Integrated CLOS and PN Guidance for Increased Effectiveness of Surface to Air Missiles

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Abstract: *In this paper, a novel approach has been presented to integrate command to line-of-sight (CLOS) guidance and proportional navigation (PN) guidance in order to reduce miss distance and to increase the effectiveness of surface to air missiles. Initially a comparison of command to line-of-sight guidance and proportional navigation has been presented. Miss distance, variation of angle-of-attack, normal and lateral accelerations and error of missile flight path from direct line-of-sight have been used as noteworthy criteria for comparison of the two guidance laws. Following this comparison a new approach has been proposed for determining the most suitable guidance gains in order to minimize miss distance and improve accuracy of the missile in delivering the warhead, while using CLOS guidance. This proposed technique is based on constrained nonlinear minimization to optimize the guidance gains. CLOS guidance has a further limitation of significant increase in normal and lateral acceleration demands during the terminal phase of missile flight. Furthermore, at large elevation angles, the required angle-of-attack during the terminal phase increases beyond design specifications. Subsequently, a missile with optical sensors only and following just the CLOS guidance has less likelihood to hit high speed targets beyond 45^o in elevation plane. A novel approach has thus been proposed to overcome such limitations of CLOS-only guidance for surface to air missiles. In this approach, an integrated guidance algorithm has been proposed whereby the initial guidance law during rocket motor burnout phase remains CLOS, whereas immediately after this phase, the guidance law is automatically switched to PN guidance. This integrated approach has not only resulted in slight increase in range of the missile but also has significantly improved its likelihood to hit targets beyond 30 degrees in elevation plane, thus successfully overcoming various limitations of CLOS-only guidance approach. Hence, proposing an approach to determine most suitable gains for CLOS guidance and integration of CLOS and PN guidance for enhanced effectiveness and accuracy of surface to air missiles are the two significant contributions of this work.*

Key Words: *guidance, command line-of-sight, proportional navigation, surface-to-air missile*

NOMENCLATURE

R_m = firing unit to missile range
R_t = firing unit to target range
θ_m = elevation plane angle of missile
θ_t = elevation plane angle of target
ψ_m = azimuth plane angle of missile
ψ_t = azimuth plane angle of target
$\frac{dR_m}{dt}$ = missile range rate
$\frac{dR_t}{dt}$ = target range rate
$\frac{d\theta_t}{dt}$ = rate of change of target elevation angle
$\frac{d\psi_t}{dt}$ = rate of change of target azimuth angle
D_ϵ = displacement from the launch site to the target
λ_ϵ = lateral displacement from the desired course in elevation plane
λ_a = lateral displacement from the desired course in azimuth plane
G_Z = normal acceleration command to missile
G_Y = lateral acceleration command to missile
t_{go} = time-to-go

1. INTRODUCTION

The strategy by which the missile is steered to ensure that it remains on a direct collision course with the approaching target and eventually leads to an interception is called guidance.

Thus to improve its accuracy in delivering the warhead, the missile responds to these steering commands.

Depending on whether these commands are internal or external, guidance is further divided into two types:

- **Homing guidance**, where the interceptor missile integrates a seeker that tracks the target and an on-board guidance computer that translates target position and missile-to-target range into steering commands. The “Fire and Forget” missile is autonomous and is independent from the control unit at launch site.
- **Command guidance**, which relies on the missile guidance commands computed by the guidance computer at launch site.

In command guidance the target tracking system and the guidance computer are not a part of the missile, instead they are a part of the firing unit at the control site. This reduces the weapon cost by the placement of the sensor and guidance computer at launch site in comparison to designed-for-single-use weapon systems with on-board seeker and guidance.

Radars, optical, laser or IR systems may be used to accomplish tracking. The tracking system tracks both the missile and the target.

The missile and target ranges, elevations, and bearings are fed to a guidance computer. The guidance computer determines the missile’s normal and lateral acceleration commands using this position and velocity information.

The acceleration commands are relayed to the missile via a command uplink. The system block diagram is shown in Figure 1. This infers that throughout its encounter, the missile is dependent on the firing unit.

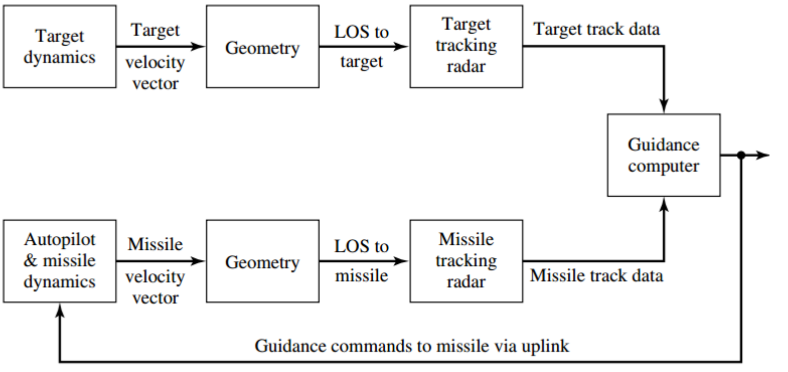


Figure 1: System Block Diagram for Command Guidance [1]

In the design of a guided missile system, the guidance law employed in the guidance loop is of vital importance. There are various types of guidance algorithms: line-of-sight guidance, pure pursuit, parallel navigation and proportional navigation etc. Beam riding and command-to-LOS are two types of command guidance and use the same basic idea i.e. the missile should be on the straight line joining the ground tracking system and the approaching target throughout the course of missile flight. Ideal line-of-sight [2] guidance implies $\theta_m = \theta_t$. Consequently in three dimensional engagements, the missile and target angles in azimuth plane must be also equal .i.e. $\Psi_m = \Psi_t$.

