Distributed H∞ State Feedback Control for Multi-Agent Systems with Imperfect Communication Networks

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Abstract: *The main objective of this paper consists in the analysis and design of distributed control systems for multi-agent systems simultaneous with various types of disturbances attenuation. The distributed structures refer to those structures for which the control laws depend only on neighbouring agents states, more precisely, communication between certain agents is missing. The design approach is based on the definition of a H∞ type cost function for the entire system, a frequently used method, especially for aerospace applications with a single agent. As a case study, a flight formation consisting of four agents with identical dynamics is used, whose time evolutions are analysed according to the effects of different communication networks failures.*

Key Words: *multi-agent systems, distributed control, H∞ design, imperfect communication networks*

1. INTRODUCTION

One of the current issues of aerospace engineering refers to the modelling and control of complex systems with different dynamics, named agents, which are characterised by the capacity of information transmission due to communication channels. The importance of this topic lies in the diversification of their missions both in the aeronautical and space fields. Therefore, the design objectives of the automatic control systems faced various challenges regarding specific requirements such as taking into account the time delays influences or certain agents failure.

The multi-agent systems, denoted MAS, applications have a rather long history, holding the researchers attention along with the significant increase in their applicability. Thus, the last decades offered a new perspective, replacing the individual agent missions with networked controlled systems, initial developments and studies being found in works such as [1], [2]. Many aspects of coordinated flight can be studied in the paper [3] which includes recent research of several multi-agent systems approaches, topics and introduced issues.

Considering that the recent period contributed to the appearance of a wide number of research works, the multi-agent systems topic can be viewed from various perspectives. The paper [4] represents an elaborated study of the MAS particularities, where preliminary elements, notions related to the applicability areas or control systems design challenges are relieved. The progress regarding the multi-agent systems coordination and control, taking into account the way of information transmission using communication networks, is described in several papers, such as [5], [6], [7].

Taking into consideration the networked control systems expertise, both centralised configurations of control systems, that require information from all agents, and distributed structures, for which the control laws depend only on neighbouring agents states, have been developed. A comparative study of these two approaches is represented by the reference [8], which includes theoretical notions related to their specific characteristics. Unlike the centralised case that involves the interconnection of all networked members, the distributed control supposes a specific structure, namely, the information transmission occurs between certain pairs of agents. Significant theoretical results of this type of control are shown in [9], and progress regarding this research direction is presented in [10].

Compared to the centralised case that presents difficulties in data processing, the efficiency of distributed control is remarked, especially for systems with a large number of agents. This type of control has been intensively treated, using various approaches, in works such as [11], [12], [13], [14]. For instance, the paper [11] focuses on the obtained results using coupled linear matrix inequalities (LMIs) for a distributed controller design to ensure $H\infty$ performances. The objective of [14] is to determine a distributed control law for an autonomous vehicles network with simple double-integrator dynamics, considering information restrictions about agents states.

The networked systems modelling consists in a combination between control and communication theories. Being a quite recent approach, the topic of these systems requires special attention in order to develop and improve existing results. The challenges introduced by the complexity of such systems refer to the influences of communication channels imperfections on their stability, leading to the appearance of a significant number of recent works and one can mention a few, such as: [15], [16], [17].

The networked agents communication has an important role in control system design because of the various imperfections of the communication channels. The present work focuses on the control of multi-agent systems with identical dynamics in which the minimization of a $H\infty$ cost function is aimed, a problem addressed in various specialized studies. For instance, distributed control system design is treated in reference [8] for interconnected identical systems based on the H_2 and $H\infty$ performance criteria. Using the $H\infty$ method, the influences of time delays on the stability of two different flight configurations is treated in paper [18]. The problem solution is reduced to solving two specific Riccati equations, noting that the structure of the obtained distributed controller corresponds to the interconnection mode of the agents. The reference [19] focuses on the problem of filtering external disturbances in the case of multi-agent systems. This can be solved using $H\infty$ control applied to independent systems.

