

A Linear Analysis of a Blended Wing Body (BWB) Aircraft Model¹

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Abstract: *In this article a linear analysis of a Blended Wing Body (BWB) aircraft model is performed. The BWB concept is in the attention of both military and civil sectors for the fact that has reduced radar signature (in the absence of a conventional tail) and the possibility to carry more people. The trim values are computed, also the eigenvalues and the Jacobian matrix evaluated into the trim point are analyzed. A linear simulation in the MatLab environment is presented in order to express numerically the symbolic computations presented. The initial system is corrected in the way of increasing the consistency and coherence of the modeled type of motion and, also, suggestions are made for future work.*

Key Words: *Lateral-directional, Blended Wing Body, Flight Control System, Stability, Simulation.*

1. INTRODUCTION

Blended Wing Body (BWB) aircraft, which are also more simply called Blended Wing aircraft, or Hybrid Wing/ Body aircraft, designates an alternative airframe design which incorporates design features from both a futuristic fuselage and flying wing design having potential advantages in aerial refueling role - less carburant consume (relative 20%) - and reduction of noise for example.

The MOB aerodynamic model used in this article is derived from the work done by Castro [5].

Several research studies indicate that the BWB concept offers a significant performance improvement as compared to conventional civil transportation aircraft due to its efficient aerodynamic configuration.

In literature, it is stated that this current BWB aircraft model is the result of a multidisciplinary design optimization performed by a consortium of companies, research institutes and universities.

The initial aircraft design was made by Cranfield University, where the European MOB (Multidisciplinary Optimisation Blended Wing Body) planform was initially analyzed by Castro [5] and later by Smith and Addasi [6].

The aim of the current research was to perform a linear analysis of a Blended Wing Body aircraft model in order to provide a basis for a future study on Pilot-Induced-Oscillations (PIO) proness of this model.

Depending on the model of the airplane used these oscillations can be studied in the longitudinal case in the lateral-directional case.

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The reader can consult, for example the master thesis [8] for the longitudinal case, and, the paper [9], for lateral directional case.

Trim analysis was done by solving the differential equations of motion (2) for a straight and level flight condition (see, for example, the algebraic relations (22) and (23)), for the lateral-directional system.

2. THE NONLINEAR MOB BWB MODEL

In order to obtain the non-linear BWB aerodynamic model (3) - which is also treated in [1], derived from the work done by Castro [5] - one must consider the the general equations of unsteady motion given bellow (see [7], p. 104):

$$\left. \begin{aligned}
 X - mg \sin \theta &= m(\dot{u}^E + qw^E - rv^E) \\
 Y + mg \cos \theta \sin \phi &= m(\dot{v}^E + ru^E - pw^E) \\
 Z + mg \cos \theta \cos \phi &= m(\dot{w}^E + pv^E - qu^E) \\
 L &= I_x \dot{p} - I_{zx} \dot{r} + qr(I_z - I_y) - I_{zx} pq + qh'_z - rh'_y \\
 M &= I_y \dot{q} + rp(I_x - I_z) + I_{zx}(p^2 - r^2) + rh'_x - ph'_z \\
 N &= I_z \dot{r} - I_{zx} \dot{p} + pq(I_y - I_x) + I_{zx} qr + ph'_y - qh'_x \\
 p &= \dot{\phi} - \dot{\psi} \sin \theta \\
 q &= \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi \\
 r &= \dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi \\
 \dot{\phi} &= p + (q \sin \phi + r \cos \phi) \tan \theta \\
 \dot{\theta} &= q \cos \phi - r \sin \phi \\
 \dot{\psi} &= (q \sin \phi + r \cos \phi) \sec \theta \\
 \dot{x}_E &= u^E \cos \theta \cos \psi + v^E (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\
 &\quad + w^E (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\
 \dot{y}_E &= u^E \cos \theta \sin \psi + v^E (\sin \phi \sin \theta \sin \psi + \cos \phi \sin \psi) \\
 &\quad + w^E (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\
 \dot{z}_E &= -u^E \sin \theta + v^E \sin \phi \cos \theta + w^E \cos \phi \cos \theta \\
 u^E &= u + W_x \\
 v^E &= v + W_y \\
 w^E &= w + W_z
 \end{aligned} \right\} \tag{1}$$

