Analytical study of joint heat transfer between a gasdynamic boundary layer and an anisotropic strip

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Abstract: *When solving the problems of coupled heat transfer between viscous flows and streamlined bodies under the conditions of aerodynamic heating of aircraft, it is necessary to overcome significant difficulties. They associated primarily with determining the boundary conditions. The paper investigates the joint (coupled) heat transfer between a heat and gas dynamic boundary layer and an anisotropic strip under conditions of aerodynamic heating based on the obtained analytical solution of the second initial boundary value problem of thermal conductivity in an anisotropic strip with arbitrary boundary conditions. Since the system of equations of the gasdynamic boundary layer is essentially nonlinear, mainly numerical methods are used to solve it. For an incompressible boundary layer near the critical point of a blunt wedge, an analytical solution is obtained to determine the components of the velocity vector, density, temperature, and heat fluxes. The closed-form solution to the conjugate problem was received in the form of a Fredholm integral equation of second kind. The results of numerical experiments are obtained and analyzed.*

Key Words: *viscosity, heat fluxes, components of the thermal conductivity tensor, conjugate problem*

1. INTRODUCTION

In solving the problems of coupled heat transfer between viscous flows and streamlined bodies under the conditions of aerodynamic heating of aircraf, it is necessary to overcome significant difficulties associated primarily with determining the boundary conditions at the gas – solid object interface [1], [2], [3], [4], [5], [6], because in coupled media, problems of different physical nature are solved and are described by various partial differential equations.

The problem is complicated if the streamlined body has anisotropy of heat transfer properties, since the anisotropic thermal conductivity equations contain mixed derivatives for the analytical solution of which it is possible to use only integral transformation methods and only for domains in which at least one of the boundaries is directed to infinity [7], [8], [9], [10], [11], [12].

Since the system of equations of the gasdynamic boundary layer is essentially nonlinear, numerical methods are mainly used to solve it [6], however, under simplifying assumptions, in particular on the incompressibility of the flow occurring in the shock layer on blunt objects, an approximate analytical solution can be obtained to determine heat fluxes to the object, which are used as boundary conditions in the thermal conductivity problem [13], [14], [15], [16], [17], [18], [19].

An approximate analytical solution of the system of equations of the thermal boundary layer is obtained in order to determine heat fluxes to the object, which are then used as boundary conditions for the analytical solution of the thermal conductivity problem in the anisotropic strip [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31]. As a result, the conjugate heat transfer problem in the boundary layer and the thermal conductivity problem in the anisotropic strip are solved, and the continuity of heat fluxes and temperatures at the gas – solid object interface is used as boundary conditions.

2. MATERIALS AND METHODS

Let us consider the problem of coupled heat transfer when flowing around a critical point of a blunt anisotropic strip (Fig. 1). According to the characteristics of the approach flow of velocity V_s, height H, Mach number M, it is necessary to determine the heat and gas dynamic characteristics of the incompressible boundary layer and the heat fluxes to the object, using them as the boundary conditions at the gas – solid object interface, the thermal conductivity problem in the anisotropic strip can be solved.

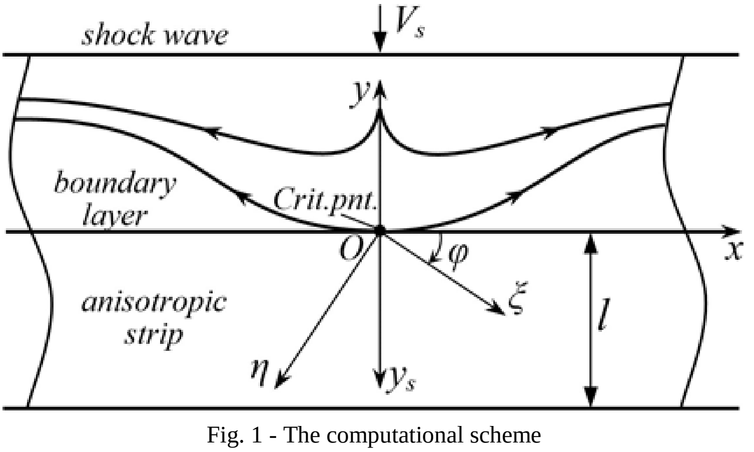

Fig. 1 - The computational scheme

The origin O of coordinates Oxy is at the gas – solid object interface, with the Ox axis directed along this boundary, the Oy axis is directed inside the boundary layer, the Oy_s axis is directed inside the anisotropic strip with thickness l, $O\boldsymbol{\xi}$, $O\boldsymbol{\eta}$ are the main axes of the thermal conductivity tensor oriented with respect to the Ox axis by the angle φ.

