Centralized and Distributed Linear Quadratic Design for Flight Formations

Serena Cristiana VOICU (STOICU)*,[1,2], Adrian-Mihail STOICA[2]

*Corresponding author
[1]INCAS – National Institute for Aerospace Research "Elie Carafoli",
Systems Department,
Bd. Iuliu Maniu 220, 061126 Bucharest, Romania,
voicu.serena@incas.ro*
[2]"POLITEHNICA" University of Bucharest, Faculty of Aerospace Engineering,
Splaiul Independentei 313, 060042 Bucharest, Romania

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Abstract: *This paper focuses on the analysis of the particularities of control for multi-agent systems. The design method is based on an optimal control approach which requires the solution of Linear Quadratic Regulator problem (LQR). The characteristics of the two types of control (centralized and distributed) for unmanned aerial vehicles flight formations are highlighted by the case studies. The dynamics of an UAV (Unmanned Aerial Vehicle) is used for the longitudinal motion. The flight formation considered as a case study consists of four identical agents.*

Key Words: *centralized control, distributed control, flight formation, linear quadratic optimization*

1. INTRODUCTION

Due to the wide range of applicability for multi-agent systems, this topic has attracted the attention of recent research. Despite the progress made in terms of stability of a vehicle or a formation of autonomous air vehicles for certain missions, this topic is still a challenge. The performances of the control law refer to the following aspects: the stability of the flight formation agents, the maintenance of a certain value imposed for altitude, velocity, or position.

Optimal control is a feature of modern methods of designing control systems. Besides obtaining the stability of the resulting system or accomplishing certain constraints, linear quadratic (LQ) methods ensure optimal control of a linear system providing excellent robustness stability margins [1].

The performance properties of the optimal control are particularly treated in [3]. Optimal control models in which all states are measurable offer better properties compared to conventional ones.

Communication through the network has become fundamental for the interconnection of the independent systems. The control of flight formations involves the way of communication between the agents, being a very important aspect for the design approaches. There are three different types of control structures for multi-agent systems (centralized, decentralized and distributed). The paper [20] presents a comparative study of the three approaches to the controller design.

The design of the centralized controller involves the interconnection of all formation agents. This aspect implies difficulties in data processing due to accessing information from all members of the formation. The simulations presented in [20] show that the use of an optimal centralized controller ensures the desired performance, but it becomes inefficient as the number of formation agents increases. This type of control requires high performances for the central controller and any error influences the behavior of the entire flight formation.

The second configuration, the decentralized controller, refers to the existence of a controller for each agent of the formation. Paper [21] notes that the term "decentralized" refers to the controller implementation and the design of the control law is done in a centralized way. Compared to the previous architecture that uses a single controller, the decentralized control introduces independent controllers without the possibility of their interconnection. This configuration is treated in works such as [11], [13]. The distributed control is characterized by the possibility that each agent being commanded individually. The exchange of information between the subsystems is ensured by the existence of communication channels. The distributed controller is based on the interconnection of the local controllers. The information transmission between them and the adjacent agents is possible. Having a widespread applicability such as satellite flight, multi-agent systems, etc., distributed control has been approached in various studies. Paper [10] considers a controller based on Linear Quadratic Regulator Proportional Integral for a flight formation of unmanned aerial vehicles. The proposed method ensures the leader tracking by the others agents of the formation. Paper [17] proposes a distributed control approach based on potential functions for multiple vehicles formations with undirected interconnection. In [18], a leader-follower formation in a noisy environment is considered. A numerical simulation is presented to show the performance of the distributed tracking control proposed. Two distributed control laws together with their effectiveness analysis through simulation example are presented in paper [19]. The issue of distributed control is also treated in [16] using optimal controllers. Paper [20] is based on satisfying certain properties of the state-space model to design distributed controllers for interconnected identical subsystems.

The present paper focuses on the analysis of the particularities of control types for multi-agent systems. The work deals with linear quadratic method in order to study the characteristics of centralized and distributed control and their impact on networked systems stability. The theoretical procedures are inspired from [2]. The features of the presented approach for automatic control systems are highlighted by case studies. The presented numerical simulations analyze both types of control for longitudinal dynamics using two configurations of flight formation. The paper is organized as follows. The first part introduces a few aspects about the types of control for flight formation. The following two sections include some preliminaries, the problem formulation and the optimal approach to solving the LQR problem. After presenting the stability properties, a case study is analyzed. The paper ends with some concluding remarks.