The present paper focuses on the analysis and design of distributed control systems for multi-agent systems simultaneous with various types of disturbances attenuation. Considering the multitude of external factors that can affect the achievement of a mission's objectives, an optimal coordination solution is needed in order to guarantee the desired performances. These situations involve the failure of certain agent for a specified period of time, the temporary communication disconnection for the entire flight formation and the effects of time delays on the vehicles behaviour.

This work is organised in several parts as follows. After presenting a few preliminaries in the second part, in the third section a distributed controller design methodology based on the $H\infty$ norm minimization is developed. The following part includes the case studies which implies a flight formation consisting of four identical agents. The comparative results analysis considering the effects of different communication networks failures is presented in the last part dedicated to the concluding remarks.

2. PRELIMINARIES

The dynamics of a flight formation with identical agents defined as follows is considered.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1u_1(t) + B_2u_2(t) \\ y_1(t) &= Cx(t) + Du_2(t) \\ y_2(t) &= x(t), t \geq 0 \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u_1(t) \in \mathbb{R}^{m_1}$ represents an exogenous input, $u_2(t) \in \mathbb{R}^{m_2}$ denotes the control variable, $y_1(t) \in \mathbb{R}^{n_1}$ is the quality output and y_2 stands for the measured output.

In this development, there are two conditions which are assumed to be true: $C^TD = 0$ and $D^TD = I$. If the previous assumptions are not accomplished, for invertible D^TD, one may consider the change of control variable u_2 as follows:

$$u_2 = -\left(D^TD\right)^{-1}D^TCx + \left(D^TD\right)^{-\frac{1}{2}}\tilde{u}_2.$$

In this case, it can be seen that the conditions hold for the new control variable $\tilde{u}_2$. As proved in Theorem 1 in [18], there exists a state-feedback gain $F \in \mathbb{R}^{m_2 \times n}$ so that the closed-loop system obtained from (1) with $u_2(t) = Fy_2(t)$ is stable and it has the property that for $x(0) = 0$ and for a given value $\gamma > 0$,

$$\int_0^\infty (|y_1(t)|^2 - \gamma^2|u_1(t)|^2)\,dt < 0 \tag{2}$$

for all $\forall u_1 \in \mathcal{L}^2([0,\infty), \mathbb{R}^{m_1})$, where $\mathcal{L}^2([0,\infty), \mathbb{R}^{m_1})$ is the space of all m_1- dimensional square-integrable functions and where $|\cdot|$ denotes the Euclidian norm, if the algebraic Riccati equation.

$$A^TX + XA + \gamma^{-2}XB_1B_1^TX - XB_2B_2^TX + C^TC = 0 \tag{3}$$

has a stabilising solution $X \geq 0$ and in this case,

$$F = -B_2^TX. \tag{4}$$

This approach requires the interaction graphs introduction that describe the information communication between agents, defined by specific matrix forms of graph theory, especially by the adjacency matrix. The control and stabilisation performances of the multi-agent system essentially depends on the interaction graphs. Therefore, it is reminded that the adjacency matrix is defined as the matrix form that indicates the existing connection between two agents, denoted $\mathcal{A}(\mathcal{G})$, expressed by the following form:

$$\mathcal{A}(\mathcal{G}) = \begin{cases} a_{ii} = 0, & \forall i \in \mathcal{V} \\ a_{ij} = 0, & (i,j) \notin E, \forall i,j \in \mathcal{V}, i \neq j \\ a_{ij} = 1, & (i,j) \in E, \forall i,j \in \mathcal{V}, i \neq j \end{cases}. \tag{5}$$

More specific, matrices properties and notions regarding graph theory are detailed in various references such as [21], [22].

3. H∞ STATE FEEDBACK CONTROL FOR MULTI-AGENT SYSTEMS

The aim of this part lies in the development of a distributed controller design methodology based on the H∞ norm minimization. The main advantages of this method consist in the attenuation of disturbances effects and in the stability robustness performances regarding uncertainties or parameter variations.