We assume that the boundary layer is quasi-steady (steady at each moment of time), and the thermal conductivity problem in the anisotropic strip is nonstationary.

In addition, the flow is symmetric relative to the Oy axis, but the thermal conductivity in the strip is not.

The system of equations of dynamic and thermal boundary layers relative to components $u(x,y)$, $v(x,y)$ of the velocity vector, temperature $T(x,y)$, density $\rho(x,y)$, pressure $p(x)$ has the form (Eqs. 1-9) [3], [5], [6], [10]:

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho u)}{\partial y}=0, 0<y<\delta(x), |x|<\infty \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right), 0<y<\delta(x), |x|<\infty \tag{2}$$

$$0=-\frac{\partial p}{\partial y}; p=p_e(x), \frac{\partial p_e}{\partial x}=-\rho_e u_e\frac{\partial u_e}{\partial x}, y=\delta(x), |x|<\infty \tag{3}$$

$$\rho u\frac{\partial I}{\partial x}+\rho v\frac{\partial I}{\partial y}=\frac{\partial}{\partial y}\left[\frac{\mu(T)}{\mathrm{Pr}}\frac{\partial I}{\partial y}+\frac{\mu(T)}{2}\left(1-\frac{1}{\mathrm{Pr}}\frac{\partial u^2}{\partial y}\right)\right], 0<y<\delta(x), |x|<\infty \tag{4}$$

$$p=\rho RT, 0<y<\delta(x), |x|<\infty \tag{5}$$

$$y=\delta(x): u\big(x,\delta(x)\big)=u_e(x); v\big(x,\delta(x)\big)=v_e(x) \tag{6}$$

$$T\big(x,\delta(x)\big)=T_e(x);\ \rho\big(x,\delta(x)\big)=\rho_e(x) \tag{7}$$

$$x=0: u(0,y)=0; p(0,0)=p_0 \tag{8}$$

$$dp_e/dx=0 \text{ at } x\to\pm\infty \tag{9}$$

where p_0 is the stagnation pressure, Pr is the Prandtl number (Eq. 10):

$$\mathrm{Pr}=\mu\cdot c_p/\lambda \tag{10}$$

I – gas enthalpy (Eq. 11):

$$I=CpT_n+u^2/2 \tag{11}$$

The index "w" refers to the gas – solid object boundary $y=0$, "e" refers to the boundary layer edge.

According to the definition of a function $T(x,y,t)$, the thermal conductivity problem in an anisotropic strip is considered with boundary conditions of the second kind, where at the boundary $y_s=0$ the heat flux (Eq. 12):

$$q_w=-\lambda\partial T/\partial y|_{y=0} \tag{12}$$

is drawn from the boundary layer (Eqs. 13-18):

$$\lambda_{11}\frac{\partial^2 T}{\partial x^2}+2\lambda_{12}\frac{\partial^2 T}{\partial x\partial y}+\lambda_{22}\frac{\partial^2 T}{\partial y^2}=c\rho\frac{\partial T}{\partial t}, -\infty<x<\infty, y_s=0, t>0 \tag{13}$$

$$-\lambda\frac{\partial T(x,y)}{\partial y}\bigg|_{y=0}=\left(\lambda_{11}\frac{\partial T(x,y_s,t)}{\partial x}+\lambda_{22}\frac{\partial T(x,y_s,t)}{\partial y}\right)\bigg|_{y_s=0}, -\infty<x<\infty, y=y_s=0, t>0 \tag{14}$$

$$T(x,y)|_{y=0}=T(x,y_s,t)|_{y_s=0}=T_w(x), -\infty<x<\infty, y=y_s=0, t>0 \tag{15}$$

$$\left(\lambda_{11}\frac{\partial T}{\partial x}+\lambda_{22}\frac{\partial T}{\partial y}\right)\bigg|_{y_s=l}=0, -\infty<x<\infty, y_s=l, t>0 \tag{16}$$

$$T(x,y_s,0)=0, -\infty<x<\infty, 0\le y_s<l, t=0 \tag{17}$$

$$T(\pm\infty,y_{s,}t)=0, \frac{\partial T(\pm\infty,y_s,t)}{\partial x}=0, \frac{\partial T(\pm\infty,y_{s,}t)}{\partial y}=0, -\infty<x<\infty, 0\le y_s<l, t>0 \tag{18}$$