2. CENTRALIZED LQR DESIGN FOR FLIGHT FORMATIONS

This part focuses on centralized control which involves communication between each pair of agents in both directions (bidirectional communication). To describe the necessary model for solving the problem, the dynamics of the formation consisting of N interconnected subsystems is defined. The dynamics of a single agent is described as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), x_i(0) = x_{i0} \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ represents the state vector and $u_i(t) \in \mathbb{R}^m$ is the input vector. The matrix $A \in \mathbb{R}^{n\times n}$ is the state matrix and the matrix $B \in \mathbb{R}^{n\times m}$ is the control matrix.
The dynamics of the formation consisting of N identical agents has the following expression:

$$\dot{x}(t) = A_f x(t) + B_f u(t) \tag{2}$$

where $A_f = I_N \otimes A$, $B_f = I_N \otimes B$ and $\otimes$ denotes the Kronecker product. The matrices A and B correspond to the dynamics of a single agent. The state vector $x(t) = [x_1{}^T(t);\quad x_2{}^T(t);\quad \ldots\quad ; x_N{}^T(t)]^T$ includes the states of the entire formation and the input vector is defined in a similar way $u(t) = [u_1{}^T(t);\quad u_2{}^T(t);\quad \ldots\quad ; u_N{}^T(t)]^T$.

The cost function of the LQR problem for N agents gathers the dynamic behavior of systems:

$$\begin{aligned} J(u, x_0) = \int_0^\infty \Bigg(& \sum_{i=1}^{N} \left(x_i{}^T(t) Q_{ii} x_i(t) + u_i{}^T(t) R_{ii} u_i(t) \right) \\ & + \sum_{i=1}^{N} \sum_{j\neq i}^{N} \left(x_i(t) - x_j(t) \right)^T Q_{ij} \left(x_i(t) - x_j(t) \right) \Bigg) dt \end{aligned} \tag{3}$$

One can directly check that the above cost function may be rewritten as:

$$J(u(t), x_0) = \int_0^\infty (x^T(t) Q_f x(t) + u^T(t) R_f u(t)) dt \tag{4}$$

where

$$Q_f = \begin{bmatrix} Q_{f_{11}} & Q_{f_{12}} & \cdots & Q_{f_{1N}} \\ Q_{f_{21}} & Q_{f_{22}} & \cdots & Q_{f_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{f_{N1}} & Q_{f_{N2}} & \cdots & Q_{f_{NN}} \end{bmatrix};\ R_f = I_N \otimes R \tag{5}$$

with $Q_{f_{ii}} = \sum_{k=1}^{N} Q_{ik}$, $i = \overline{1, N}$; $Q_{ii} = Q_{ii}{}^T \geq 0$, $\forall i$; $Q_{f_{ij}} = -Q_{ij}$, $i, j = \overline{1, N}$, $i \neq j$; $R_{ii} = R_{ii}{}^T > 0$, $\forall i$; $Q_{ij} = Q_{ij}{}^T = Q_{ji} \geq 0$, $\forall i \neq j$.

According with the well-known linear quadratic control theory, the control law that minimizes the cost function (4) is given by:

$$u(t) = -R_f^{-1} B_f{}^T P_f x(t) \tag{6}$$

where P_f represents the symmetric positive definite stabilizing solution of the algebraic Riccati equation:

$$A_f^T P_f + P_f A_f - P_f B_f R_f{}^{-1} B_f{}^T P_f + Q_f = 0 \tag{7}$$

Choosing the elements of Q_f in (5) as:

$$\begin{aligned} Q_{f_{ii}} &= Q_1, \quad i = \overline{1, N} \\ Q_{f_{ij}} &= Q_2, \quad i = \overline{1, N}, \qquad i \neq j \end{aligned} \tag{8}$$

some interesting properties of the solution to the above Riccati equation are derived in [2].