In order to develop the H∞ design method, it is necessary to define the dynamics for a network consisting of N identical agents as:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{B}_1\tilde{u}_1(t) + \tilde{B}_2\tilde{u}_2(t) \\ \tilde{y}_1(t) &= \tilde{C}\tilde{x}(t) + \tilde{D}\tilde{u}_2(t) \\ \tilde{y}_2(t) &= \tilde{x}(t), t \geq 0 \end{aligned} \tag{6}$$

where $\tilde{A} = I_N \otimes A$, $\tilde{B}_1 = I_N \otimes B_1, \tilde{B}_2 = I_N \otimes B_2, \tilde{C} = I_N \otimes C$, the matrices A, B_1, B_2, C correspond to a single agent dynamics, $\tilde{x} = \left[x_1^T \dots x_N^T\right]^T$, $\tilde{u}_1 = \left[u_{1_1}^T \dots u_{1_N}^T\right]^T$, $\tilde{u}_2 = \left[u_{2_1}^T \dots u_{2_N}^T\right]^T$, $\tilde{y}_1 = \left[y_{1_1}^T \dots y_{1_N}^T\right]^T$, $\tilde{y}_2 = \left[y_{2_1}^T \dots y_{2_N}^T\right]^T$ and $\otimes$ denotes the Kronecker product. In addition, the two above-mentioned conditions are satisfied.

For $\gamma > 0$, the following cost function can be defined:

$$\begin{aligned} J\left(u_{1_1}, \dots, u_{1_N}, u_{2_1}, \dots, u_{2_N}\right) = \int_0^\infty [\textstyle\sum_{i=1}^N (|y_{1i}(t)|^2 - \gamma^2 |u_{1i}(t)|^2) \ + \\ + \tfrac{1}{2}\textstyle\sum_{i=1}^N \sum_{j=1, j\neq i}^N (x_i(t) - x_j(t))^T Q_{ij} (x_i(t) - x_j(t))] dt \end{aligned} \tag{7}$$

and using the results from [18], the cost function can be directly written as:

$$\begin{aligned} J(\tilde{u}_1, \tilde{u}_2) &= \int_0^\infty \left(|\tilde{y}_1(t)|^2 - \gamma^2 |\tilde{u}_1(t)|^2 + \tilde{x}^T(t)\tilde{Q}\tilde{x}(t)\right) dt \\ &= \int_0^\infty \left(\tilde{x}^T(t)\tilde{Q}\tilde{x}(t) - \gamma^2 \tilde{u}_1^T(t)\tilde{u}_1(t) + \tilde{u}_2^T(t)\tilde{u}_2(t)\right) dt \end{aligned} \tag{8}$$

where $\tilde{Q}$ is positive semidefinite weighting matrices and

$$\begin{aligned} \tilde{Q}_{ii} &= C^T C + \sum_{j=1, j\neq i}^N Q_{ij} \\ \tilde{Q}_{ij} &= -Q_{ij}, i \neq j, i, j = 1, \dots, N \end{aligned} \tag{9}$$

Taking into consideration the choosing form of $Q_{ij} = P^T P, i, j = 1, \dots, N, i \neq j, P \geq 0$, the expression of $\tilde{Q}$ is:

$$\tilde{Q} = \begin{bmatrix} C^T C + (N-1)P^T P & -P^T P & \cdots & -P^T P \\ -P^T P & C^T C + (N-1)P^T P & \cdots & -P^T P \\ \vdots & \vdots & \ddots & \vdots \\ -P^T P & -P^T P & \cdots & C^T C + (N-1)P^T P \end{bmatrix} \tag{10}$$