The principle of command to line-of-sight (CLOS) guidance [3-6] is to make the missile fly as near as possible along the instantaneous line joining the tracker and the target, called the line-of-sight. The target and the missile are tracked by the ground tracking system in CLOS guidance. The ground tracking system is also responsible for measuring the angular difference between the missile and the threat. Based on the angles and angular difference computations that are fed into the guidance computer positioned near the firing unit at control site, steering commands generated and are relayed to the in-flight missile.

The performance of CLOS guidance is known to be typically good for short-range engagements. For medium and long-range engagements the performance is limited by the tracker at control site. The missile acceleration capability is strongly reliant on the guidance law used [7]. In command to line-of-sight guidance, normal and lateral acceleration commands are relayed to the in-flight missile to make the displacement of the missile from the direct line-of-sight to the target as small as possible, in elevation (pitch) and azimuth (yaw) planes. For low speed targets, the performance of CLOS is known to be good [6]. But upon increasing the target velocity, the demanded acceleration increases as a result of increasing the curvature of missile trajectory.

A comprehensive review of the literature shows that in past, modern stochastic optimal control theory [8] and feedback linearization [3] techniques have been applied to CLOS guidance. Further, CLOS has been used for midcourse guidance because of very high acceleration demand as the missile closes on to the target. To use CLOS for homing guidance [5], a supplementary feed-forward acceleration command is required. In the years to follow, it was established that Self organizing fuzzy logic control (SOFLC) design method when applied to CLOS guidance [9] produces better performance than a fuzzy logic control (FLC) and an optimal learning FLC. High probability of the missile flying out of the beam is a major shortcoming of lead angle method. A supervisory CLOS guidance law with lead angle [6] was proposed to address this limitation of lead angle method and keep the missile flight within the tracking beam. Later, a new optimal-fuzzy two-phase CLOS guidance law

[10] was proposed to reduce the acceleration demands in the midcourse phase of missile flight and coupling of supervisory and main tracking controller was introduced to ensure missile flight within the beam. For the terminal phase, CLOS guidance law without lead angle was employed and a sliding mode controller was proposed, and the parameters of the proposed controllers were optimized by Ant Colony Optimization. Chao, Lee and Soong presented an analytical study of the aspect angle of the interceptor at lock-on, lateral acceleration and normalized missile acceleration of command to line-of-sight guidance against head-on high speed maneuvering targets in their paper [11].

Variants of Proportional Navigation have been widely used in the history of missile guidance. One such research [12] suggests using variable structure control for design of augmented true PN guidance law that makes use of the information of target acceleration bound only. Another research [13] evaluates the performance for a short range homing surface-to-air missile by using PN guidance as the guidance law and Coefficient Diagram Method to design normal acceleration and roll angle control system.

In modern missile guidance, Linear Quadratic Guidance laws [14] that enable imposing a predetermined intercept angle have been presented. They are capable of achieving negligible miss distance and interceptor angle error is trivial, even if the target maneuvers. Another research [15] suggests an observer-based optimal zero-sliding midcourse guidance for missiles with thrust vector control and divert control system. A multi-objectives evolutionary algorithm based approach has also been proposed in recent research [16] for developing an integrated fuzzy guidance law with 3 fuzzy controllers for aerodynamic missiles, each of which is activated in a different region of interception. Hou, Liang and Duan suggested a novel integrated missile guidance and autopilot design method based on the adaptive block dynamic surface control approach [17] and verified the accuracy of target interception by 6DOF simulation. A different scheme for integrated missile guidance [18] also utilizes adaptive dynamic surface control but with input saturation. Furthermore, it has also been proposed that for a maneuvering missile, the body angle at which the target is intercepted is the sum of the flight path angle and the angle of attack [19] and therefore it should not be disregarded because it is significant in several cases.

Another research [20] puts forward a 3D nonlinear guidance law with acceleration command saturation constraint and exhibits excellent performance in terms of miss distance. Terminal guidance laws with constrained impact angle and impact velocity [21] for hypervelocity descent to a static target and those [22] that can recompense for elusive target maneuvers without acceleration estimation have also been proposed in recent research. Literature further affirms the application of optimization techniques to missile guidance for optimal gain evaluation [23] and trajectory optimization [24, 25].

This work develops a 3 DOF simulation of point mass model of missile in Simulink MATLAB. Complete physics of the missile which includes rocket motor parameters, i.e. thrust and burn time, mass variations with time, and the missile aerodynamics have been modeled. CLOS guidance is referred to as three point guidance. It aims to minimize the displacement of the missile from the direct line-of-sight joining the tracker at control site and the approaching target. The concept involved is since the three points are always on the same line the distance-to-go and consequently the time until impact decrease over time. The command to the missile from the guidance computer is acceleration. A simplest implementation of a CLOS law computes missile acceleration proportional to the displacement from the direct line-of-sight.