Indeed, let $P_f \in \mathbb{R}^{nN\times nN}$ be the stabilizing solution of the equation (7) where the form of the individual elements is considered ${P_f}_{ij} = P_f[(i-1)n+1; (j-1)n+1]$, $i, j = \overline{1, N}$. Then, the following properties are true [14]:

- $\sum_{j=1}^{N} {P_f}_{ij} = P$, $i = \overline{1, N}$, where $P \in \mathbb{R}^{n\times n}$ is the symmetric positive definite stabilizing solution of the algebraic Riccati equation (ARE) corresponding to the single agent LQR (Linear Quadratic Regulator) problem:

$$A^T P + PA - PBR^{-1}B^T P + Q_1 = 0 \tag{9}$$

- $\sum_{j=1}^{N} {K_f}_{ij} = K$, $i = \overline{1, N}$ where $K = R^{-1}B^T P$ is the gain matrix of the LQR problem associated with a single agent.
- The off-diagonal elements of P_f matrix denoted ${P_f}_{ij}$ for $i \neq j$ are negative semi-definite equal matrices ${P_f}_{12} \leq 0$. Additionally, the matrix ${P_f}_{12}$ is the negative semi-definite solution of ARE:

$$A_{cl}^T P_{f_{12}} + P_{f_{12}} A_{cl} + N P_{f_{12}} BR^{-1}B^T P_{f_{12}} - Q_2 = 0 \tag{10}$$

where $A_{cl} = A - BR^{-1}B^T P$. The equation (10) is the Riccati equation corresponding to the LQR problem for the stable system (A_{cl}, B).

- The gain matrix K_f for the centralized case is given by:

$$K_f = \begin{bmatrix} {K_f}_{11} & {K_f}_{12} & \cdots & {K_f}_{12} \\ {K_f}_{12} & {K_f}_{11} & \cdots & {K_f}_{12} \\ \vdots & \vdots & \ddots & \vdots \\ {K_f}_{12} & {K_f}_{12} & \cdots & {K_f}_{11} \end{bmatrix} \tag{11}$$

where the matrices ${K_f}_{11}$ and ${K_f}_{12}$ depend on N, A, B, Q_1, Q_2, R.

- The solution of ARE (7) has the following structure:

$$P_f = \begin{bmatrix} {P_f}_{11} & {P_f}_{12} & \cdots & {P_f}_{12} \\ {P_f}_{12} & {P_f}_{11} & \cdots & {P_f}_{12} \\ \vdots & \vdots & \ddots & \vdots \\ {P_f}_{12} & {P_f}_{12} & \cdots & {P_f}_{11} \end{bmatrix} \tag{12}$$

where ${P_f}_{11} = P - (N-1){P_f}_{12}$ (for more details, see [2]) (13)

3. DISTRIBUTED LQR DESIGN FOR FLIGHT FORMATIONS

Compared to the centralized case, the distributed control supposes a certain structure of communication. The information exchange exists between certain agents, which is possible due to the interconnection of the controllers. This type of control involves the existence of a controller for each system that can be independently commanded. For interconnected multi-agent systems, a data communication network is established between the agents, defined as a graph. Some theoretical notions of graphs are detailed, for instance, in [5], [9]. Any graph is defined by specific matrix forms used in the analysis of the flight formation stability [12].

The degree matrix, $D(G)$, is defined as a diagonal matrix that includes the number of connections for each agent.

The adjacency matrix, $A(G)$, indicates if the pair of agents is connected, defining the way of interconnections. The Laplacian matrix, $L(G)$, denotes the connection mode of the graph given by $L(G) = D(G) - A(G)$.

$$D(G) = \begin{cases} \deg(V_i), i = j \\ 0, i \neq j \end{cases}; \qquad A(G) = \begin{cases} a_{ii} = 0, & \forall i \in V \\ a_{ij} = 0, & (i,j) \notin E, \forall i,j \in V, i \neq j; \\ a_{ij} = 1, & (i,j) \in E, \forall i,j \in V, i \neq j \end{cases}$$

$$L(G) = \begin{cases} \deg(V_i), & i = j \\ -1, & i \neq j, \quad (V_i, V_j)\ adjacent \\ 0, & otherwise \end{cases}$$

The state-space formulation for the formation dynamics consisting of N agents is defined as:

$$\dot{x}_F(t) = A_{F_D} x_F(t) + B_{F_D} u_F(t), \qquad x_F(0) = x_{F0} \tag{14}$$

where the states and the inputs of all agents are concentrated in x_F and u_F vectors and the matrices A_{F_D} and B_{F_D} have the following expressions: $A_{F_D} = I_N \otimes A$; $B_{F_D} = I_N \otimes B$.