We assume that the gas is perfect, the equation of state of which satisfies the Mendeleev's-Clapeyron equation (5), the thicknesses of the dynamic $\delta(x)$ and thermal $\delta_B(x)$ boundary layers are equal (ie, the number Pr = 1), and thermal conductivity of the gas are determined by the Sutherland's formula [15].

We express the derivative $\partial u/\partial x$ on the left side of the momentum conservation equation (2) from the continuity equation (1) for an incompressible gas ($\rho = const$), we obtain the (Eq. 19):

$$-\rho\frac{\partial(u\cdot v)}{\partial y}+2\rho v\frac{\partial u}{\partial y}=-\frac{dp_e}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{19}$$

integrating it twice in a variable y under the assumptions made, and also setting (Eq. 20):

$$\mu(T)\approx\mu(T_w)=const \tag{20}$$

we come to the expression (the index "m" means averaging over the thickness of the boundary layer) (Eq. 21):

$$-(\rho uv)_m\,y+2-(\rho vu)_m\,y=-\frac{dp_e}{dx}\frac{y^2}{2}+\mu_w u+C_1(x)y+C_2(x) \tag{21}$$

in which $C_1(x)$ and $C_2(x)$ are determined from the boundary conditions (Eqs. 22-24):

$$y=0: u(x,0)=0\cdot C_2(x)=0 \tag{22}$$

$$y=\delta_e: u(x,\delta)=u_e(x) \tag{23}$$

$$C_1(x)=(\rho uv)_{cp}+\frac{dp_e}{dx}\frac{\delta}{2}-\mu_w\frac{u_e}{\delta} \tag{24}$$

Given these relations from (21) we obtain the longitudinal component of speed $u(x,y)$ (Eq. 25):

$$u(x,y)=\frac{1}{2\mu_w}\frac{dp_e}{dx}(y^2-\delta y)+\frac{u_e}{\delta}y \tag{25}$$

The transverse component of the velocity vector can be found from the continuity equation (1) for an incompressible flow (Eq. 26):

$$\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y} \tag{26}$$

with the boundary condition (Eq. 27):

$$v(x,0)=0 \tag{27}$$

Substituting the distribution of the longitudinal velocity (25) in (26) and integrating the obtained expression over the variable taking into account condition (27), we obtain (Eq. 28):

$$v(x,y)=-\frac{1}{2\mu_w}\frac{d^2p_e}{dx^2}\left(\frac{y^3}{3}-\frac{\delta}{2}y^2\right)-\frac{du_e}{dx}\frac{y^2}{2\delta} \tag{28}$$

where the derivative du_e/dx is determined from the Bernoullis equation in the form (3) and in the form (Eqs. 29-30):

$$p_e+\rho_e u_e^2/2=p_0 \tag{29}$$

$$\frac{du_e}{dx}=-\frac{1}{\sqrt{2\rho_e(p_0-p_e)}}\frac{dp_e}{dx} \tag{30}$$

so that (Eq. 31):

$$v(x,y)=-\frac{1}{2\mu_w}\frac{d^2p_e}{dx^2}\left(\frac{y^3}{3}-\frac{\delta}{2}y^2\right)+\frac{1}{\sqrt{2\rho_e(p_0-p_e)}}\frac{dp_e}{dx}\frac{y^2}{2\delta} \tag{31}$$

To clarify the longitudinal $u(x,y)$ and transverse $v(x,y)$ components of the velocity vector, expressions (25), (31) can again be substituted into the left side of equation (2) and into the derivative $\partial u/\partial x$ of equation (1).