In the present study, the guidance gains have been optimized to minimize miss distance i.e. the missile-to-target range at the Closest Point of Approach. Optimization is done using gradient descent method in MATLAB environment. Minimization of miss distance is defined as an objective for constrained nonlinear optimization. By setting up a problem statement, a solution for $f(x)$ is sought such that x is a local minimum subject to the constraints allowable on x i.e. $min_x f(x)$. In nonlinear programming, Sequential Quadratic Programming methods represent the state of the art and make use of dense linear algebra [26]. A version was implemented and tested by Schittkowski [27] which outdoes all other tried methods in terms of efficiency, accuracy and percentage of successful solutions. Fletcher [28] and Powell [29] present a summary of the Sequential Quadratic Programming methods. The paper finally discusses the limitations of the CLOS guidance and presents an alternative solution that suggests switching of the guidance law to proportional navigation after rocket motor burnout to reach a compromise solution for improved results. The idea is summarized in the system block diagram in Figure 2. The data from search radar is displayed on Plan Position Indicator (PPI) radar display. The missile-target information received from the tracking radar is displayed as a bull's-eye view of azimuth vs. elevation on the C-scope. The tracking information from the tracking system is used to compute the deviation of the missile from the line-of-sight and is fed to a guidance computer. The guidance computer is programmed to compute acceleration commands with CLOS-only guidance during initial thrust phase and with PN guidance after rocket motor burnout. The commands are delivered to the in-flight missile via a command uplink.

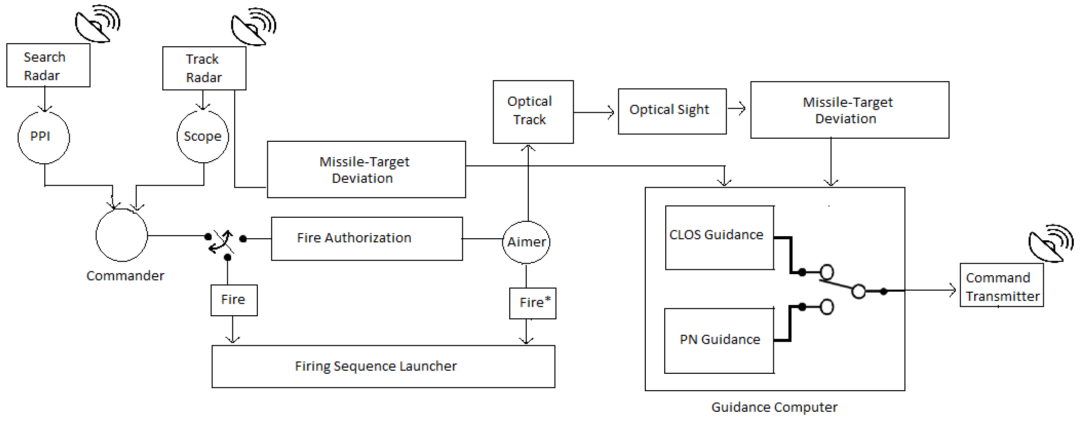


Figure 2: System Block diagram for Integrated Guidance

2. METHODOLOGY

A 3DOF simulation of a point mass model of the missile and a maneuvering target has been designed and simulated in Simulink MATLAB environment.

Implementation Platform

Simulink is a block diagram environment with a graphical editor and predefined libraries. It has been integrated with MATLAB and is extensively used for model-based design and multi-domain simulation of physical systems. A simulation engine with ordinary differential equation solvers computes system dynamics by numerical integration. System requirements for Simulink MATLAB (R2016a) include any x86-64 bit processor with 4 to 6 GB of free disk space and 4 GB RAM. The model comprises of target dynamics, LOS computation of target and missile, missile's point mass model and missile guidance. The detail of each block is discussed below:

Target Dynamics

Target flight path and velocity is computed by twice integrating the instantaneous inertial acceleration from a predefined initial velocity vector. It is assumed that the target can accelerate without loss of speed. 2 g laterally accelerating target with target speeds of 40 to 400 m/s and dive angle of 20° have been selected for defining the target simulation environment.

Missile Dynamics

Missile acceleration in axial, normal and lateral direction is computed at each time instant using forces acting on the missile body. The missile acceleration in body axis is converted to inertial axis system followed by superimposition of gravity vector. The missile acceleration vector is integrated to yield velocity and position vectors in inertial frame of reference using initial conditions. The physical properties of the generic missile are summarized in Table 1.

Table 1: Physical Properties of the Missile

Mass at launch	100 kg
Mass at burn-out	50 kg
Propellant mass	50 kg
Burn time	~3.3 s
Total impulse	10,000 daN.s
Cross sectional diameter	6.5 inch
Axial acceleration	40-g
Normal/ lateral acceleration	25-g

Aerodynamic Forces

The sum of normal forces and tangential forces acting on the surface due to fluid motion around the missile are resolved into three components along axes parallel and perpendicular to the free-stream velocity direction. These forces are known as lift (L), drag (D), and side force (Y) and are shown in Figure 3 in stability axis system. The forces computed for the purpose of simulation are in body axis system and account for the change in drag force because of canard deflection.