The following problem of distributed optimal control is studied in [2]:

$$\min_{K_D} J_D\left(u_{F_D}, x_{F_{D0}}\right) = \int_0^\infty \left(x_{F_D}^T(t) Q_{F_D} x_{F_D}(t) + u_{F_D}^T(t) R_{F_D} u_{F_D}(t)\right) dt \tag{15}$$

where $u_{F_D} = K_D x_{F_D}$ with an imposed structure of K_D given by the formation geometry.

The paper proposes a method to determine a suboptimal distributed controller for which the definition of networked system is necessary. Therefore, the symmetric positive definite matrix $M \in \mathbb{R}^{N \times N}$ is introduced, establishing several ways for defining this matrix in order to ensure system stability.

It is necessary to specify the minimum size of the linear quadratic problem to be solved, given by $N_\ell = d_{max} + 1$, where d_{max} is the maximum degree of an agent. Reference [2] proves different conditions for matrix M. If M has the property:

$$\lambda_i(M) > \frac{N_\ell}{2}, \qquad \forall \lambda_i(M) \in S(M) \backslash \{0\} \tag{16}$$

where $\lambda_i(M)$ denotes the eigenvalues of matrix M, $S(M)$ is the spectrum of M and N_ℓ defines the minimum number of agents for which the problem needs to be solved. The following distributed feedback controller

$$K_D = -I_N \otimes R^{-1} B^T P + M \otimes R^{-1} B^T P_{\ell_{12}} \tag{17}$$

guarantees asymptotic stability of the closed loop system having the state equation

$$A_{cl} = I_N \otimes A + (I_N \otimes B) K_D. \tag{18}$$

In order to determine the conditions for choosing the matrix M necessary to guarantee the stability of the controller, it is required to set the following condition [8].

Condition 1: $A - XP + \alpha N_\ell X P_{\ell_{12}}$ is a Hurwitz matrix for all $\alpha \in \left[0, \frac{1}{2}\right]$. Checking this condition for given matrices P and $P_{\ell_{12}}$ can be performed checking the stability of an affine parameter-depending model $\dot{x} = \underbrace{(A_0 + \alpha A_1)}_{A(\alpha)} x$, where $A_0 = A - XP$, $A_1 = N_\ell X P_{\ell_{12}}$, $0 \leq \alpha \leq \frac{1}{2}$. This test may be formulated as a LMI (Linear Matrix Inequality) problem.

If M has the property:

$$\lambda_i(M) \geq 0, \qquad \forall \lambda_i(M) \in S(M) \tag{19}$$

the distributed controller is defined as in (17) and the closed-loop system is asymptotically stable. Considering that the interconnection of N agents is defined by means of a graph G with the associated Laplacian matrix whose eigenvalues are $0 = \lambda_1(G) < \lambda_2(G)\ldots \leq \lambda_N(G)$ and the maximum vertex degree d_{max}, different structures of the matrix M depending on the flight formation configuration can be chosen [2]. These features will be shown in the case study.

- If $M = aL(G)$ with $a > \frac{N_\ell}{2\lambda_2(G)}$, the closed-loop system is asymptotically stable when the controller has the expression (17). In addition, if Condition 1 is satisfied, the closed-loop system is asymptotically stable for $a \geq 0$.
- Let $M = aI_N - bA(G)$, $b \geq 0$. To achieve asymptotic stability of closed-loop system, the choice of parameters a and b satisfies $a - bd_{\max} > \frac{N_\ell}{2}$. In addition, the constraint for both parameters reduces to $a - bd_{\max} \geq 0$ when Condition 1 holds.
- Let $M = aI_N - A^w(G)$ with $A^w(G)$ - weighted adjacency matrix given by $A^w(G) = \begin{cases} A^w_{i,j} = 0, i = j, (i,j) \notin \mathcal{A} \\ A^w_{i,j} = w_{i,j,}(i,j) \in \mathcal{A}, \forall i,j = 1,\ldots,N, i \neq j \end{cases}, w_{\max} = \max_i \{\sum_j w_{ij}\}$.
  With K_D defined by expression (17), the closed-loop system is asymptotically stable if $a > w_{\max} - \frac{N_\ell}{2}$. If Condition 1 holds, the stability constraint becomes $a \geq w_{\max}$.