Under these assumptions, we integrate the energy conservation equation (4) with respect to the variable y, assuming that the derivative $\partial I/\partial x$ weakly depends on the variable y, we obtain (Eq. 32):

$$(\rho u)_{cp}\frac{\partial I}{\partial x}y+(\rho v)_{cp}I=\frac{\mu_w}{\mathrm{Pr}}\frac{\partial I}{\partial x}+\frac{\mu_w}{2}\left(1-\frac{1}{\mathrm{Pr}}\right)\frac{\partial u^2}{\partial y}+D_1(x) \tag{32}$$

We integrate the resulting expression one more time with respect to the variable y (Eq. 33):

$$(\rho u)_{cp}\frac{\partial I}{\partial x}\frac{y^2}{2}+(\rho v)_{cp}\int_0^y I dy=\frac{\mu_w}{\mathrm{Pr}}I+\frac{\mu_w}{2}\cdot k\cdot u^2+D_1(x)\cdot y+D_2(x) \tag{33}$$

where (Eq. 34):

$$k=1-1/\mathrm{Pr}\approx -0.4 \tag{34}$$

(with the number $\mathrm{Pr}=0.71$) (Eqs. 35-37):

$$y=0: u=0, I(x,0)=I_w(x), D_2(x)=-\frac{\mu_w}{\mathrm{Pr}}I_w \tag{35}$$

$$y = \delta : u = u_e, I = I_e \tag{36}$$

$$D_1(x) = -\frac{\mu_w}{\mathrm{Pr}}\frac{I_e - I_w}{\delta} - \frac{\mu_w \cdot 0.4}{2}\frac{u_e^2}{\delta} + \frac{(\rho u)_{cp}}{\delta}\frac{\partial I_e}{\partial x}\frac{\delta}{2} + \frac{(\rho v)_{cp}}{\delta}\int_0^\delta I dy \tag{37}$$

Substituting $D_1(x)$, $D_2(x)$ in (33), we obtain (Eq. 38):

$$\begin{aligned} -\frac{\mu_w}{\mathrm{Pr}}(I - I_w) + \frac{\mu_w}{\mathrm{Pr}}\frac{I_e - I_w}{\delta}y - 0.2\mu_w\left(\frac{u_e^2}{\delta}y - u^2\right) \\ = (\rho u)_{cp}\left(\frac{1}{\delta}\frac{\partial I_e}{\partial x}\frac{\delta^2}{2}y - \frac{\partial I}{\partial x}\frac{y^2}{2}\right) + (\rho v)_{cp}\left(\frac{1}{\delta}\int_0^\delta I dy - \int_0^y I dy\right) \end{aligned} \tag{38}$$

The terms on the right side of expression (38) are of the order of the square of the boundary layer thickness and, as a first approximation, they can be neglected, thus (Eq. 39):

$$I(x, y) = I_w(x) + \frac{I_e(x) - I_w(x)}{\delta(x)}\mathrm{y} - 0.2 \cdot \mathrm{Pr}\left[\frac{u_e^2(x)}{\delta(x)}y - u^2(x, y)\right] \tag{39}$$

We differentiate expression (39) with a variable y, substituting $y = 0$ (Eq. 40):

$$q_w(x) = \lambda_w \frac{\partial T}{\partial y}\bigg|_w = \lambda_w \frac{T_e(x) - T_w(x)}{\delta} - 0.2\mu_w \frac{u_e^2(x)}{\delta} \tag{40}$$

Because (Eq. 41):

$$u_e^2(x) = \frac{2}{\rho_e}(p_0 - p_e(x)) \tag{41}$$

then (Eq. 42):

$$\begin{aligned} q_w(x) = -\lambda_w \frac{\partial T}{\partial y}\bigg|_w = \lambda_w \frac{T_w(x) - T_e(x)}{\delta(x)} + \frac{0.4\mu_w}{p_e(x)\delta(x)}\big((p_0 - p_e(x)\big) \\ = T_w(x)\left(\frac{\lambda_w}{\delta(x)}\right) + \left(\frac{0.4\mu_w}{p_e(x)\delta(x)}\big((p_0 - p_e(x)\big) - \frac{\lambda_w T_e(x)}{\delta(x)}\right) \end{aligned} \tag{42}$$