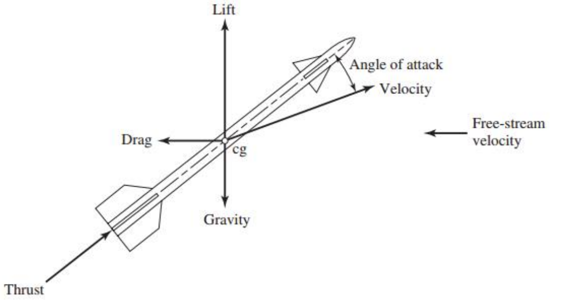


Figure 3: Missile Thrust and Aerodynamic Forces [1]

A drag coefficient curve is specified in the missile model and is shown in Figure 4. Missile drag is then computed using these drag coefficients. Required normal & lateral force is computed from normal and lateral acceleration demand. The angle-of-attack requirement is computed using dynamic pressure, missile cross sectional area and normal force verses angle-of-attack slope. Limitations are imposed on angle-of-attack to confine it within physical limits.

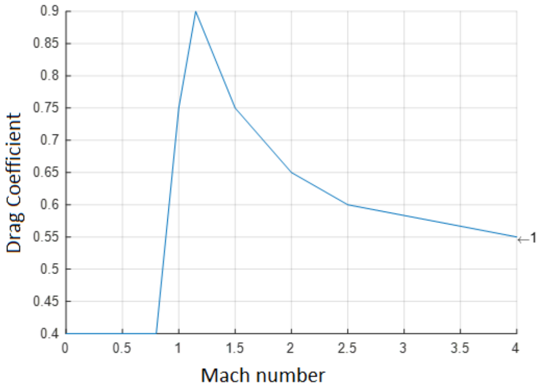


Figure 4: Drag Coefficient Variation with Mach number

Missile Guidance

During the midcourse phase of missile flight, the target and missile velocities and position are fed to the guidance computer in polar co-ordinate system: R (range), θ (elevation), ψ (azimuth). The information provided to the guidance computer is the position vector of the target [R_t θ_t ψ_t] and the missile [R_m θ_m ψ_m] as well as the rates [$\frac{dR_t}{dt}\frac{d\theta_t}{dt}\frac{d\psi_t}{dt}$]. The missile range rate $\frac{dR_m}{dt}$ is also measured. Line-of-sight rate is measured for PN guidance. The guidance objective is to minimize the miss distance for both maneuvering and non-maneuvering targets. The guidance computer receives this information from tracking system and then issues command to the firing unit to set launch angle. The algorithm developed for missile guidance is based on the switching of two guidance laws, namely CLOS and PN guidance. Figure 5 illustrates the lateral error components in the elevation plane for command guidance.

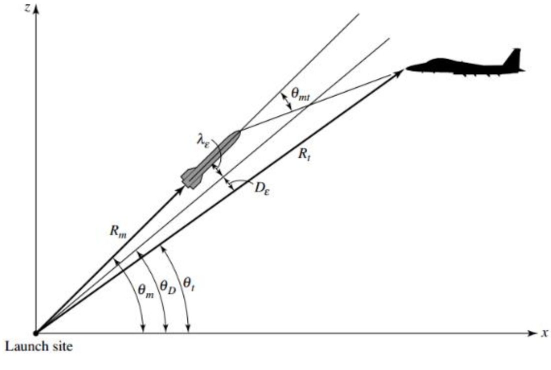


Figure 5: Missile-Target Geometry [1]

Cross Range Error

Mathematical model to follow has been used to compute the guidance commands in elevation and azimuth planes. The azimuth plane is defined as the xy-plane and the elevation plane is defined as the xz-plane. R_m is the missile range and R_t is the range of the approaching target. D_ϵ is the displacement from the launch site to the target and λ_ϵ is the lateral displacement from the desired course or the cross range error. Their relationships are given by equations 1 and 2, respectively.

$$D_\epsilon = R_m(\theta_D - \theta_t) \tag{1}$$

$$\lambda_\epsilon + D_\epsilon = R_m(\theta_m - \theta_t) \tag{2}$$

Subtracting equations (1) and (2) gives the missile's lateral error from the desired course.

$$\lambda_\epsilon = R_m(\theta_m - \theta_t) - R_m(\theta_D - \theta_t) \tag{3}$$

The missile and target range must be the same when time-to-go is zero to ensure interception. This is true only if

$$\theta_D = \theta_t + \frac{d\theta_t}{dt} t_{go} \Rightarrow \theta_D - \theta_t = \frac{d\theta_t}{dt}\left(\frac{R_t - R_m}{\dot{R_m} - \dot{R_t}}\right) \tag{4}$$

where $t_{go} = \frac{R_t - R_m}{\dot{R_m} - \dot{R_t}}$

Substituting these values in equation 3 gives a generic equation of lateral displacement from desired course in rad m.

$$\lambda_\epsilon = R_m(\theta_m - \theta_t) - K_G R_m \frac{d\theta_t}{dt}\left(\frac{R_t - R_m}{\dot{R_m} - \dot{R_t}}\right) \tag{5}$$

where K_G is the proportionality constant used to tune the command guidance system. Similarly, it can be shown that the lateral error for the azimuth plane is given by equation 6.

$$\lambda_\mathrm{A} = R_m(\psi_m - \psi_t)\cos\theta_t - K_G R_m \frac{d\psi_t}{dt}\left(\frac{R_t - R_m}{\dot{R_m} - \dot{R_t}}\right)\cos\theta_t \tag{6}$$

As the missile approaches the target, the second term in equations 5 and 6 approach zero i.e. the distance-to-go and consequently, the time until impact approach zero.

