The kinematics of the rigid feedback linkage, the impedance of the hydraulic servomechanism and the flutter occurrence

Ioan URSU*

*Corresponding author
*INCAS - National Institute for Aerospace Research “Elie Carafoli”
B-dul Iuliu Maniu 220, Bucharest 061126, Romania
iursu@incas.ro

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Abstract: *The paper brought to light a previous result of the author, used in the design of hydraulic servos actuating flight controls of the airplanes IAR 93 and IAR 99. The results highlights the importance of the the kinematics of the rigid feedback linkage of the hydraulic servo in an aeroservoelastic frame.*

Key Words: *aeroservoelasticity, hydraulic servomechanism, impedance, flutter prevention, IAR 99, IAR 93 .*

1. PROBLEM BACKGROUND

The servomechanism impedance, also called dynamic stiffness, determines its behavior in the presence of output load variations. Formally, the impedance is defined as the ratio, in dynamic regime, of the output exerted force F and the displacement z induced by this force, in the context of a blocked input ($x = 0$). The impedance is a complex variable function and it is calculated based on the following simplified mathematical model (see Fig. 1 and [1])

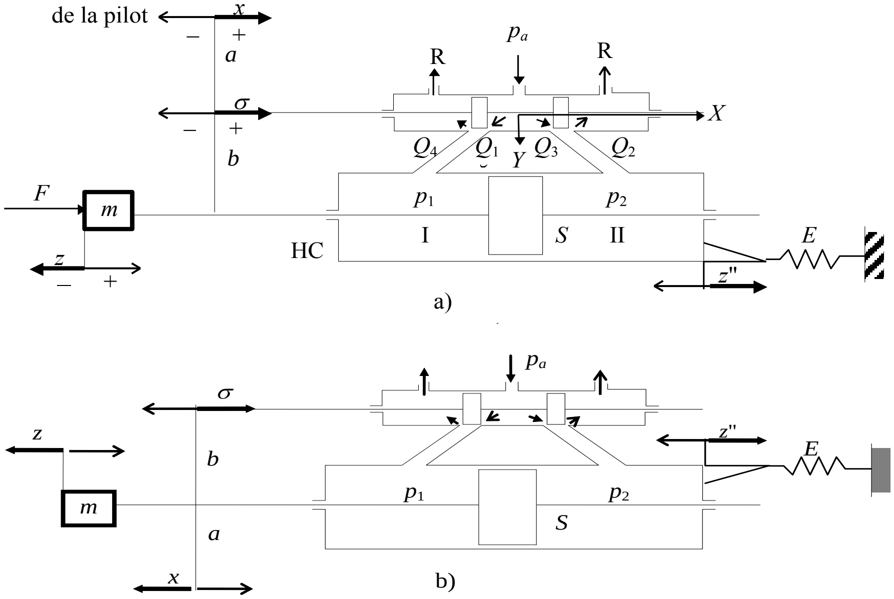


Fig. 1− Sketches for mathematical modeling of hydraulic servomechanism: a) mechano-hydraulic servomechanism ensuring a pseudo-active flutter compensation; b) mechano-hydraulic servomechanism favorable to flutter occurrence

$$S\left(\dot{z}-\dot{z}''\right)+k_c\dot{p}=-k_Q^*\sigma-k_{Qp}^*p,\sigma=\lambda_2 z-\lambda_3 z'',F=-Sp=Ez'' \tag{1}$$

Some of notations are deductible from the figure; k_Q^*,k_{Qp}^* are flow gain, respectively, flow-pressure gain, and λ_2,λ_3 are kinematic coefficients involving the values *a*, *b*. Noting with *F* (*s*) and *z* (*s*) the Laplace transforms of the variables *F* (*t*) and z (*t*), it results the expression of the impedance function ($s = \mathrm{i}\omega$):

$$\mathrm{Imp}(\mathrm{i}\omega):=\frac{F(s)}{z(s)}=r_d\,\frac{s+a_1}{s+a_2} \tag{2}$$

$$a_1=\lambda_2 k_Q^*/S,a_2=r_d\left(\lambda_3 k_Q^*/(ES)+k_{Qp}^*/S^2\right),1/r_d=1/E+k_c/S^2 \tag{2'}$$

The impedance function of the servomechanism highlights its ability to absorb or promote the flutter oscillations of the primary flight controls which are aerodynamically triggered. The theoretical and experimental analysis of this feature is required by the Aviation Regulations since the 80s.