Following the results in [20], for the Riccati type equation of form

$$\tilde{A}^T\tilde{X} + \tilde{X}\tilde{A} + \gamma^{-2}\tilde{X}\tilde{B}_1\tilde{B}_1^T\tilde{X} - \tilde{X}\tilde{B}_2\tilde{B}_2^T\tilde{X} + \tilde{Q}^T\tilde{Q} = 0 \tag{11}$$

with the stabilising solution $\widetilde{X} \geq 0$, the obtained controller is expressed as:

$$\tilde{F} = -\tilde{B}_2^T\tilde{X}. \tag{12}$$

Moreover, the structure of the stabilising solution of Riccati type equation (11) is:

$$\tilde{X} = \begin{bmatrix} \tilde{X}_1 & \tilde{X}_2 & \cdots & \tilde{X}_2 \\ \tilde{X}_2 & \tilde{X}_1 & \cdots & \tilde{X}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{X}_2 & \tilde{X}_2 & \cdots & \tilde{X}_1 \end{bmatrix} \tag{13}$$

for which $\widetilde{X}_1 = X_1 + (N-1)X_2$, being the stabilising positive semidefinite solution of the Riccati equation (14)

$$A^TX_1 + X_1A + X_1(\gamma^{-2}B_1B_1^T - B_2B_2^T)X_1 + C^TC = 0 \tag{14}$$

and $\widetilde{X}_2 = X_2$, being the stabilising solution of the following Riccati equation

$$\begin{gathered}(A + (\gamma^{-2}B_1B_1^T - B_2B_2^T)X_1)^TX_2 + X_2(A + (\gamma^{-2}B_1B_1^T - B_2B_2^T)X_1) \\ + NX_2(\gamma^{-2}B_1B_1^T - B_2B_2^T)X_2 + P^TP = 0.\end{gathered} \tag{15}$$

Thus, the H∞ state-feedback gain of the multi-agent system is obtained. This structure corresponds to a configuration that allows total agents interconnection, the centralised one, and has the following expression:

$$\tilde{F} = \begin{bmatrix} F_1 & F_2 & \cdots & F_2 \\ F_2 & F_1 & \cdots & F_2 \\ \vdots & \vdots & \ddots & \vdots \\ F_2 & F_2 & \cdots & F_1 \end{bmatrix}, \tag{16}$$

with

$$F_1 = -B_2^T(X_1 + (N-1)X_2);\ F_2 = B_2^TX_2. \tag{17}$$

Considering that the attention is focused on distributed control, the existence of null terms in the adjacency matrix, corresponding to the positions where the connections between agents are missing, is observed. This aspect determines the state-feedback gain definition according to the adjacency matrix as follows:

$$\tilde{F}_D = I_N \otimes F_1 + \mathcal{A}(\mathcal{G}) \otimes F_2. \tag{18}$$

The introduction of the null terms in the adjacency matrix raises a new problem regarding the controller capacity to guarantee the system stability. Adopting the parameterization in [23] and following the detailed development in [20], the state-feedback gain expression becomes:

$$\tilde{F}_D = -I_N \otimes (B_2^TX_1) - \mathcal{N}_{a,b} \otimes (B_2^TX_2) \tag{19}$$

where $\mathcal{N}_{a,b} = (N_L - 1 - a)I_N - b\mathcal{A}(\mathcal{G})$ with $N_L = 1 + d_{max}$, d_{max} - the maximum number of an agent connections and the parameters a and b to be determined in order to obtain the closed-loop system stability. As expressed in [20], the state matrix corresponding to the closed-loop system is defined as:

$$\tilde{A}_D = I_N \otimes (A - B_2 B_2^T X_1) - \mathcal{N}_{a,b} \otimes (B_2 B_2^T X_2). \tag{20}$$

With the aim of determining the pair (a, b) for which the above matrix is Hurwitz, the following algorithm proposed in [24] is used.

The first step assumes the determination of $\delta_1 < 0$ and $\delta_2 > 0$ so that $\Lambda(\tilde{A}_D) \in \mathbb{C}^-, \forall \delta \in [\delta_1, \delta_2]$, where $\Lambda(\cdot)$ denotes the spectrum of $(\cdot)$.