Missile Acceleration Commands Computed from CLOS Guidance

Simplest implementation of CLOS computes missile acceleration commands proportional to the lateral displacements from the direct line-of-sight in elevation and azimuth plane. The normal acceleration command G_z relayed to the in-flight missile is given by equation 7.

$$G_z = K_1 R_m(\theta_m - \theta_t) + K_2 R_m \frac{d\theta_t}{dt}\left(\frac{R_t - R_m}{\dot{R_m} - \dot{R_t}}\right) \tag{7}$$

And correspondingly the lateral acceleration command G_Y is given by equation 8

$$G_Y = K_3 R_m(\psi_m - \psi_t)\, cos\,\theta_t + K_4 R_m \frac{d\psi_t}{dt}\left(\frac{R_t - R_m}{\dot{R_m} - \dot{R_t}}\right)\cos\theta_t \tag{8}$$

where K_1, K_2, K_3 and K_4 are the guidance gains.

Optimization of Guidance Gains

At high speed, the probability of hitting a target if it maneuvers becomes small. Such a scenario is selected where the target is travelling at high speed and missile guidance gains are optimized. The objective function is defined as minimization of miss distance at the Closest Point of Approach and a constraint of miss distance ≤ 1 meter is given to formulate a merit function. The guidance gains K_1, K_2, K_3 and K_4 are defined as optimization parameters. The optimization using SQP algorithm converged within 118 seconds and 16` iterations. The optimization makes use of *fmincon* function at the backend to internally create full matrices.

Final values of guidance gains K_1, K_2, K_3 and K_4 are 0.7675, -0.2311, 1.8747 and -3.3979. SQP satisfies bound at all iterations.

Moreover, it can recover from *NaN* and *Inf* results when implemented in MATLAB environment.

Missile Acceleration Commands after Switching to PN Guidance

After rocket motor burn-out, the guidance law is switched to PN guidance. According to PN law, the commanded acceleration is proportional to the line-of-sight rate; the proportionality constant is broken down into the product of the effective navigation ratio N and the relative missile-to-target closing velocity.

$$\vec{a} = N v_{cl} \dot{\vec{\lambda}} \tag{9}$$

Where N is the proportionality constant (N=5), v_{cl} is the missile-to-target closing velocity and $\dot{\lambda}$ is the LOS rate.

Closing velocity is the negative derivative of the range-vector and equals the difference between missile and target velocity.

$$v_{cl} = -\dot{\boldsymbol{R}} \tag{10}$$

where $\dot{R}$ is rate of change of range.

The acceleration commands are relayed to the in-flight missile auto-pilot where these steering commands are converted to turning moments via actuators and deflection of the control surfaces.

The switching of guidance law from CLOS to PN is accomplished with a switch programmed to switch the guidance law at t_{switch}. It is the expressed by the following equation:

$$t_{switch} = t_{delay} + t_b \tag{11}$$

where t_{delay} is the time of missile launch and t_b is the burn time.

3. RESULTS

The paper discusses results for a 2g maneuvering target moving at 400 m/sec and 20º dive angle test scenario. Figure 6 shows the projections of missile and maneuvering target's intercept trajectory in elevation and azimuth planes, respectively.

Elevation plane is defined as xz-plane and azimuth plane is defined as the xy-plane. The launcher elevation limit is 45º and elevation limit of tracking radar at launch site is 70º. The miss distance obtained as a result of switching methodology is less than 1 m.

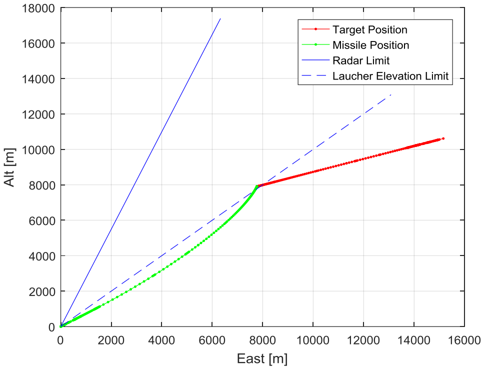
The variation of mass of missile with time is shown in Figure 7. The missile is launched at 0.5 seconds. Initial mass of the missile is 100 kilogram.

The rocket motor has a burn time of 3.3 seconds. Final mass of the missile after rocket-motor burnout is 50 kilogram.

The missile flight is divided into three phases: initial thrust phase, midcourse phase and terminal phase.

The missile achieves a Mach number of 3.5 at the end of the burnout phase after which it glides for approximately 17 seconds during the midcourse and terminal phases. The Mach number steadily decreases to Mach 0.8 because of drag and gravity momentum loss as shown in Figure 8.

During the rocket motor burnout phase the missile accelerates to 40 g. Graph of axial acceleration with time is plotted against simulation time in Figure 9.