Graphically, the expression (2) is a semi-circle in the (Re(Imp(ω), Im(Imp(ω)) complex plane (Fig. 2 a, b); more specifically, the semi-circle is located in quadrant I or IV, according to the existing order relation established between the a_1 and a_2 parameters in the expression of the impedance function, namely $a_1 < a_2$, or $a_1 > a_2$ respectively; the first case corresponds to the favorable situation when the actuator absorbs the flutter oscillations (indicating the presence of an effective positive damping in the system), thus creating a pseudo-active control, while the second corresponds to the unfavorable situation when the actuator can contribute to enhance the flutter oscillations under a "negative" damping in the system.

In this way, the real part of the complex number represented by the impedance function is a measure of the system rigidity and the imaginary part is a measure of the system damping (or lack of damping); dimensionally, both components are rigidities. These properties are listed in the Av. Regulation P. 970, developed in the early 80s, but their physic and mathematical proof is not known in the literature of specialty.

2. MAIN RESULT

In the following, the energy foundation of the anti-flutter stability relationship $a_1 < a_2$ is demonstrated. This relationship can be interpreted as ensuring the flutter pseudo-active control by the hydraulic servomechanism, through constructive and functional design conditions.

Proposition. *A sufficient condition for the anti-flutter stability of the (mechanical) hydraulic servomechanism* is $a_1 < a_2$.

Proof. The energy balance method is used. If *F* is the effort needed to get the $z = z_0\sin\omega t$ output displacement while the servomechanism is powered and the input blocked (Fig. 3b), then $\Phi = -F$ is the servomechanism response effort, made by means of energy received from outside (hydraulic energy); this effort verifies the equation

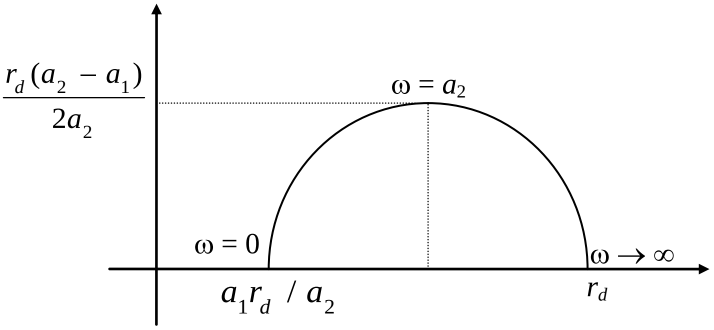
$$\dot{\Phi}=-a_2\Phi-r_\mathrm{d}z_0\left(\omega\cos\omega t+a_1\sin\omega t\right)=-a_2\Phi+A_1\sin\omega t+B_1\cos\omega t. \tag{3}$$

The produced effort Φ can be considered as being consumed by the forces of inertia, viscous and elastic friction, thus

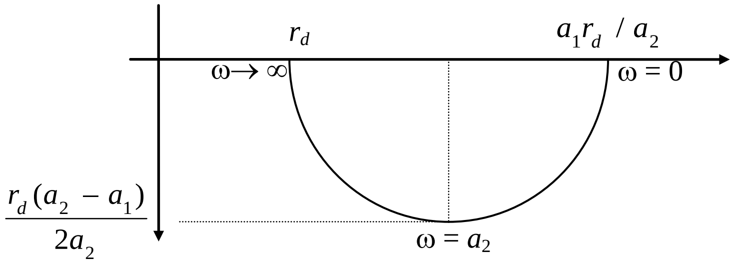
$$\Phi = m\ddot{z} + f\dot{z} + kz = A_2 \sin\omega t + B_2 \cos\omega t \tag{4}$$