Remark 1. Through this article the following unit measure are used:

- m - meter (for length)
- m^2 - square meter (for surface)
- s - second (for time)
- kg – kilogram (for mass)
- N - newton (for force)

In the system (1) we have:

- S - wing reference area (m^2)
- b - wing span (m)
- I_{xx} - product of inertia about X axis
- I_{yy} - product of inertia about Y axis
- I_{zz} - product of inertia about Z axis
- m - aircraft mass (kg)
- v - aircraft speed along Y body axis ($\frac{m}{s}$)
- p - variation of the ϕ roll angle (roll rate)
- r - variation of the yaw angle (yaw rate about the yaw axis)
- ϕ - roll angle of the aircraft (rad)
- θ, ψ - is the pitch and respectively the yaw angle
- q - variation of the pitch angle (pitch rate)
- (u^E, v^E, w^E) velocity vector relative to Earth frame of reference
- $qh'_z - rh'_y, rh'_x - ph'_z, ph'_y - qh'_x$ - gyroscopic couples
- L, M, N - rolling, pitching and yawing moments (in Nm)
- X, Y, Z - axial, lateral and normal force components (in N)

Remark 2. Nm represent the product between 1 newton multiplied with 1 meter and it is used for expressing the moment.

Considering $I_{xx} = I_x, I_{yy} = I_y, I_{zz} = I_z, I_{xz} = I_{zx} = 0$, the gyroscopic couples close to zero, for the lateral directional motion, in the case of the BWB aircraft we obtain the following system of differential equations:

$$\begin{cases} \dot{Y} = \frac{Y}{m} + pW - rU + g \cos \theta \sin \phi \\ \dot{p} = \frac{L}{I_{xx}} - qr \frac{I_{zz} - I_{yy}}{I_{xx}} \\ \dot{r} = \frac{N + N_E}{I_{zz}} - pq \frac{I_{yy} - I_{xx}}{I_{zz}} \\ \dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta \end{cases} \quad (2)$$

Remark 3. N_E denotes the yawing moment induced by the engine relative to the center of gravity of the aircraft and this term should be zero if all the engines work at the same thrust amount.

Remark 4. In the system bellow, taken from [1], in the first equation, actually we must have the term $g \cos \theta \sin \phi$ with the "+" sign, taking into account the fact that in [1] the Jacobian matrix shows this term derived with plus and also because in [2], p. 95 we have also the term with "+" sign.

Because of these considerations, the system (3) must be written as (2).

$$\begin{cases} \dot{V} = \frac{Y}{m} + pW - rU - g \cos \theta \sin \phi \\ \dot{p} = \frac{L}{I_{xx}} - qr \frac{I_{zz} - I_{yy}}{I_{xx}} \\ \dot{r} = \frac{N + N_E}{I_{zz}} - pq \frac{I_{yy} - I_{xx}}{I_{zz}} \\ \dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta \end{cases} \quad (3)$$

For system (2) we consider the numerical values given below

$$\begin{cases} S = 841.70 \text{ m}^2 \\ I_{xx} = 4.7032 \text{ e}07 \text{ kg} - \text{m}^2 \\ I_{yy} = 2.5069 \text{ e}07 \text{ kg} - \text{m}^2 \\ I_{zz} = 9.9734 \text{ e}07 \text{ kg} - \text{m}^2 \\ b = 80.00 \text{ m} \\ m = 371280 \text{ kg} \end{cases} \quad (4)$$

One can denote the fact that the mass and inertias parameters for the MOB BWB aircraft are relatively similar with the Boeing 747 (see [4], p. 210).

The side force along positive Y body axis and the roll/yaw moments about the X and Z axis respectively are given by

$$\begin{cases} Y = \bar{q} S C_y \\ L = \bar{q} S C_l b \\ N = \bar{q} S C_n b \end{cases} \quad (5)$$

where we have the following aerodynamic coefficients which are expressed bellow

- C_y - aerodynamics side force
- C_l - rolling moment coefficient
- C_n - yawing moment coefficient
- \bar{q} - dynamic pressure ($\frac{N}{m^2}$)

Remark 4. $\frac{N}{m^2}$ represent 1 newton per 1 square meter and it used for expressing the dynamic pressure.