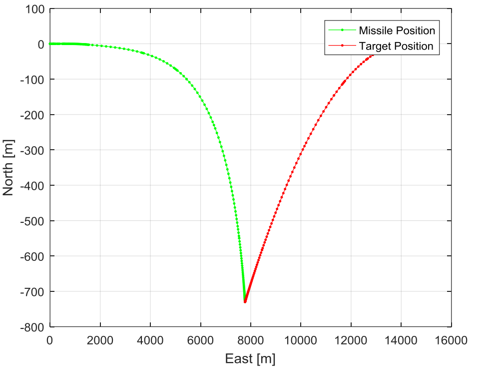


Figure 6: Interception Trajectory in (a) Elevation and (b) Azimuth Plane for a High Speed Maneuvering Target

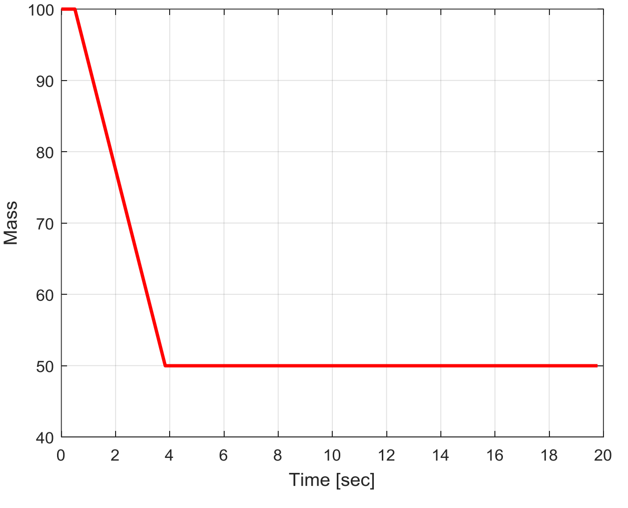


Figure 7: Mass Variation with Time

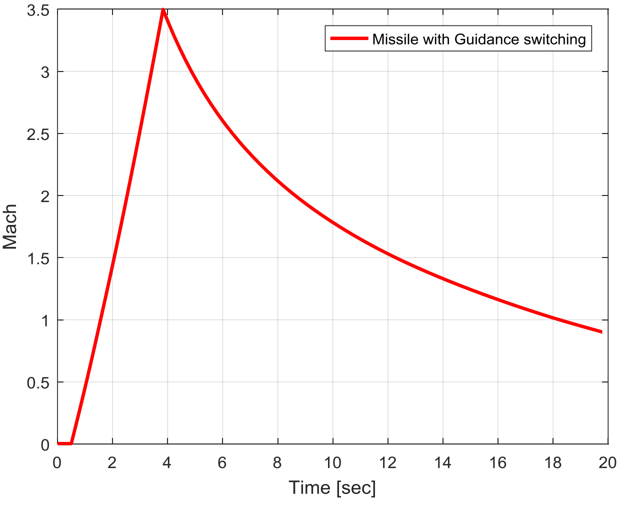


Figure 8: Mach number Variation with Time

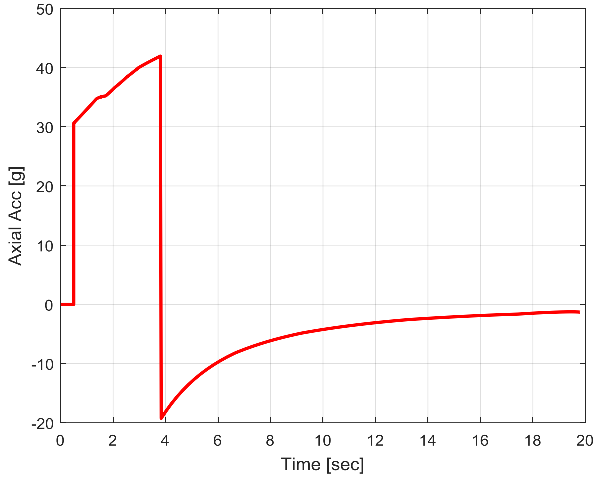


Figure 9: Axial Acceleration Variation with Time

Figure 10 illustrates a comparison of the angle-of-attack computed from PN and CLOS guidance, for a high speed target. The angle-of-attack in CLOS guidance increases rapidly beyond ±10º in the terminal phase of missile flight due to rapid change in LOS rate during the terminal phase [Figure 10].

When a limit is placed on achievable CN_{max} it results in a miss distance of more than 20 meters. Switching the guidance law to PN after rocket motor burnout addresses the issue, as the missile maneuvers towards the target during the initial phase of the trajectory resulting in miss distance of less than a meter.

Figure 11 shows a comparison for missile's normal acceleration computed using PN and command-to-LOS guidance.

The strategy results in nominal terminal maneuvering requirement and therefore modest angle-of-attack requirement during the terminal phase resulting in low miss distance.

Further CLOS guidance results in sinusoidal g- requirement during the midcourse phase ensuing greater energy loss and therefore slightly smaller range.