where

$$A_1 = -r_\mathrm{d} z_0 a_1,\ B_1 = -r_\mathrm{d} z_0 \omega,\ A_2 = z_0\left(k - m\omega^2\right),\ B_2 = z_0 f\omega. \tag{4'}$$



a) $a_1 < a_2$



b) $a_1 > a_2$

Fig. 2 – Graph of the impedance function
a) the case of flutter damping (pseudoactive control); b) the case of flutter enhancing

The solution of the equation (4), in initial condition Φ(0) = 0, is written:

$$\Phi(t) = \int_0^t e^{a_2\tau}\left(A_1\sin\omega\tau + B_1\cos\omega\tau\right)\mathrm{d}\tau e^{-a_2 t} \tag{5}$$

Thus the consumed energy W_2 will be compared with the energy W_1 received during a time period

$$W_1 = \int_0^{2\pi/\omega}\int_0^t e^{a_2 t}\left(A_1 \sin\omega t + B_1 \cos\omega t\right)\mathrm{d}t e^{-a_2 t}\ \omega\, z_0 \cos\omega t\, \mathrm{d}t \tag{6}$$

$$W_2 = \int_0^{2\pi/\omega}\left(A_2 \sin\omega t + B_2 \cos\omega t\right)\omega\, z_0 \cos\omega t\, \mathrm{d}t \tag{6'}$$

where expressions were obtained from the general relation

$$W = \int \Phi \frac{\mathrm{d}z}{\mathrm{d}t}\,\mathrm{d}t$$

By doing the calculations we obtain

$$W_2 = \pi z_0^2 f\omega \quad \text{– received (consumed) energy}$$

$$W_1 = \frac{r_\mathrm{d}\,\omega z_0^2\left(a_1 - a_2\right)\left[\pi - a_2\omega\left(1 - \mathrm{e}^{-2\pi a_2/\omega}\right)\Big/\left(a_2^2 + \omega^2\right)\right]}{a_2^2 + \omega^2} \quad \text{– delivered energy.}$$

The energy condition of stability in the presence of flutter, $W_2 > W_1$, returns to

$$r_d\left(a_1 - a_2\right)\left[\pi - a_2\omega\left(1 - \mathrm{e}^{-2\pi a_2/\omega}\right)\Big/\left(a_2^2 + \omega^2\right)\right]\Big/\left(a_2^2 + \omega^2\right) < \pi f. \tag{7}$$

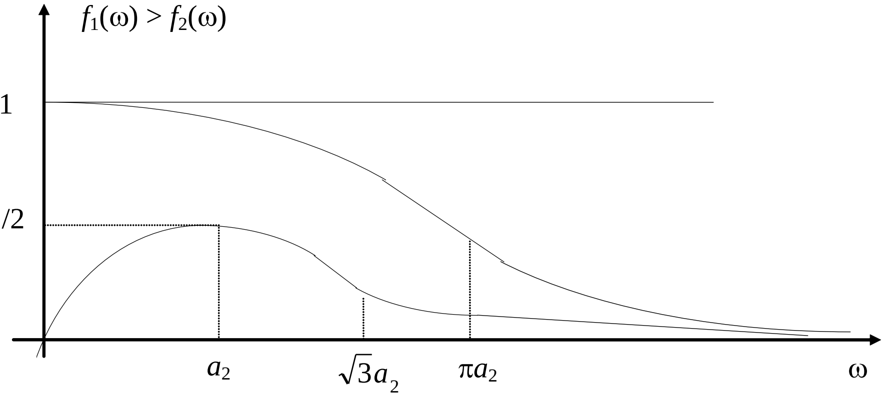


Fig. 3 – The graphs of functions $f_1(\omega)$, $f_2(\omega)$

The graphs of functions

$$f_1(\omega) = 1 - \mathrm{e}^{-2\pi a_2/\omega}, \quad f_2(\omega) = a_2\omega\Big/\left(a_2^2 + \omega^2\right)$$

are shown in Fig. 3; the product $f_1(\omega)\,f_2(\omega)$ is hence sub-unitary and positive, so that an increase in the left side of inequality (17), leading to the stability covering condition, is justified.