These non-dimensional terms are in general functions of center of gravity (CG) position and angle of attack of a Blended Wing Body aircraft.

$$C_y = C_{y\beta} \beta + \frac{b}{V_t} (C_{y_r} r + C_{y_p} p) + C_{y_{\delta r}} \delta r \quad (6)$$

$$C_l = C_{l\beta} \beta + \frac{b}{V_t} (C_{l_r} r + C_{l_p} p) + C_{l_{\delta r}} \delta r + C_{l_{\delta \alpha}} \delta \alpha \quad (7)$$

$$C_n = C_{n_\beta} \beta + \frac{b}{V_t} (C_{n_r} + C_{n_p} p) + C_{n_{\delta r}} \delta r + C_{n_{\delta a}} \delta a \tag{8}$$

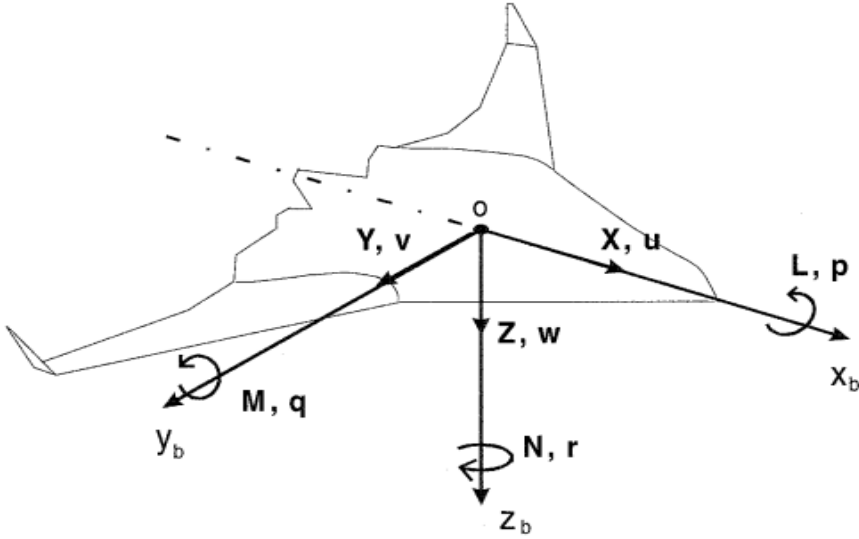


Fig. 1 – Body reference axis orientation

From [3], p. 74, assuming $\sin \beta \approx \beta$, in the context of relations between $V_t \cos \alpha \cos \beta$, $V_t \sin \beta$, $V_t \sin \alpha \cos \beta$ it follows that

$$v \approx V_t \beta \tag{9}$$

where V_t represents the total velocity.

The input vector U is two-dimensional for the model presented is given by

$$U = (\delta a \quad \delta r)^T \tag{10}$$

in which

- T denotes transpose since U is a column vector;
- δa is the aileron deflection angle;
- δr is the rudder deflection angle.

3. LATERAL-DIRECTIONAL NORMALISED DERIVATES

Body axis systems are always fixed to the body - they move and rotate with it. The orientation of the body axes is shown in Fig. 1, along with the notation for positive linear force (X, Y, Z), velocity (u, v, w), angular velocity components (p, q, r) and moment (L, M, N). The X axis is measured positive forward of midships, and negative aft.

The Y axis is positively measured to starboard, and negative to port. The Z axis is measured as positive down, and negative up.