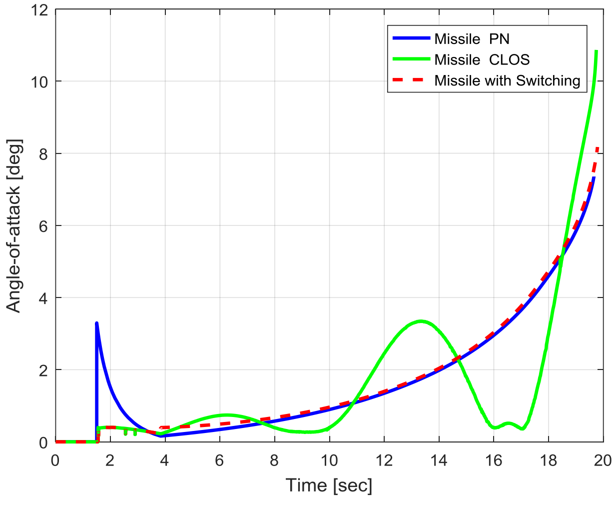


Figure 10: Angle-of-Attack Variation with Time

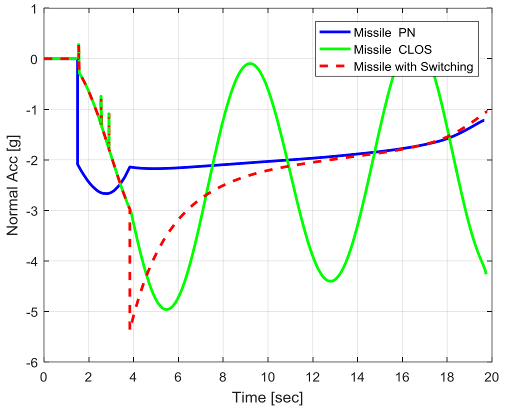


Figure 11: Normal Acceleration vs. Time

Figure 12 shows a comparison for missile's lateral acceleration computed using PN and command-to-LOS guidance. The lateral acceleration command from the guidance computer with switching of guidance law from CLOS to PN is almost same in azimuth as that obtained in case of pure PN guidance command. The lateral acceleration remains within limits of the missile lateral maneuvering envelope, which is 25-g at peak velocity.

A cause of concern may be that by switching the guidance law to PN guidance does the missile maneuver out of the radar beam? It is true that mathematically no effort has been made as part of this study, to ensure that the missile remains within the radar beam width. The deviation of the missile from LOS has been observed as part of the current study.

Error of missile flight path, in degrees, from desired line-of-sight is computed in elevation and azimuth planes during missile flight using both PN and CLOS guidance. After, the missile is launched at 0.5 seconds the line-of-sight error in elevation and azimuth plane is plotted in Figures 13 (a) and (b) respectively. The error of missile path from direct LOS when PN guidance is used is approximately 2.2º in elevation plane for the selected scenario however using optical sensors only, as in CLOS guidance, it results in an error within ±1.25º. Switching of the guidance mode from CLOS to PN does not result in the deterioration of LOS error as shown in Figure 13 (a). In azimuth plane, the lateral error of the missile, in degrees, from the direct line-of-sight is comparatively large when using optical sensors only. However, when switching of the guidance law is implemented maximum deviation from the direct LOS is within ±0.4º of the beam [Figure 13 (b)].

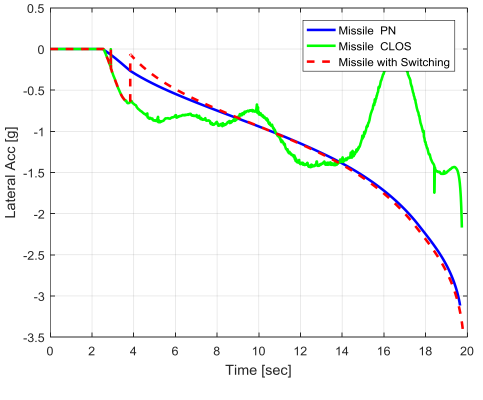


Figure 12: Lateral Acceleration vs. Time

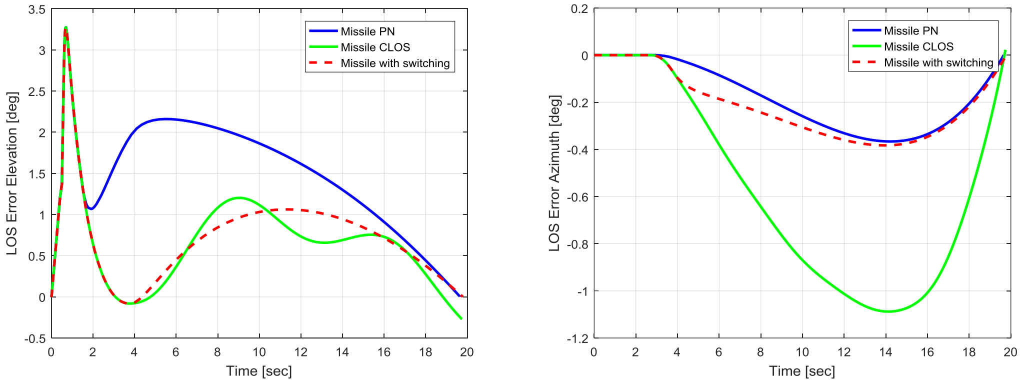


Figure 13: LOS Error in (a) Elevation and (b) Azimuth Plane

Altitude vs. range plot of the missile using optical sensors only is plotted for non-maneuvering high speed targets in Figure 14.

