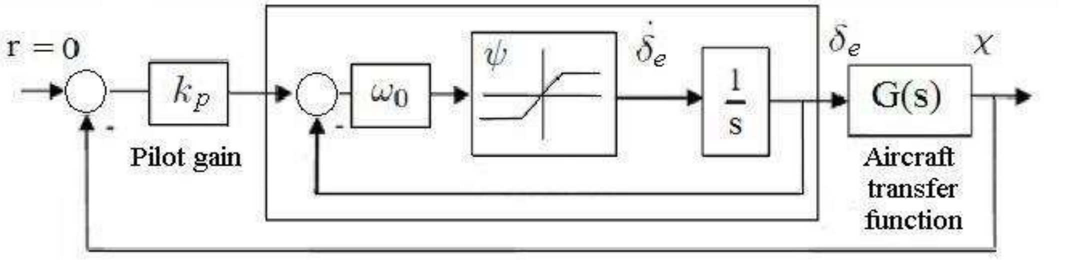

Figure 2 – Block diagram of the pilot-aircraft coupled system with rate limiter

In this paper δ_e represents the elevator deflection (the flight control surface is the elevator).

The rate saturation of the of the longitudinal flight control surface, δ_e, is defined as

$$\dot{\delta}_e = \begin{cases} \left|\dot{\delta}_e\right|, \; if \;\; \left|\dot{\delta}_e\right| < V_L \\ V_L, \; if \;\; \dot{\delta}_e \geq V_L \\ -V_L, \; if \;\; \dot{\delta}_e \leq -V_L \end{cases} \tag{1}$$

where V_L is the rate limit value.

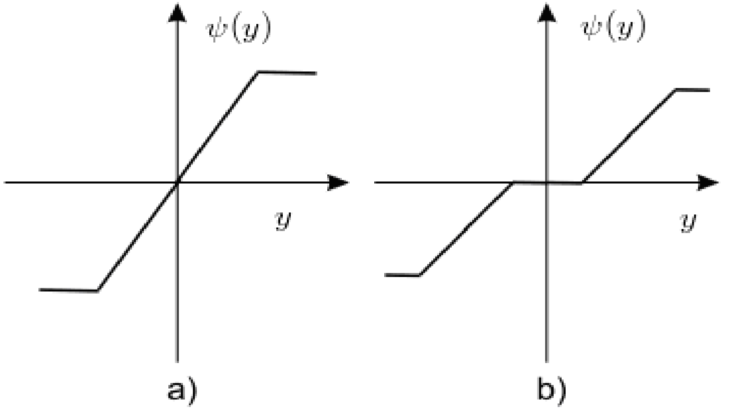

Figure 3 – Saturation nonlinearities: a) standard; b) with deadzone

The airplane dynamics is represented by the open-loop transfer function $G(s)$, expressed by

$$G(s) = c^T(sI - A)^{-1}b \tag{2}$$

and, throughout this paper, by ψ a nonlinear function is denoted (a saturation, like in Figure 3, which fulfills the following sector condition:

$$0 \le \underline{\psi} \le \frac{\psi(y)}{y} \le \overline{\psi} \le \infty,\ \psi(0) = 0 \tag{3}$$