$$r_d\left(a_1 - a_2\right)\Big/\left(a_2^2 + \omega^2\right) < f. \tag{8}$$

It follows that if $a_1 < a_2$, the damping effect exerted by the servomechanism on flutter oscillations is ensured to any pulsation ω; the condition is written as follows:

$$\frac{\lambda_2 k_Q^*}{S} < \left(\frac{k_Q^*}{E} + \frac{k_{Qp}^*}{S}\right)\frac{SE}{S^2 + k_c E}, \tag{9}$$

The relationship is relevant, highlighting the importance both of the coefficient k^*_{Qp} and the *aeroservoelastic* coefficients λ_2 and E within the flutter analysis. ♦

Performing a calculation using the nominal data of the servomechanism SMH (see also [2]-[6]) from the aileron chain of IAR 99 shows that inequality (9) is satisfied, which is not the case for the servomechanism with λ_2 greater than 1,.(see Fig. 1b).

Such a servomechanism with a improper structure in terms of impedance function cannot induce a pseudo-active control of flutter.

2. HISTORICAL COMMENTS

Although a diagnosis of the situation was correctly formulated [7], the ignorance, the underestimation or perhaps the risk taking by decision makers (however in regular terms, since the current regulation Av. P. 970 did not provide the criterion of impedance) led to a dramatic outcome, namely the IAR 93 crash in the late 70s, caused by a tailplane flutter. Eventually, this catastrophe resulted in the replacement of the MIG 21 type BU 51 servomechanisms (improper in terms of aeroservoelastic properties) mounted on the IAR 93 with actuators which basically were compatible from the viewpoint of aeroservoelasticity, provided under a collaborative program by Dowty British Company.

Since the impedance tests performed by the Romanian party on some Dowty actuator specimens proved the existence of irregular segments of the experimental impedance function, located in quadrant IV, the English partner at that time (the early 80s) in the manufacture of the electrohydraulic servomechanisms for IAR 93 assumed the responsibility of location on board the airplane of such exemplars of servomechanisms (fortunately, flight "events" were no longer been reported in this context).

For a proper understanding of the situation, it should be added that the sentence demonstrated above gives only a sufficient condition for stability.

Failure to meet this requirement does not necessarily entail the occurrence of instability. Here the relationship between certainty and risk assessment of decisions taken at a time can be highlighted.

The impedance or dynamic stiffness, can also be defined for electrohydraulic servomechanisms [8].

Unlike the case of frequency response test, the experimental determination of the servomechanism (SMH) impedance – the schematic diagram of the system is given in Fig. 4b - requires the introduction of sinusoidal signals in force at its output through an electrohydraulic generator of signals (GEHS), the servomechanism input being blocked.

For a signal of amplitude force and data frequency, the ratio of the amplitude of the input force and the output displacement induced by this force is measured as well as the phase difference between these signals (measurements being made under stability conditions).

The procedure is repeated with keeping the chosen force amplitude for a number of frequencies in a band from the neighborhood of zero to a high enough frequency for impedance measurement to be relevant (i.e., until normal frequency levels of aerodynamic disturbances –forces- at the control surfaces - or of mechanical vibrations of the servomechanism output adjacent structure are exceeded).