4. CASE STUDY

After the development of the equations of motion, the linear system of equations that defines the motion of the air vehicles is obtained. In order to study the evolutions of the formation agents, the decoupled dynamics of the system is needed. Thus, the performances of the multi-agent system for the longitudinal dynamics are analyzed.

In the case study, the state space system is used for the longitudinal motion of air vehicles. The longitudinal dynamics is described by the decoupled equations for $x = [u \quad w \quad q \quad \theta \quad h]^T$ whose expression is given by: $[\dot{u} \quad \dot{w} \quad \dot{q} \quad \dot{\theta} \quad \dot{h}]^T = A_{long}[u \quad w \quad q \quad \theta \quad h]^T + B_{long}[\delta_e \quad \delta_T]^T$.

4.1 Centralized control

A flight formation control approach is to design a centralized controller when all the vehicles are interconnected. The complete mathematical model of an air vehicle is detailed in [4], [6], [7] and, in the following case study, the numerical values of the UAV linearized model are the ones given in [15]. The analyzed flight formation consists of four identical agents where the control vector is $u = [\delta_e \quad \delta_T]^T$ and the state vector is $x = [u \quad w \quad q \quad \theta \quad h]^T$.

The flight formation dynamics has the state-space formulation $\dot{x}_F = A_F x_F + B_F u_F$, with $A_F = I_4 \otimes A$, $B_F = I_4 \otimes B$. The weighting matrices are defined as in (5). Solving the linear quadratic problem involves minimizing the cost function (4) which supposes determining the stabilizing solution of ARE (7). Taking into account the expression of centralized controller (11), the matrices satisfy the specified properties and $P_{f_{11}} = P - (N-1)P_{f_{12}}$. In the case study, each vehicle is considered to have different initial conditions (different values for u and h). It is desired for the formation to reach a certain altitude $H = 30$m and a specific velocity $u = 15$ m/s. After determining the solution of ARE P_f and the gain matrix K_f, the time responses for the closed-loop system are obtained using MATLAB. Analyzing the eigenvalues of matrix A_{cl_f} corresponding to the closed-loop system for the formation, $\mathrm{Re}(\lambda) < 0$, achieving the system stability.

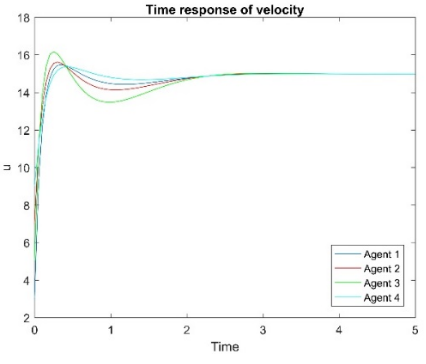

Fig. 1 Time response of velocity – centralized control

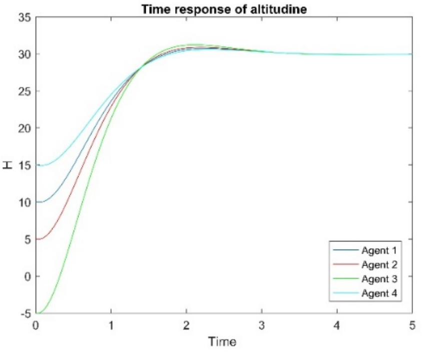

Fig. 2 Time response of altitude – centralized control

The simulations presented in Fig. 1 and Fig. 2 illustrate the responses of the controlled system using the linear quadratic method considering different initial conditions for each agent. From Fig. 1 and Fig. 2, one can see that the formation members reach the desired altitude and velocity, corresponding to the trimming conditions, when they start from different initial conditions.