Table 1 – Lateral-directional normalised derivatives

Dimensionless coefficient	Multiplier	Dimensional
$C_{y\beta}$	$\frac{\bar{q}S}{mV_t}$	Y_v
C_{yp}	$\frac{\bar{q}Sb}{mV_t}$	Y_p
C_{yr}	$\frac{\bar{q}Sb}{mV_t}$	Y_r
$C_{y\delta\alpha}$	$\frac{\bar{q}S}{m}$	$Y_{\delta\alpha}$
$C_{y\delta r}$	$\frac{\bar{q}S}{m}$	$Y_{\delta r}$
$C_{l\beta}$	$\frac{\bar{q}Sb}{I_{xx}V_t}$	L_v
C_{lp}	$\frac{\bar{q}Sb^2}{I_{xx}V_t}$	L_p
C_{lr}	$\frac{\bar{q}Sb^2}{I_{xx}V_t}$	L_r
$C_{l\delta\alpha}$	$\frac{\bar{q}Sb}{I_{xx}}$	$L_{\delta\alpha}$
$C_{l\delta r}$	$\frac{\bar{q}Sb}{I_{xx}}$	$L_{\delta r}$
$C_{n\beta}$	$\frac{\bar{q}Sb}{I_{zz}V_t}$	N_v
C_{np}	$\frac{\bar{q}Sb^2}{I_{zz}V_t}$	N_p
C_{nr}	$\frac{\bar{q}Sb^2}{I_{zz}V_t}$	N_r
$C_{n\delta\alpha}$	$\frac{\bar{q}Sb}{I_{zz}}$	$N_{\delta\alpha}$
$C_{n\delta r}$	$\frac{\bar{q}Sb}{I_{zz}}$	$N_{\delta r}$

For the lateral-directional movement we have:

- Lateral force component/ Side force (Y)
- Rolling moment (L)
- Yawing moment (N)
- U – Axial velocity/ Total linear velocity
- V – Lateral velocity components of the cg
- W – Normal velocity

The expressions for the Y, L, N dimensional derivatives are given in Table 1.

4. BWB - TRIM VALUES COMPUTATION

For the system (2) we make the notations that can be taken from the table above (Table 1).

For the trim state we have $\bar{X} = 0_4$, where, for the system (2), the trim vector is given by

$X = (v \quad p \quad r \quad \phi)^T$, where T denotes the transpose of the column vector X.

Using the Table 1, the system (2) transforms itself into the following system:

$$\begin{cases} \dot{v} = Y_v v - ru + pW + Y_p p + Y_r r + g \cos \theta_1 \sin \phi + Y_{\delta r} \delta r \\ \dot{p} = L_v v + L_p p + L_r r + L_{\delta a} \delta a + L_{\delta r} \delta r \\ \dot{r} = N_v v + N_p p + N_r r + N_{\delta a} \delta a + N_{\delta r} \delta r \\ \dot{\phi} = p + r \cos \phi \tan \theta_1 \end{cases} \quad (11)$$

The homogenous system associated to the system (11) is given bellow:

$$\begin{cases} L_v \bar{v} + (W_1 + Y_p) \bar{p} + (Y_r - U_1) \bar{r} + g \cos \theta_1 \sin \bar{\phi} + Y_{\delta r} \bar{\delta r} = 0 \\ L_v \bar{v} + L_p \bar{p} + L_r \bar{r} + L_{\delta a} \bar{\delta a} + L_{\delta r} \bar{\delta r} = 0 \\ N_v \bar{v} + N_p \bar{p} + N_r \bar{r} + N_{\delta a} \bar{\delta a} + N_{\delta r} \bar{\delta r} = 0 \\ \bar{p} + \bar{r} \cos \bar{\phi} \tan \theta_1 = 0 \end{cases} \quad (12)$$

where the barred symbols $(\bar{v} \quad \bar{p} \quad \bar{r} \quad \bar{\phi})$ represents the trim state components.

From [1], p. 3, it follows that

$$\bar{\phi} = 0 \quad (13)$$

From the last equation of the system (12) and (13), we have the following relation

$$\bar{p} = -\tan(\theta_1) \bar{r} \quad (14)$$

From relations (12), (13) and (14), we have the results given by

$$\bar{r} = \frac{N_{\delta a} \bar{\delta a} + N_{\delta r} \bar{\delta r} + N_v \bar{v}}{N_p \tan(\theta_1) - N_r} \quad (15)$$

$$\bar{v} = \frac{1}{Y_r} [(W_1 + Y_p) \tan(\theta_1) - Y_r + U_1] \frac{N_v \bar{v} + N_{\delta a} \bar{\delta a} + N_{\delta r} \bar{\delta r}}{N_p \tan(\theta_1) - N_r} - \frac{Y_{\delta r}}{Y_v} \bar{\delta r} \quad (16)$$

$$\bar{v} = \frac{Y_v}{Y_v - N_v} h_1 h_2 \left[\frac{h_1 h_2}{Y_r} N_{\bar{\alpha}\alpha} \bar{\delta}a + \left(\frac{h_1 h_2}{Y_r} N_{\bar{\delta}r} - \frac{Y_{\bar{\delta}r}}{Y_v} \right) \bar{\delta}r \right] \tag{17}$$