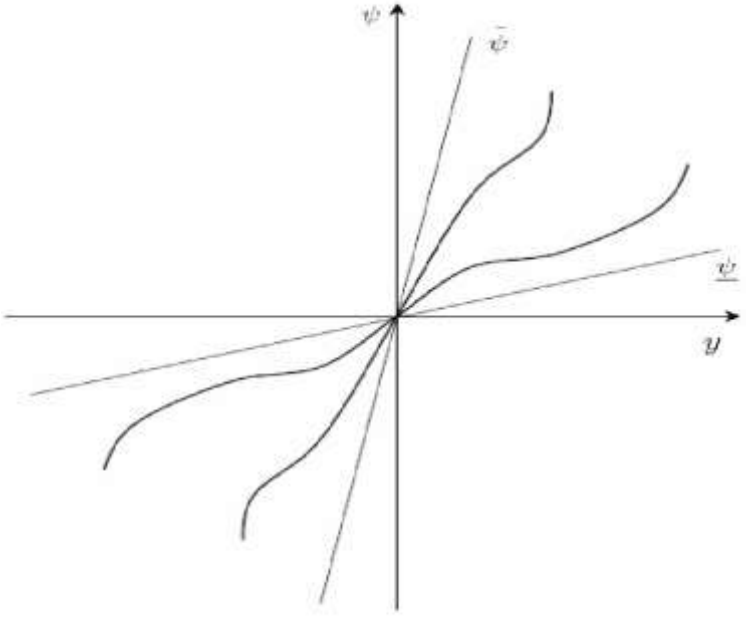

Figure 4 – Sector restricted nonlinearity

were $\overline{\psi}$, $\underline{\psi}$ and y can be seen in Figure 4.

2.1 Absolute Stability Problem

Absolute stability problem [11] refers to the global asymptotic stability of the zero equilibrium of the general nonlinear system

$$\dot{x}(t) = Ax(t) - b\psi(c^T x(t)) \tag{4}$$

having sector restricted nonlinearities of the form (3) and the property of the equilibrium being valid for all the linear and nonlinear functions verifying (4).

2.1.1 Lurie Control Problem

The following transformation (adapted from [12], pp. 57-58) of the pilot-aircraft system who has rate saturation can be written as bellow (see Figure 5), where the feedback structure which involves the absolute stability problem contains the saturation nonlinearity ψ , with L a linear block and N a non-linear one.

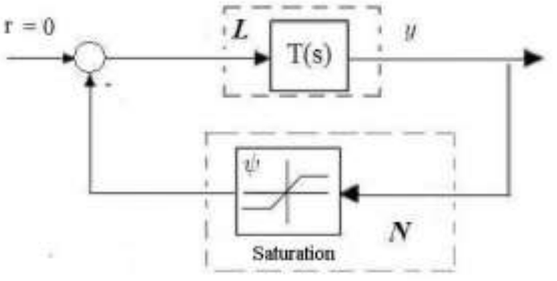

Figure 5 – Absolute stability feedback structure

In the example considered in this paper, the following equivalence between the systems (5)-(6) and (7) is used.

Further, the pilot-aircraft coupled system which has rate saturation, represented in Figure 2, is rewritten as a Lurie system.

$$\dot{x}_\alpha(t) = A_\alpha x_\alpha(t) - b_\alpha \psi(y(t)) \tag{5}$$

where

$$y(t) = c_\alpha{}^T x_\alpha(t) \tag{6}$$

and

$$\begin{cases} \dot{x}_\beta(t) = A_\alpha x_\beta(t) + b_\beta(t)\delta_e(t) \\ \dot{\delta}_e(t) = -\psi(\omega_0(c_\beta{}^T x_\beta(t) + \delta_e(t))) \end{cases} \tag{7}$$

δ_e was introduced as a state and the following notations were used:

$$\begin{cases} y(t) = \omega_0\left(c_\beta{}^T x_\beta(t) + \delta_e(t)\right) \\ u(t) = -\psi(y(t)) \end{cases} \tag{8}$$

Applying the Laplace transformation, the following system is obtained

$$\begin{cases} \tilde{x}_\beta(s) = \left(sI - A_\beta\right)^{-1} b_\beta \tilde{\delta}_e(s) \\ \tilde{\delta}_e(s) = \dfrac{1}{s}\tilde{u}(s) \end{cases} \tag{9}$$

From the above system one can obtain

$$\tilde{x}_\beta(s) = \frac{1}{s}\left(sI - A_\beta\right)^{-1} b_\beta \tilde{u}(s) \tag{10}$$

From (8), (9) and (10) it results

$$\tilde{y}(s) = \frac{\omega_0}{s}\tilde{u}(s)\left(c_\beta{}^T\left(sI - A_\beta\right)^{-1} b_\beta + 1\right) \tag{11}$$