The second step implies solving the following systems of inequalities:

$$\begin{array}{lcl} \delta_1 + 1 - N_L + a + b\mu_2 < 0 & & \delta_1 + 1 - N_L + a + b\mu_1 < 0 \\ \delta_2 + 1 - N_L + a + b\mu_1 > 0 & \text{and} & \delta_2 + 1 - N_L + a + b\mu_2 > 0 \\ b > 0 & & b < 0 \end{array} \tag{21}$$

with $\mu_1 = \min_i \mu_i$ and $\mu_2 = \max_i \mu_i$, where $\mu_i, i = 1, \dots, N$ represent the eigenvalues of the adjacency matrix $\mathcal{A}(\mathcal{G})$.

Further demonstrations and detailed developments can be consulted in [20].

4. CASE STUDIES

This part is dedicated to the case studies using H∞ design approach for a flight configuration consisting of four aerial vehicles with identical dynamics for which the limited communication is considered.

After linearization and decoupling the equations of motion, the matrices corresponding to the longitudinal dynamic model of an unmanned aerial vehicle (UAV) is obtained.

The elaborated mathematical model can be found in dedicated references such as [25], [26] and the analysis in the present case studies use the numerical values according to [27]. The theoretical results of the described approach are applied to a distributed type configuration illustrated in Fig. 1.

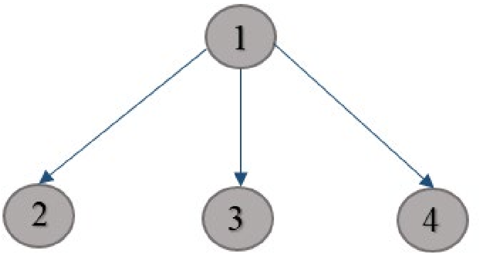

Fig. 1 Flight formation configuration

In order to obtain the state space model of form (6) corresponding to the studied configuration, defining the state vector as $x = [u \quad w \quad q \quad \theta \quad h]^T$, the longitudinal dynamics can be approximated by the linear system defined below:

$$[\dot{u} \quad \dot{w} \quad \dot{q} \quad \dot{\theta} \quad \dot{h}]^T = A_{long}[u \quad w \quad q \quad \theta \quad h]^T + B_{long}[\delta_E \quad \delta_T]^T. \tag{22}$$

The introduction of the integrators with the states η_1 and η_2 is realised to guarantee the agents capacity to maintain the preset values of the analysed states (the velocity u and the altitude h).

Thus, the complete system of form (1) becomes the system (23).

$$\begin{bmatrix} \dot{x} \\ \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A_{long} & 0 & 0 \\ -C_u & 0 & 0 \\ -C_h & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x \\ \eta_1 \\ \eta_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B_1} \begin{bmatrix} u_{com} \\ h_{com} \end{bmatrix} + \underbrace{\begin{bmatrix} B_{long} \\ 0 \\ 0 \end{bmatrix}}_{B_2} \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix}$$

$$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \delta_E \\ \delta_T \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix} \tag{23}$$

$$y_2 = [x^T \quad \eta_1 \quad \eta_2]^T$$

The interconnections between the four agents of the above configuration are described by the adjacency matrix given in (24). The network stability is proved by $\mathrm{Re}(\lambda) < 0$ for the eigenvalues of matrix (20) corresponding to the closed-loop system.

$$\mathcal{A}(\mathcal{G}) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

After obtaining the stabilising solutions of the two Riccati equations, the distributed controller is obtained, using the algorithm (21) and the expression (19), whose structure is expressed by (25).

$$\tilde{F} = \begin{bmatrix} F_1 & F_2 & F_2 & F_2 \\ F_2 & F_1 & 0 & 0 \\ F_2 & 0 & F_1 & 0 \\ F_2 & 0 & 0 & F_1 \end{bmatrix} \tag{25}$$

Analysing the distributed controller structure related to the adjacency matrix, it can be seen that it reproduces the interconnection mode of the four agents. Rather, agent 1 is connected with all the other agents, the missing interconnections being represented by the null terms. In order to obtain the numerical simulations, null initial conditions of velocity and altitude are considered and it is required to maintain the preset values $h = 12$ m and $u = 3$ m/s.