We have used the following notations:

$$h_1 = (W_1 + Y_p) \tan(\theta_1) - Y_r + U_1 \tag{18}$$

$$h_2 = \frac{1}{(N_p \tan(\theta_1) - N_r)} \tag{19}$$

$$\bar{\delta}a = \frac{1}{L_{\bar{\delta}r} N_{\bar{\alpha}\alpha} - N_{\bar{\delta}r} L_{\bar{\alpha}\alpha}} (q_1 \bar{v} + q_2 \bar{p} + q_3 \bar{r}) \tag{20}$$

where

- $q_1 = N_{\bar{\delta}r} L_v - N_v L_{\bar{\delta}r} N_{\bar{\alpha}\alpha} + N_v N_{\bar{\delta}r} L_{\bar{\alpha}\alpha}$
- $q_2 = N_{\bar{\delta}r} L_p - N_p L_{\bar{\delta}r} N_{\bar{\alpha}\alpha} + N_p N_{\bar{\delta}r} L_{\bar{\alpha}\alpha}$
- $q_3 = N_{\bar{\delta}r} L_r - N_r L_{\bar{\delta}r} N_{\bar{\alpha}\alpha} + N_r N_{\bar{\delta}r} L_{\bar{\alpha}\alpha}$

$$\bar{\delta}r = \frac{-1}{L_{\bar{\delta}r} N_{\bar{\alpha}\alpha} - N_{\bar{\delta}r} L_{\bar{\alpha}\alpha}} (q_4 \bar{v} + q_5 \bar{p} + q_6 \bar{r}) \tag{21}$$

where

- $q_4 = L_{\bar{\alpha}\alpha} N_v - L_v N_{\bar{\alpha}\alpha}$
- $q_5 = L_{\bar{\alpha}\alpha} N_p - L_p N_{\bar{\alpha}\alpha}$
- $q_6 = L_{\bar{\alpha}\alpha} N_r - L_r N_{\bar{\alpha}\alpha}$

From symbolic computations (making substitutions into the 2nd and 3rd equations of the system (12), we have obtained that

$$\bar{\delta}a = 0 \tag{22}$$

$$\bar{\delta}r = 0 \tag{23}$$

The result obtained from relations (22), (23), (14), (15) and (17), is given by the following trim point:

$$\bar{X} = \{0,0,0,0\} \tag{24}$$

In the numerical analysis bellow we consider the following initial point, representing an arbitrary perturbed state:

$$\bar{X} = \{0.01,0.04,0.06,0\} \tag{25}$$

Remark 5. The $\bar{\delta}r$ and $\bar{\delta}a$ are given analytically in (20) and (21), however a numeric evaluation of them, for the arbitrary perturbed state has not been made since, using the (22) and (23), (25) converge to (24).

The values in (18) are as follows

- $V_t = U_1 = 102.8889 \frac{m}{s}$
- $\theta_1 = 0.1907$ (rad)

The Jacobian matrix of the system (2) is

$$\begin{pmatrix} Y_v & Y_p + W_1 & Y_r - U_1 & g \cos(\theta_1) \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & \tan(\theta_1) & 0 \end{pmatrix}$$

where the terms (U_1, W_1) are steady state X and Z axis body velocities.

The Jacobian matrix evaluated into the initial point (24) is

$$\begin{pmatrix} -0.0246999780998 & 22.3920241159172 & -102.6600452942782 & 0.096321622923765 \\ -0.0075135424154 & -1.9804481742410 & 0.7910211128694 & 0 \\ -0.0013858736331 & -0.4068870370948 & -0.0486079816127 & 0 \\ 0 & 10000000000000 & 0.1930458262534 & 0 \end{pmatrix}$$

The eigenvalues of the Jacobian matrix are:

- -1.8259
- -0.0572 + 0.2488i
- -0.0572 - 0.2488i
- -0.1134

Remark 6. From eigenvalues we have obtained that the linearized system around the trim point is stable.