Using the notation

$$G(s) = c_\beta^{\,T}\left(sI - A_\beta\right)^{-1} b_\beta \tag{12}$$

from (11) and (12) the following relation is determined

$$\tilde{y}(s) = \omega_0 \frac{G(s)+1}{s}\tilde{u}(s) \tag{13}$$

which is equivalent to

$$\tilde{y}(s) = T(s)\tilde{u}(s) \tag{14}$$

The transfer function $T(s)$ (in the case of rate limiter) is

$$T(s) = \frac{\tilde{y}(s)}{\tilde{u}(s)} = \omega_0\left(\frac{1}{s} + \frac{G(s)}{s}\right) \tag{15}$$

The relation (15) shows that the transfer function is in the critical case of a single zero root (the denominator of the transfer function that describes the characteristic polynomial which, in turn, describes the longitudinal stability characteristics of the airplane, has all the roots in C^{-} with the exception of one which is zero).

3. THE LOW-ORDER LONGITUDINAL BWB SYSTEM

The aerodinamic BWB model presented in this section was obtained using the paper [13], as the main reference, and also, taking into consideration [8] and [9]. The simplified linear system for the uncoupled longitudinal dynamics was considered taking into account the case of short-period approximation.

3.1 The Linear Model

The following short-period system with unsaturated actuator model is considered:

$$\begin{cases} \dot{\alpha} = q \\ \dot{q} = M_q q + M_{\delta_e}\delta_e \\ \dot{\delta}_e = (-\omega_0)\left[\left(k_\alpha + k_p\right)\alpha + k_q q + \delta_e\right] \end{cases} \tag{16}$$

where:

- α - incidence angle [rad];
- δ_e - elevator deflection [rad];
- q - pich rate [rad/s].

The aerodinamic constants and global gains of the system are given bellow:

- $M_q = -0.1556,\ M_{\delta_e} = -1.3495$
- $k_\alpha = 0.9,\ k_q = -1.27$

For the above system the elements from the state - space representation are:

$$x = \begin{bmatrix} \alpha \\ q \\ \delta_e \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & M_q & M_{\delta_e} \\ -\omega_0(k_\alpha + k_p) & -\omega_0 k_q & -\omega_0 \end{bmatrix}$$

By using the Routh-Hurwitz criterion can be verified the stability of the linear system (16).

Remark 3.1.1. The cut-off frequency of the first order actuator model $\omega_0 = 20\,\frac{\text{rad}}{\text{sec}}$ and the pilot gain $k_p = 1$.

The characteristic polynomial is

$$P(s) = s^3 + (\omega_0 - M_q)s^2 + \omega_0\left(k_q M_{\delta_e} - M_q\right) + \omega_0\left(k_\alpha + k_p\right)M_{\delta_e} \tag{17}$$

and the Routh-Hurwitz criterion gives the following conditions:

$$\begin{gathered} \omega_0 - M_q > 0 \\ \omega_0\left(k_\alpha + k_p\right)M_{\delta_e} > 0 \\ \left(\omega_0 - M_q\right)\left(k_q M_{\delta_e} - M_q\right) - \left(k_\alpha + k_p\right)M_{\delta_e} > 0 \end{gathered} \tag{18}$$

which are fulfilled for the given aerodynamic constants and global gains of the system.

3.2 The Nonlinear Model

The following pilot-aircraft nonlinear system is obtained by adding the rate limited actuator and the stability augmentation system (SAS) to the system (16):

$$\begin{cases} \dot\alpha = q \\ \dot q = M_q q + M_{\delta_e}\delta_e \\ \dot\delta_e = -\psi(\sigma) \end{cases} \tag{19}$$

with the following output of the linear system

$$\sigma = \omega_0\left[\left(k_\alpha + k_p\right)\alpha + k_q q + \delta_e\right] \tag{20}$$