4.2 Distributed control

The challenge regarding the control of networked control system appears in case of limited transmission information. This situation cannot be ensured by the existence of a centralized controller. The capacity of an air vehicle to receive or transmit information regarding the behavior of a limited number of agents introduces new features in the problem of designing the control law. This part considers two configurations of a formation consisting of four agents.

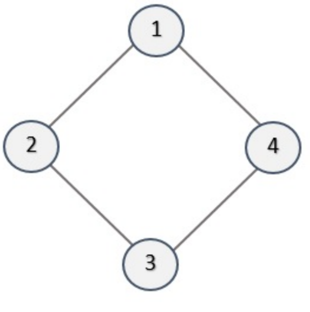

Fig. 3 Flight formation - Configuration I

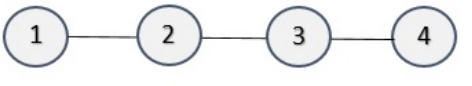

Fig. 4 Flight formation – Configuration II

4.2.1 Configuration I

For the analysis of the longitudinal dynamics in case of distributed control, a flight formation consisting of four air vehicles with the configuration in Fig. 3 is considered. The interconnection of the agents is defined by the corresponding adjacency matrix

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{20}$$

To solve the linear quadratic problem, it is necessary to solve ARE associated with centralized problem for $N_\ell = 3$ agents. The weighting matrices are defined in a same way as in centralized control. Furthermore, the solution of the LQR problem corresponding to a single agent P is determined. Compared to centralized control, the distributed gain matrix depends on the way of agent's interconnection given by matrix M. Thus, the definition of M and choosing the necessary parameters is important. In this case, M is defined $M = aI_N - bA(G)$ where $a = 2$ and $b = 1$, satisfying $a - bd_{max} \geq 0$ condition. After obtaining the distributed controller and checking the corresponding properties, the expression of K_D is:

$$K_D = \begin{bmatrix} K_1 & K_2 & 0 & K_2 \\ K_2 & K_1 & K_2 & 0 \\ 0 & K_2 & K_1 & K_2 \\ K_2 & 0 & K_2 & K_1 \end{bmatrix} \tag{21}$$

Regarding the configuration in Fig. 3, it can be seen that there is connection between the pair of agents (1,2) and (1,4). There is no communication between agent 1 and agent 3. Thus, the null term corresponding to the connection between agent 1 and agent 3 is explained. The structure of K_D stands for all positions where interconnection between agents is not possible.

To study the stability of closed-loop system, the eigenvalues of matrix $A_{F_D cl}$ are determined. $\mathrm{Re}(\lambda) < 0$ for all eigenvalues, so the obtained distributed controller ensures the formation stability. The numerical simulations illustrate the time responses of the four agents for the considered formation. As in the previous type of control, different initial conditions are considered for each agent. The performances of distributed control are to achieve the desired altitude $H = 30\ m$ and velocity $u = 15 m/s$ and to maintain these constraints during flight.

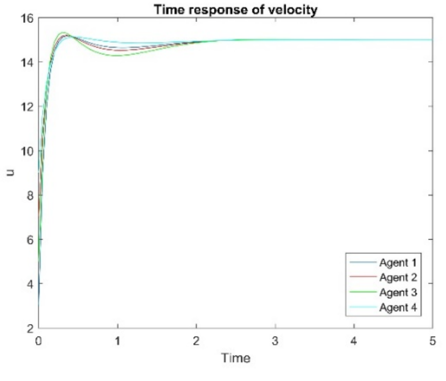

Fig. 5 Time response of velocity-distributed control-config. I

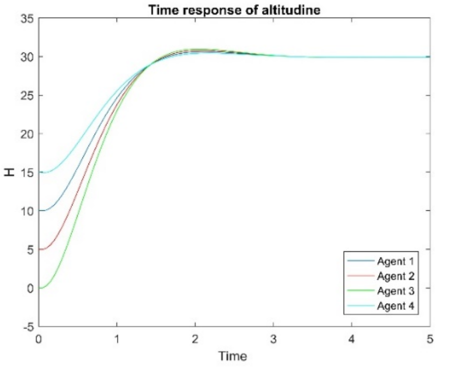

Fig. 6 Time response of altitude-distributed control-config. I

Fig. 5 and Fig. 6 demonstrate the desired performances for all air vehicles in the flight formation.