5. MATLAB LINEAR SIMULATION WITH ZERO INPUT

In this section we present a MatLab linear simulation for system

$$\Delta \dot{x} = A \Delta x + B \Delta u \tag{26}$$

where we have made the following notations:

- A is the Jacobian matrix of system (2) evaluated in the initial point (25);
- $\Delta x = x - \bar{x}$;
- $\Delta u = u - \bar{u}$;
- $B = \begin{pmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{pmatrix}$.

Numerically,

$$B = \begin{pmatrix} -0.1038 & 2.753624111735078 \\ -0.52124276547619 & 0.580912161830243 \\ 0.004214512263421 & -0.200189332512483 \\ 0 & 0 \end{pmatrix}$$

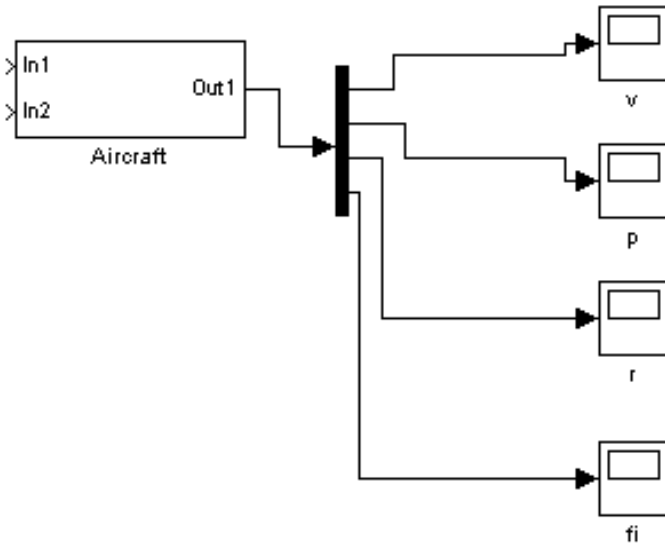


Fig. 2: Simulink scheme of the system (25)

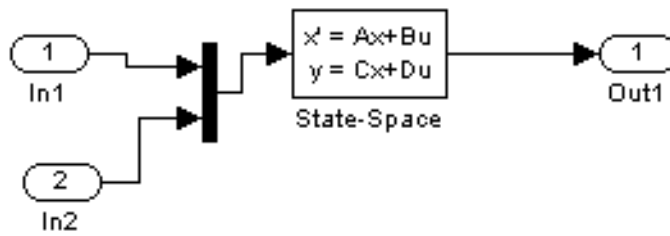


Fig. 3: The aircraft subsystem of the system (25)

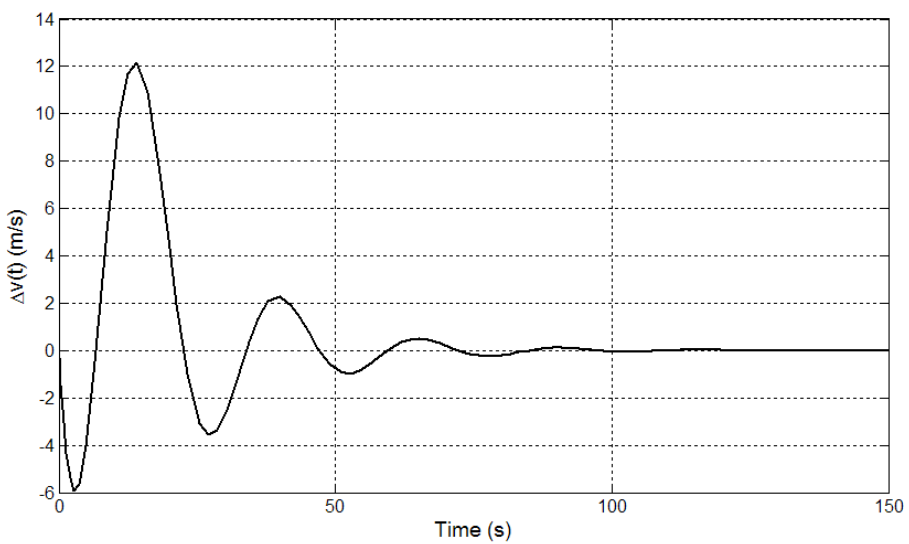


Fig. 4: Δv output of the system (25)