To solve the conjugate problems, it is now necessary to substitute heat fluxes in the form (42) into the analytical solution of the anisotropic thermal conductivity problem, (Eqs. 11-18). The solution of the second initial boundary value problem of thermal conductivity in an anisotropic strip was first obtained by the authors using the Green's function method and is given in the monograph [15]. Therefore, it is given here without conclusion. With zero heat current at the inner border of the strip $y_s = l, (q_2 = 0)$, this solution has the form (Eq. 43):

$$\begin{aligned} T(x, y_s, t) = \frac{1}{2\gamma l \lambda_{22}\sqrt{\pi}}\int_0^t \left\{\left[1 + 2\sum_{k=1}^{\infty}\cos\left(k\pi\frac{l - y_s}{l}\right)\exp\left(-\frac{k^2\pi^2}{\gamma l^2}(t - \tau)\right)\right] \right. \\ \left. \times \int_{-l1}^{l1}\left[\frac{q_w(\xi)}{\sqrt{\beta(t - \tau)/\gamma}}\exp\left(-\frac{(\xi + \alpha y_s - x)^2}{4\beta(t - \tau)/\gamma}\right)\right]\right\} d\xi d\tau \end{aligned} \tag{43}$$

where $q_w(x)$ is the heat flux from the boundary layer, defined by formula (40), l_1 is the distance along the Ox axis from the critical point $x = 0$, where $q_w(x) \neq 0$ but outside of it $q_w(x) \approx 0$, (Eqs. 44-46):

$$\alpha = \lambda_{12}/\lambda_{22} \tag{44}$$

$$\beta = (\lambda_{11}\lambda_{22} - \lambda_{22}^2)/\lambda_{22}^2 = \lambda_\xi \lambda_\eta/\lambda_{22}^2 \tag{45}$$

$$\gamma = cp/\lambda_{22} \tag{46}$$

The components of the thermal conductivity tensor of the anisotropic strip are determined by the relations (Eqs. 47-49) [15]:

$$\lambda_{11} = \lambda_\xi \cos^2\varphi + \lambda_\eta \sin^2\varphi \tag{47}$$

$$\lambda_{22} = \lambda_\xi \sin^2\varphi + \lambda_\eta \cos^2\varphi \tag{48}$$

$$\lambda_{12} = \lambda_{21} = (\lambda_\xi - \lambda_\eta)\sin\varphi\cos\varphi \tag{49}$$

where $\lambda_{\xi,}$ λ_η, are the main components of the thermal conductivity tensor, acting in the direction of the principal axes $O_\xi \cdot O_\eta$ oriented by an angle φ relative to the Ox axis (Fig. 1).

It remains to substitute in (43) either the heat flux $q_w(x)$ from (42), then at $y_s = 0$ on the left side of (43) $(T_w(x,0,t) = T_w(x))$ we obtain the inhomogeneous Fredholm integral equation of the second kind with respect to temperature $T_w(x)$ or the temperature $T_w(x)$ from expression (42) into the left side of function (43), we obtain the Fredholm equation of second kind relative to heat flux $q_w(x)$.

Substituting (42) in (43) instead of $q_w(\xi)$, we get at $y_s = 0$ (Eq. 50):

$$T(x,t) = \frac{1}{2\gamma l \lambda_{22}\sqrt{\pi}} \int_0^t \left\{ \left[1 + 2\sum_{k=1}^{\infty} (-1)^k \cos\left(k\pi \frac{l - y_s}{y_s}\right) \exp\left(-\frac{k^2\pi^2}{\gamma l^2}(t-\tau)\right) \right] \right.$$
$$\times \int_{-l1}^{l1} \left[\left(T_w(\xi)\frac{\lambda_w}{\delta(\xi)} + \frac{0.4\mu_w}{p_e(\xi)\delta(\xi)}\left((p_0 - p_e(\xi)\right) - \frac{\lambda_w T_e(\xi)}{\delta(\xi)} \right) \right.$$
$$\left.\left. \times \frac{\exp\left(-\frac{(\xi + \alpha y_s - x)^2}{4\beta(t-\tau)/\gamma}\right)}{\sqrt{\beta(t-\tau)/\gamma}} \right] \right\} d\xi d\tau \tag{50}$$

is solved by the iterative method. This closed-form solution (one function) is a solution to the conjugate problem (Eqs. 1-18) of heat transfer between the temperature gasdynamic boundary layer and the anisotropic strip. Substituting the obtained distribution of temperatures $T(x,t)$ of the interface in (39), (42), we find the distribution of temperatures (enthalpies) in the boundary layer and heat fluxes q_w to the wall.