Although the interconnection between certain pairs of agents is limited, the way of communication does not influence the desired objectives of the formation.

4.2.2 Configuration II

This part uses a new configuration consisting of an equal number of agents. Their architecture and way of communication are different from the previous case. The analyzed configuration is illustrated in Fig. 4 and the expression (22) defines the interconnection of the agents. For numerical simulations, the same longitudinal dynamics is used.

$$L(G) = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \tag{22}$$

The weighting matrices necessary to minimize the cost function (15) have a similar form as in the previous configuration. The determination of controller (17) assumes solving the two ARE, but the expression of matrix M is different for this case. M is described by $M = aL(G)$ with $a = 1.5$, satisfying the stability condition $a \geq 0$.

After obtaining the gain matrix, the structure of the controller is analyzed where $K_{D_{13}} = K_{D_{14}} = 0$, $K_{D_{31}} = K_{D_{24}} = 0$ and $K_{D_{41}} = K_{D_{42}} = 0$. Fig. 4 specifies that the only connection for agent 1 is agent 2 and communication between the pairs of agents (1,3) and (1,4) is not possible. Thus, the terms corresponding in the controller form are null. This property is proven for each agent of the formation.

For stability analysis, the closed-loop system defined by A_{F_Dcl} is constructed. After obtaining the eigenvalues of A_{F_Dcl}, it can be seen that $Re(\lambda) < 0$, which demonstrates the closed-loop system stability.

In order to study the time responses of the four air vehicles, the same initial conditions as in the previous analysis are set.

Fig. 7 proves that all agents maintain the desired velocity value during the simulation and the time response of altitude (Fig. 8) shows the formation behavior for which a desired altitude is required.

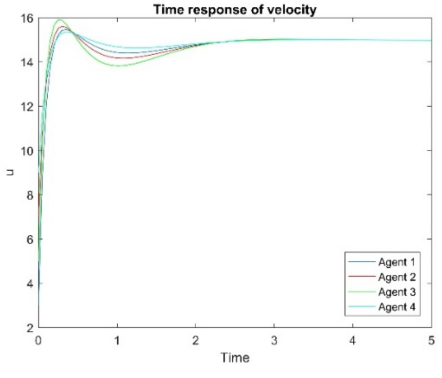

Fig. 7 Time response of velocity-distributed control-config. II

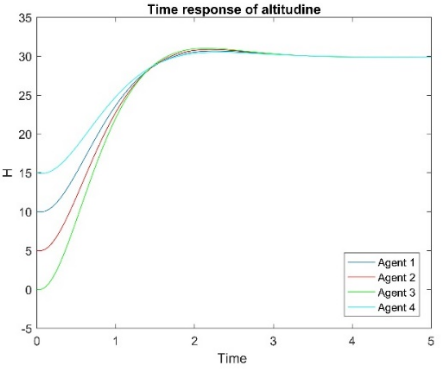

Fig. 8 Time response of altitude-distributed control-config. II

5. CONCLUDING REMARKS

As a case study of the paper, the longitudinal dynamics of an UAV is used and the analyzed formation consists of four identical agents. To study the design characteristics of the two types of controller (centralized and distributed), different multi-agent system configurations are used. The impossibility of interconnection between certain agents introduces a new problem, namely, whether the control system design guarantees the system stability. In the case of centralized control, the bidirectional communication for all agents reduces the complexity of this issue. Therefore, it is necessary to meet the different stability conditions in order to define the appropriate agent interconnection matrix.

The results offered by the distributed controller ensure the system stability, being an efficient solution in solving the problems introduced by the communication restrictions. This aspect is highlighted by the null terms in the gain matrix associated with the impossibility of interconnection between certain agents.

REFERENCES

[1] B. D. O. Anderson, J. B. Moore, *Optimal Control. Linear Quadratic Methods*, ISBN-10: 9780486457666, ISBN-13: 978-0486457666, Dover Publications, 2007.