Firstly, the results correspond to the hypothesis that communication channels that allows the information transmission is ideal. Further, various situations of imperfect communication channels are treated. The cases refer to the effect of the temporary interruption of communications for certain agents, the complete loss of communication for the entire network, considering a preset period of time, and the influence of time delays on the response capacity of the flight formation. In order to emphasize the performances of the obtained distributed controller, the communication interruption is considered firstly for 2 s, being after extended to 6 s. Regarding the last case, the information transmission is realised using the time delay value $\tau = 0.5$s.

4.1 The case of a single agent failure

The first analysed case considers the temporary loss of communication for agent 2 in the time interval $t = 1:3$ s. After studying this situation, the interruption is extended to $t = 1:7$ s in order to analyse the comparative time response of the flight formation. Thus, for the time period when the connection of agent 2 with the entire formation is compromised, the distributed controller form becomes:

$$\tilde{F} = \begin{bmatrix} F_1 & 0 & F_2 & F_2 \\ 0 & F_1 & 0 & 0 \\ F_2 & 0 & F_1 & 0 \\ F_2 & 0 & 0 & F_1 \end{bmatrix}. \tag{26}$$

Taking into account the above observations, the time evolutions of the two analysed states are obtained and represented in Fig. 2 and Fig. 3. It is observed that the missing connection of agent 2 for 2 s has no significant effects on the system stability or on the achievement of imposed objectives, this aspect being highlighted by the minor differences in the time evolutions for the considered time interval.

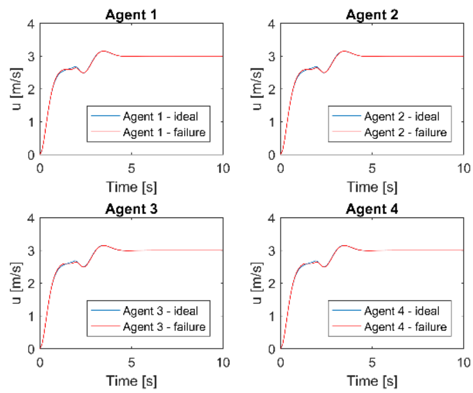

Fig. 2 Comparative time response of velocity

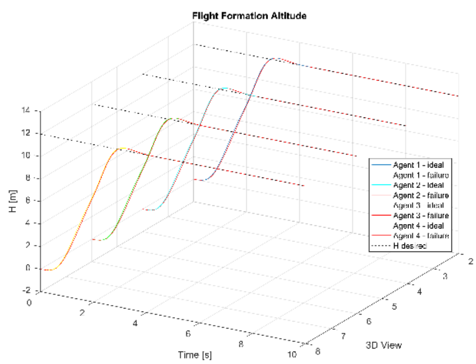

Fig. 3 Comparative time response of altitude

For the case where the failure time of agent 2 is extended, the time evolutions of the agents are presented in Fig. 4 and Fig. 5. These results indicate changes in the agents behaviour compared to the ideal communication case, but without affecting the stabilising and controlling capacity of the obtained controller.

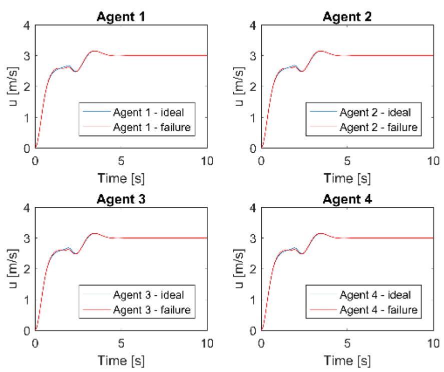

Fig. 4 Comparative time response of velocity

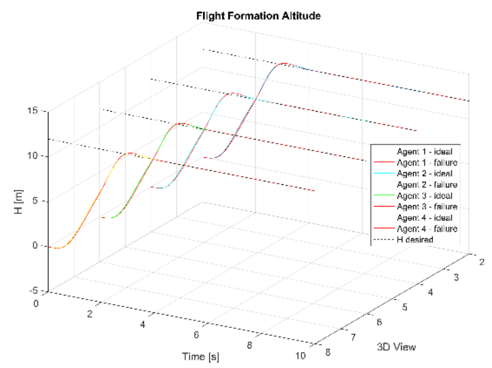

Fig. 5 Comparative time response of altitude

4.2 The case of network communication failure

The second scenario deals with the case of the total loss of network connections for the two different time periods.