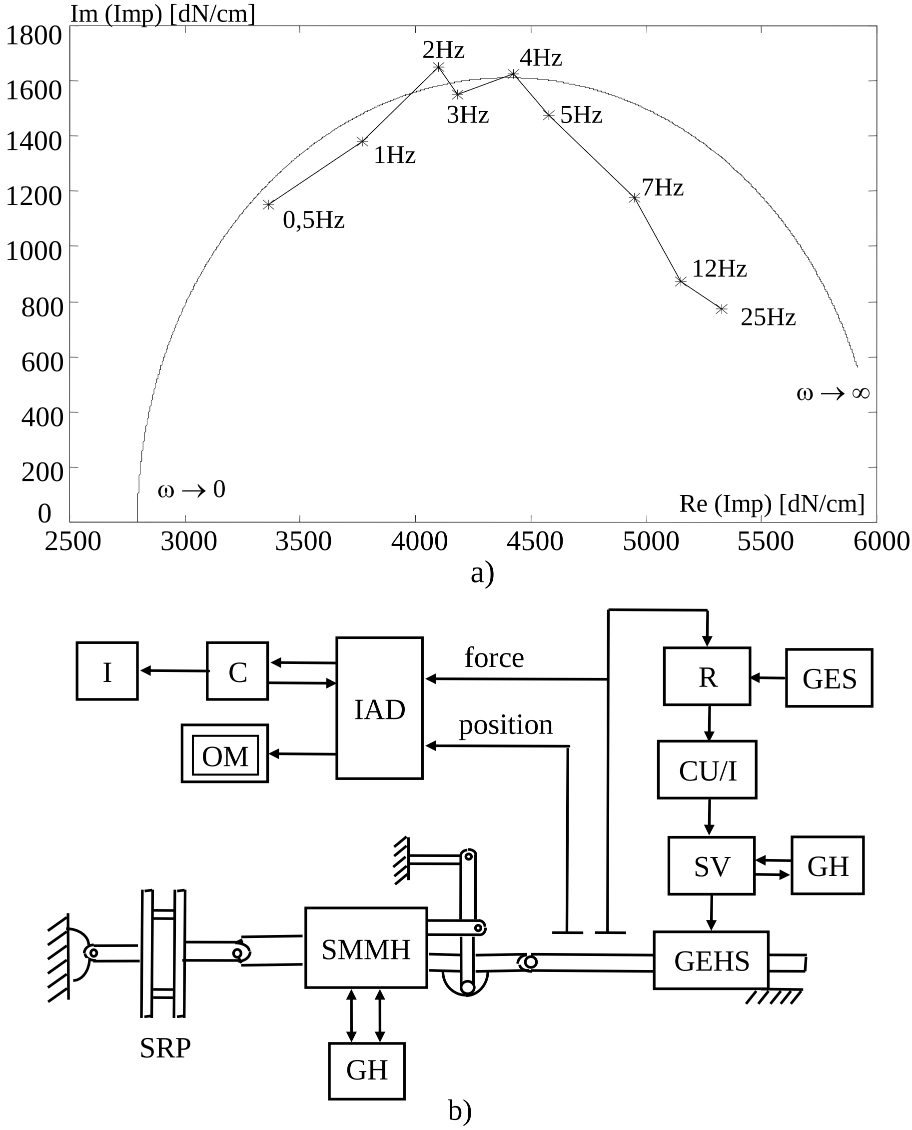


Fig. 4 − Impedance function definition: a) comparison between theoretical and experimental impedance curves; b) block diagram of the impedance test system for mechanical-hydraulic servomechanisms

For different values of force signal amplitude different impedance curves are obtained. Basically, the bench scheme has the same devices and appliances as for the frequency response bench: a structure clamping rigidity simulator (SRP), a regulator (R), a current strength to voltage converter, (CU / I) GEHS supply group consisting of a servo-valve (SV) and a hydrostatic generator (GH) a data acquisition system (IAD), printer peripherals (I) and a memory oscilloscope (OM) as well as the PC (C). An experimental impedance curve determined for the output force amplitude of 125 daN is reproduced from the reference [9] in Fig. 4a) and is part of the approval programs for the servomechanism SMH and IAR 99 Hawk aircraft. The impedance test compliance with regulatory requirements and the theoretical analysis is obvious.

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