Remark 3.2.1. The nonlinear function ψ denotes a saturation function and the following notation is made

$$u(t) = -\psi(\sigma(t)) \tag{21}$$

Using the Laplace transform, the following transfer function of the linear part of the system is obtained:

$$T(s) = \frac{\tilde\sigma(s)}{\tilde u(s)} = \omega_0\left[\frac{\left(k_\alpha + k_p\right)M_{\delta_e}}{s^2\left(s - M_q\right)} + \frac{k_q M_{\delta_e}}{s\left(s - M_q\right)} + \frac{1}{s}\right] \tag{22}$$

4. THE POPOV CRITERION, APPLICATION AND CONCLUSIONS

The Popov criterion [11] will be applied to the longitudinal BWB mathematical model with rate saturation of the actuator, in order to investigate the absolute stability of the pilot-airplane system, in the mentioned conditions.

We can apply the Popov criterion if the sector conditions (3) are satisfied. From [14] (pg. 246), for practical considerations, the following theorem is used (with $\underline{\psi} = 0$ and $k = \overline{\psi} - \underline{\psi}$):

Theorem 4.1 *Consider a Lurie system with a nonlinearity* ψ *in the sector* $[0, k]$. *The equilibrium in the origin is globaly asymptotically (exponentially) stable, provided that there exists the constant* $\xi > 0$ *such that the following inequality is true:*

$$\mathrm{Re}\left[(1 + j\omega\xi)T(j\omega)\right] > -\frac{1}{k}, \ \forall\omega \in \Re \tag{23}$$

The Popov criterion express a frequency condition for the global asymptotic stability property of a dynamical system, in the conditions of the Lurie problem ([15], pp. 263-264).

The frequency domain Popov condition can be writen as

$$\frac{1}{k} + \mathrm{Re}\left[(1 + j\xi\omega)\left(\mathrm{Re}(T(j\omega)) + j\,\mathrm{Im}(T(j\omega))\right)\right] > 0, \ \forall\omega \in \Re \tag{24}$$

which is equivalent with

$$\frac{1}{k} + \mathrm{Re}(T(j\omega)) - \xi\omega\,\mathrm{Im}(T(j\omega)) > 0, \ \forall\omega \geq 0 \tag{25}$$

In the relation (25) ω was restricted to positive values, given the properties of the transfer functions. Indeed, the real part of a transfer function is even (in ω, which means that $\mathrm{Re}(T(j\omega)) = \mathrm{Re}(T(-j\omega))$) and the imaginary part is odd (but when multiplied with ω it is also even, which means that $\omega\mathrm{Im}(T(j\omega)) = -\omega\mathrm{Im}(T(-j\omega))$).

If relation (25) is multiplied with $\frac{1}{\xi}$ (if $\xi > 0$), the following inequality results:

$$\frac{1}{k\xi} + \frac{1}{\xi}\mathrm{Re}(T(j\omega)) - \omega\,\mathrm{Im}(T(j\omega)) > 0, \forall\omega \geq 0 \tag{26}$$

For the above formally obtained relation, applying the limit $\xi \to \infty$, yields to:

$$-\omega\,\mathrm{Im}\left[\mathrm{T}(j\omega)\right] > 0, \forall\omega \geq 0 \tag{27}$$

which is the case of the "infinite Popov parameter".

Relations (25) and (27) are useful forms of applying the Popov criterion, using the above theorem.

4.1 The Popov Criterion Applied to BWB Model

From the last relation of subsection 3.2, the open-loop transfer function associated to the Lurie problem in the case of the feedback structure that contains the saturation nonlinearity is given by

$$T(s) = \omega_0 \frac{s^2 + \left(k_q M_{\delta_e} - M_q\right)s + \left(k_\alpha + k_p\right)M_{\delta_e}}{s^2\left(s - M_q\right)} \tag{28}$$

By making the substitution $s = j\omega$ in relation (28), the BWB transfer function becomes

$$T(j\omega) = \omega_0 \frac{-\omega^2 + \left(k_q M_{\delta_e} - M_q\right)j\omega + \left(k_\alpha + k_p\right)M_{\delta_e}}{\omega^2\left(M_q - j\omega\right)} \tag{29}$$