The controller structure (25) is modified for the time interval corresponding to the network communication failure, rewritten as:

$$\tilde{F} = \begin{bmatrix} F_1 & 0 & 0 & 0 \\ 0 & F_1 & 0 & 0 \\ 0 & 0 & F_1 & 0 \\ 0 & 0 & 0 & F_1 \end{bmatrix}. \tag{27}$$

The numerical simulations presented in Fig. 6 and Fig. 7 show more pronounced changes in the agents evolutions compared to the ideal case for the considered time period. The effects of communication failure are observed in Fig. 8 and Fig. 9 for the extended time period ($t = 1{:}\,7\ s$), while maintain the imposed values of velocity and altitude.

Furthermore, minor changes in the agent 1 behaviour are identified, being the single agent connected to the entire network ($\mathrm{d_{max}} = 3$).

The effects caused by the communication failure persist for a short time after the communication recovery, emphasizing the designed controller capacity to achieve the required performances.

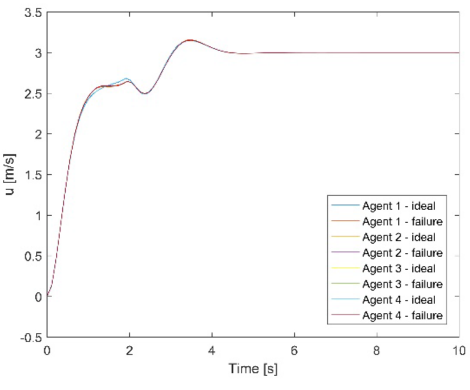

Fig. 6 Comparative time response of velocity

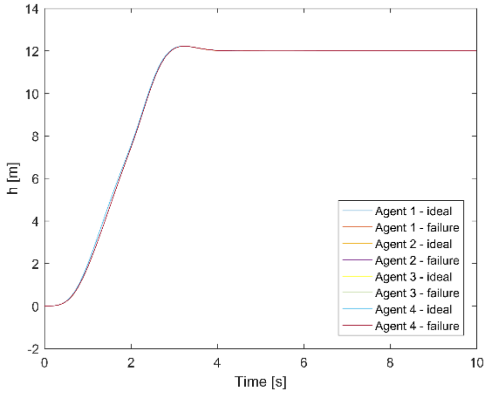

Fig. 7 Comparative time response of altitude

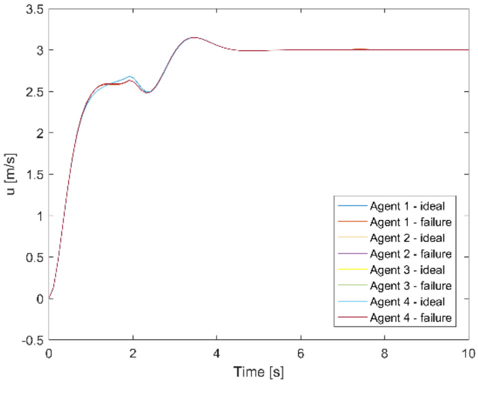

Fig. 8 Comparative time response of velocity

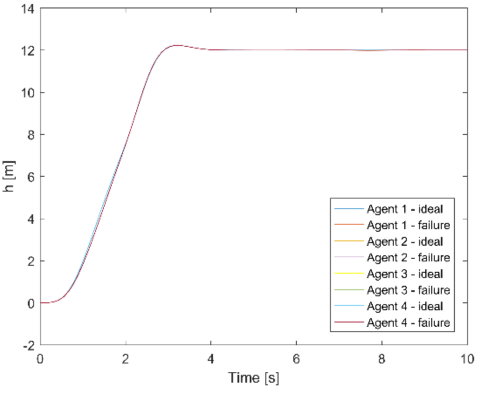

Fig. 9 Comparative time response of altitude

Remark: for this configuration, considering that agent 1 is the only agent interconnected with the entire network, its failure would be equivalent to the scenario of the complete communication loss.