Interception zone is 10 km and engagement zone is approximately 17 km for high speed targets. Utilization of CLOS algorithm for engagement of high speed target beyond 45° zone of engagement has been shown as a limitation.

In this region the LOS rate changes rapidly resulting in high g-maneuvers at a point once the missile is already in a low energy state.

Altitude vs. range plot of the missile using switching of the guidance laws is plotted for high speed non-maneuvering targets in Figure 15.

Interception zone is approximately 10.8 km and engagement zone is 18 km for high speed targets.

The figure shows that the missile is capable of hitting all targets within the tracking radar elevation limit.

This concludes the missile's range and likelihood to hit targets beyond 45º in elevation plane has improved substantially while remaining within physical limits of the ground tracking radar.

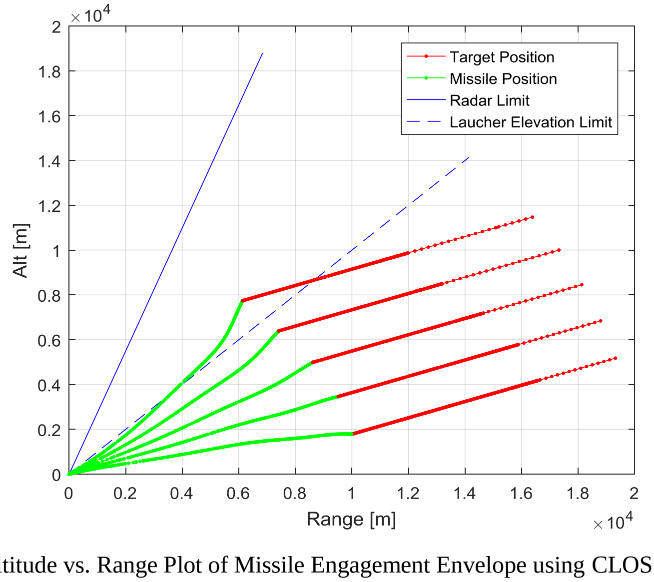


Figure 14: Altitude vs. Range Plot of Missile Engagement Envelope using CLOS Guidance Only

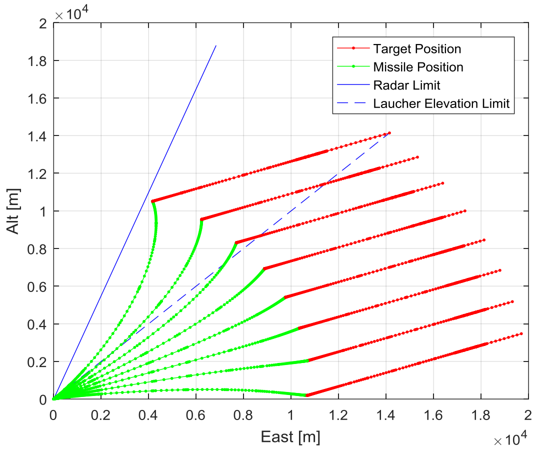


Figure 15: Altitude vs. Range Plot of Missile with Combine CLOS & PN Guidance

4. CONCLUSIONS

A comparison of command to line-of-sight guidance and proportional navigation has been presented of a generic Surface-to-Air Missile in this paper for a special case of a high speed maneuvering target. It covers a comprehensive comparison of miss distance, variations of angle-of-attack, normal and lateral accelerations and error of missile flight path from direct line-of-sight for the two guidance laws.

Further, the approach to determine the optimum guidance gains for CLOS guidance using constrained nonlinear minimization using miss distance as the objective function is good way to obtain the optimum CLOS guidance gains. The paper further proposes a guidance algorithm to overcome the limitations of CLOS-only guidance. It suggests switching of the guidance law after rocket motor burnout to proportional navigation for improved results.

Following conclusions are drawn based on the results presented in this document:

(a) the analysis suggests that normal and lateral acceleration increase during the terminal phase of missile flight when only optical sensors are used.

(b) At large elevation angle initial condition, the angle-of-attack corresponding to g-command increases beyond ±10º which becomes a violation of the design specifications resulting in a miss distance.

(c) The missile with optical sensors only i.e. with CLOS-only guidance implemented, is unable to hit high speed targets beyond 45º in elevation plane.

(d) The error of missile path from direct LOS for PN guidance exceeds ±2º in elevation plane.

Switching of guidance law after rocket motor burnout does not compromise deviation from LOS angle and also results in smaller miss distance.

Moreover, the normal and lateral acceleration demands with switching are comparatively small.

The error from the direct LOS in elevation plane when switching of guidance law is implemented also remains within ±1º of the beam which implies that PN guidance in conjunction with CLOS guidance can be used for command guidance. Additionally, the missile's accuracy increases for intercept condition beyond 45º in elevation by utilizing combined CLOS and PN guidance algorithm.

5. LIMITATIONS

The conclusions drawn above are based on assumption the target in the same plane as that of the missile at the launch instance. This is a valid assumption as most CLOS missiles function in the same manner. The conclusions drawn are based on basis of limited number of cases. It is recommended that further studies may be carried out to pinpoint any weakness in the study.

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