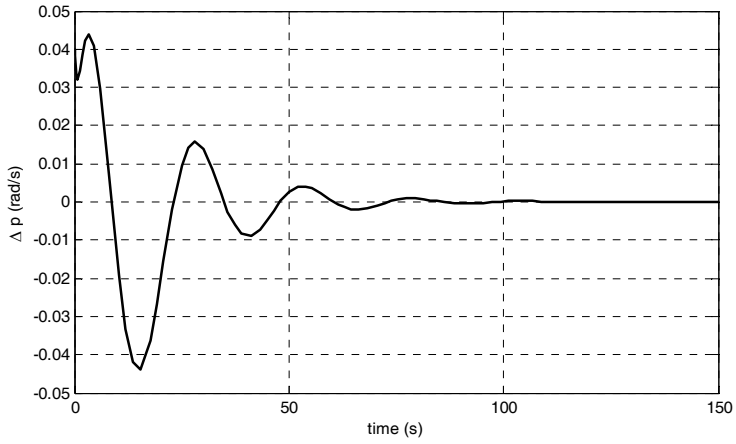


Fig. 5: Δp output of the system (25)

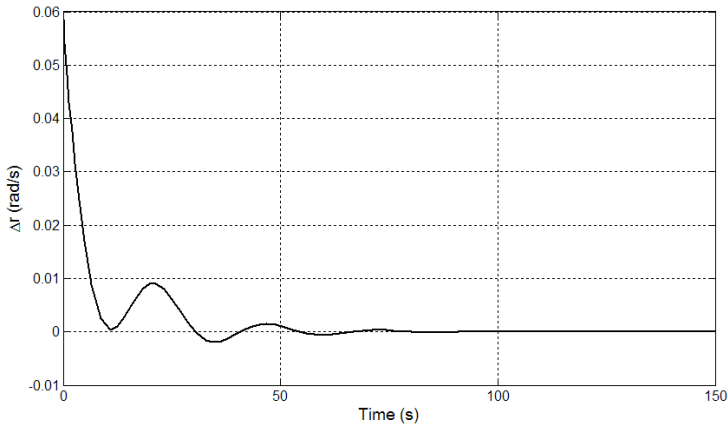


Fig. 6: Δr output of the system (25)

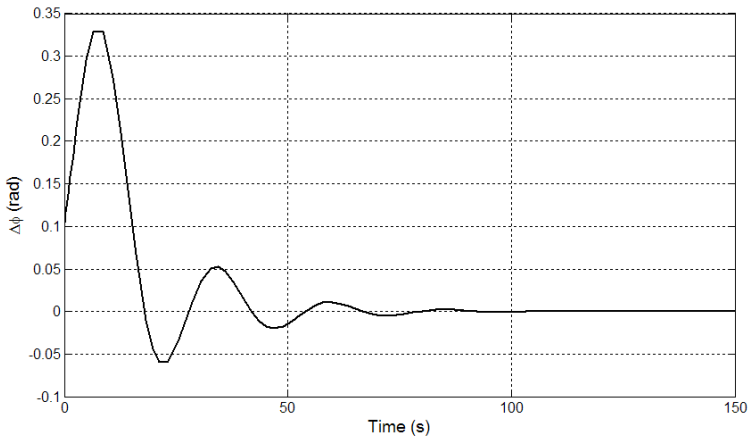


Fig. 7: $\Delta \phi$ output of the system 25

6. CONCLUSIONS AND FUTURE WORK

In this paper corrections have been made to the original system proposed in [1] (we took the term $g \cos \theta \sin \phi$, correctly with the "+" sign) and, like in the original article, the stability of the system is highlighted (the real parts of the eigenvalues associated to the the linearized system around the trim point are negative).

In the case of the numerical simulations it is shown the fact that the system (2) converge from the initial point (25) to the trim point (24) (Figures 4-7).

The results obtained are based on correct assumptions (correct sign, correlations with other references, for example [2]) and, as a future work a PIO susceptibility analysis should be performed together with a more extended numerical simulation regarding the non-linear BWB model.

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