The frequency-domain transfer function is rewritten as follows:

$$T(j\omega) = \mathrm{Re}\left[T(j\omega)\right] + j\,\mathrm{Im}\left[T(j\omega)\right] \tag{30}$$

where

$$\mathrm{Re}\left[T(j\omega)\right] = \omega_0 \frac{-k_q M_{\delta_e}\omega^2 + \left(k_\alpha + k_p\right)M_{\delta_e}M_q}{\omega^2\left(\omega^2 + M_q^{\,2}\right)} \tag{31}$$

$$\mathrm{Im}\left[T(j\omega)\right] = \omega_0 \frac{-\omega^2 + k_q M_{\delta_e}M_q - M_q^2 + \left(k_\alpha + k_p\right)M_{\delta_e}}{\omega\left(\omega^2 + M_q^{\,2}\right)} \tag{32}$$

Considering the Popov inequality (25), in the case of the finite Popov parameter, and introducing the notations below

$$P_k = -\left(k_\alpha + k_p + k_q M_q\right)M_{\delta_e} \tag{33}$$

the frequency domain condition (25) in the presented case leads to the following inequality:

$$\left(1 - \omega_0 k\right)\omega^4 + \left[M_q^2 - \omega_0 k\left(k_q M_{\delta_e} + \xi\left(P_k + M_q^2\right)\right)\right]\omega^2 + \omega_0 k\left(k_\alpha + k_p\right)M_{\delta_e}M_q > 0 \tag{34}$$

Using the notation

$$\begin{aligned} f(\omega,\xi) = {} & \left(1 - \omega_0 k\right)\omega^4 + \left[M_q^2 - \omega_0 k\left(k_q M_{\delta_e} + \xi\left(P_k + M_q^2\right)\right)\right]\omega^2 + \\ & \omega_0 k\left(k_\alpha + k_p\right)M_{\delta_e}M_q \end{aligned} \tag{35}$$

the relation (34) becomes

$$f(\omega,\xi) > 0, \quad \forall\omega \ge 0 \tag{36}$$

Further, analyzing the monotony of the function $f(\omega,\xi)$, imposing the condition for positivity, the below limit is considered

$$\lim_{\omega\to\infty} f(\omega,\xi) = \infty \tag{37}$$

which is true for $k < \frac{1}{\omega_0}$.

For

$$\lim_{\omega \to 0} f(\omega, \xi) = \omega_0 k (k_\alpha + k_p) M_{\delta_e} M_q \tag{38}$$

it results

$$(k_\alpha + k_p) M_{\delta_e} M_q > 0 \tag{39}$$

For the finite Popov parameter the following condition is obtained

$$\xi < \frac{1}{P_k + M_q^2} \left(\frac{M_q^2}{\omega_0 k} - k_q M_{\delta_e} \right) \tag{40}$$

Numerically, for $k = \frac{1}{25}$

$$\xi < 0.61 \tag{41}$$

So, it is obtained a finite Popov parameter condition for the asymptotic stability of the trivial equilibrium point for the system (19).

In the case of the infinite Popov parameter, the following limits are computed

$$\lim_{\omega \to \infty} - \omega \operatorname{Im}[T(j\omega)] = \omega_0 > 0 \tag{42}$$

$$\lim_{\omega \to 0} - \omega \operatorname{Im}[T(j\omega)] = \omega_0 \left(1 + \frac{P_k}{M_q^2} \right) > 0 \tag{43}$$

It follows that the Popov frequency domain inequality is satisfied. So, the longitudinal BWB model is absolutely stable with the influence of actuator nonlinearity in the mentioned conditions, using the Popov Criterion for both finite and infinite parameter ξ.

5. CONCLUSIONS

The absolute stability of the presented longitudinal BWB model, with rate limiting in the closed loop pilot-vehicle system, was proved in the specified conditions, using the Popov criterion – for both finite and infinite parameter ξ – in the context of the associated Lurie problem. As a future work it can be considered the analysis of the models that are more complex in representation, in the presence of the delay asociated to the coupled pilot-aircraft system.

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