4.3 The case of time delay presence

This section treats the situation when the information transmission is realised in the presence of time delays.

Both the desired performances and the initial conditions are maintained for the case when time delays are introduced.

Thus, with the aim of analyse their influences, one can consider the first-order delay Padé approximation with the following transfer function:

$$e^{-\tau s} \simeq \frac{2-\tau s}{2+\tau s} \tag{28}$$

where τ represents the time delay.

The introduction of the state vector of the Padé approximation for agent i, denoted x_{p_i}, with $\dot{x}_{p_i}(t) = -\frac{2}{\tau}x_{p_i}(t) + x_i(t)$, determines the below new expression:

$$x_i(t-\tau) = \frac{4}{\tau}x_{p_i}(t) - x_i(t). \tag{29}$$

The two figures below (Fig. 10 and Fig. 11) describe the time responses of the agents states for the case of ideal communication channels compared to the scenario of information transmission with the time delay $\tau = 0.5$s, indicated by the subscript D. Although the value of the chosen time delay does not affect the stability of the flight formation, the analysis of the simulation results highlights a slight degradation of the control performance, shown by the evolutions offset corresponding to the imperfect communication. The consequence of this fact implies an increase of the required time to reach the imposed states values.

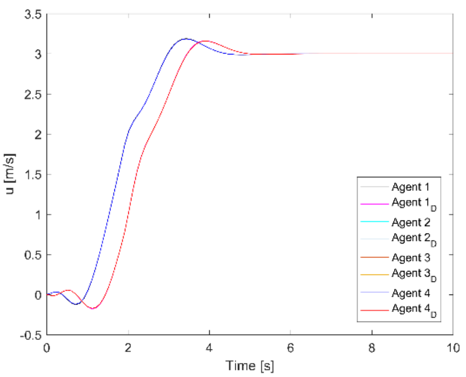

Fig. 10 Comparative time response of velocity

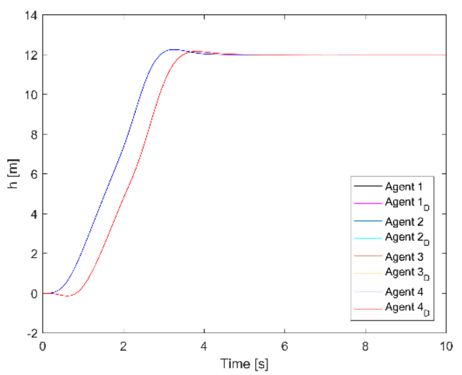

Fig. 11 Comparative time response of altitude

5. CONCLUDING REMARKS

One of the specific challenges faced by the multi-agent systems concerns the information transmission between agents, which is possible due to communication channels. On the other hand, this particularity implies several communication channels imperfections that can have as effects the desired performances reduction or the influences on the interconnected systems stability.

The presented case studies integrate the comparative results offered by H∞ approach for various situations of imperfect transmission channels. To obtain these results, the decoupled dynamics of a flight configuration consisting of four identical agents is used. The attention is directed towards the stability problem study of a flight formation simultaneously with the convergence of certain states towards a common imposed value.

The comparative analysis of the obtained results relieve the fact that the desired performances are ensured for all considered scenarios that involve imperfect communication channels. The numerical simulations of the first two scenarios indicate that the control performances remain acceptable both for the case of the temporary interruption of an agent connection and for the complete communication failure for a preset time period. Moreover, the latest obtained results indicate the presence of time delays through the offset of the time evolutions compared to the ideal communication case. Hence, considering all analysed scenarios of imperfect communication channels, the capacity of the distributed designed controller to guarantee the flight formation stability and the imposed performances is remarked.

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