[2] F. Borrelli, T. Keviczky, *Distributed LQR design for identical dynamically decoupled systems*, Proceedings of the 45th IEEE Conference on Decision & Control, USA, 2006.

[3] K. Zhou, J. C. Doyle, K. Glover, *Robust and Optimal Control*, 1st edition, ISBN-10: 0134565673, ISBN-13: 978-0134565675, Prentice Hall, 1995.

[4] M. V. Cook, *Flight Dynamics Principles*, Elsevier Aerospace Engineering Series, 2nd Edition, Hardcover ISBN: 9780750669276, eBook ISBN: 9780080550367, 2007.

[5] J. A. Fax, R. M. Murray, *Graph Laplacians and Stabilization of Vehicle Formations*, 15th Triennial World Congress, Spain, 2002.

[6] D. McLean, *Automatic Flight Control Systems*, Prentice Hall International Series in Systems and Control Engineering, ISBN: 0130540080, 9780130540089, 1990.

[7] B. Etkin, L. D. Reid, *Dynamics of Flight. Stability and Control*, Wiley, 3rd edition, ISBN-10: 0471034185, ISBN-13: 978-0471034186, 1995.

[8] C. Scherer, S. Weiland, *Linear Matrix Inequalities in Control*, 2000.

[9] M. Meshabi, M. Egerstedt, *Graph Theoretic Methods in Multiagent Networks*, ISBN-10: 0691140618, ISBN-13: 978-0691140612, Princeton Series in Applied Mathematics, 2010.

[10] Q. Ali, S. Montenegro, *Explicit Model Following Distributed Control Scheme for Formation Flying of Mini UAVs*, 2016.

[11] S.-H. Wang, E. Davison, On the stabilization of decentralized control systems, *IEEE Transactions on Automatic Control*, vol. **AC-18**, No. 5, pp. 473–478.1973.

[12] J. Fax, *Optimal and Cooperative Control of Vehicle Formations*, Thesis, California Institute of Technology, doi:10.7907/M4N7-AK02, 2002.

[13] J. P. Corfmat, A. S. Morse, Decentralized control of linear multivariable systems, *Automatica*, Volume **12**, Issue 5, Pages 479-495, September 1976.

[14] I. Tomic, *Distributed LQR Control of Multi-Agent Systems*, A dissertation submitted for the degree of Doctor of Philosophy in Control Theory, City, University of London School of Mathematics, Computer Science and Engineering, October 2016.

[15] C. Hajiyev, H. E. Soken, S. Y. Vural, *State Estimation and Control for Low-cost Unmanned Aerial Vehicles*, Hardcover ISBN: 978-3-319-16416-8, Softcover ISBN: 978-3-319-36396-7, Springer International Publishing, 2015.

[16] B. Bamieh, F. Paganini, M. A. Dahleh, Distributed Control of Spatially Invariant Systems, *IEEE Transactions on Automatic Control*, Volume **47**, Issue 7, Jul 2002.

[17] R. Olfati-Saber, R. M. Murray, Distributed Cooperative Control of Multiple Vehicle Formations using Structural Potential Functions, *IFAC Proceedings Volumes*, Volume **35**, Issue 1, Pages 495-500, 2002.

[18] J. Hu, G. Feng, Distributed tracking control of leader-follower multi-agent systems under noisy measurement, *Automatica*, **46**(8):1382-1387, DOI:10.1016/j.automatica.2010.05.0202011

[19] W. Ren, Distributed Cooperative Attitude Synchronization and Tracking for Multiple Rigid Bodies, *IEEE Transactions on Control Systems Technology*, Volume **18**, Issue 2, Pages: 383 - 392, March 2010.

[20] P. Massioni, M. Verhaegen, Distributed Control for Identical Dynamically Coupled Systems: A Decomposition Approach, *IEEE Transactions on Automatic Control*, Volume **54**, Issue 1, Pages: 124 - 135, Jan. 2009.

[21] Z. Jin, *Coordinated Control for Networked Multi-Agent Systems*, Dissertation (Ph.D.), California Institute of Technology, doi:10.7907/ZRAD-XN95, 2007.