3. RESULTS AND DISCUSSIONS

The solution obtained is applicable to the problem of flowing around a critical point of a cone (wedge) blunt in radius $R_0 = 0.05\ m$ with a semivertex angle $\theta_0 = 10°$ behind the normal part of the shock wave, where the speed is subsonic (incompressible flow). Approach flow speed $V_s = 3000\ m/s$, (Mach number $M = 10$), thermal and physical characteristics of the approach flow, ρ_s, T_s, a_F, p_s, μ_s, λ_s are determined from the International Standard Atmosphere for altitude $H = 12000\ m$.

The dynamic coefficient of viscosity and thermal conductivity of the gas were determined according to the Sutherland's formula (Eq. 51) [15]:

$$\frac{\mu_s}{\mu_n} = \frac{\lambda_w}{\lambda_n} = \left(\frac{T_w}{T_n}\right)\frac{T_n + 110}{T_w + 110} \tag{51}$$

At the outer boundary layer edge, the gasdynamic characteristics $T_e(x)$, $\rho_e(x)$, $u_e(x)$ are calculated by the gas pressure $p_e(x)$, which is determined on the blunt conical bodies by the interpolation formula (Eqs. 52-53) [15]:

$$p_e(\theta) = p_0(1 - 1.17\sin^2\theta + 0.225\sin^6\theta) \cdot |\theta| \leq \frac{\pi}{2} - \theta_0 \tag{52}$$

$$p_e(x) = p_0(A\ln(x + B) + C), \left(\frac{\pi}{2} - \theta_0\right)R_0 < |x| \leq \left(\frac{\pi}{2} - \theta_0\right)R_0 + L \tag{53}$$

where (Eq. 54):

$$\theta = x/R_0 \tag{54}$$

L is the length of the wedge tail, $A = 0.028904$, $B = -1.238331$, $C = 0.017189$ for the semivertex angle $\theta_0 = 10°$.

Using pressure according to formulas (52-53), temperature $T_e(x)$ and density $\rho_e(x)$ are determined from the relations (Eq. 55):

$$\frac{T_e(x)}{T_o} = \frac{p_e(x)^{\frac{k-1}{k}}}{p_0}, \frac{p_e(x)}{p_0} = \left(\frac{p_e(x)^{1/k}}{p_0}\right) \tag{55}$$

and speed $u_e(x)$ is determined from the Bernoullis equation (Eqs. 56-58) [15]:

$$p_e u_e \frac{du_e}{dx} = -\frac{dp_e}{dx} \tag{56}$$

$$|x| \leq \left(\frac{\pi}{2} - \theta_0\right)R_0 + L \tag{57}$$

$$u_e(x) = \sqrt{\frac{2k}{k-1}\frac{p_0^{1/k}}{\rho_0}\left(p_0^{\frac{k-1}{k}} - (p_e(x))^{\frac{k-1}{k}}\right)} \tag{58}$$

The thickness of the anisotropic strip $l = 0.005\ m$ in the calculations was assumed to be metal (with high thermal conductivity) (Eqs. 59-60):

$$T_w(x,t)\frac{\lambda_w}{\delta(x)} + \frac{0.4\mu_w}{\rho_e(x)\delta(x)}\left(p_0 - p_e(x)\right) - \frac{\lambda_w T_e(x)}{\delta(x)} = (c\rho l)_s\frac{\partial T_w(x,t)}{\partial t} \tag{59}$$

$$T_w(x,0) = T_{w0} \tag{60}$$

where $T_{w0} = 300\ K$, and the heat-sink capacity and density of the metal took the values: $c_s = 10^3 kg \cdot K$, $\rho_s = 4000\ kg/m^3$.

The solution to the initial problem (59) is the function (Eq. 61):

$$T_w(x,t) = \left(T_{w0} + \frac{B}{A}\right)\exp\left(\frac{At}{(c\rho l)_s}\right) - \frac{B}{A} \tag{61}$$

where (Eq. 62):

$$A = -\frac{\lambda_w}{\delta(x)}, B = \frac{0.4\mu_w}{\rho_e(x)\delta(x)}\left(p_0 - p_e(x)\right) + \frac{\lambda_w T_e(x)}{\delta(x)} \tag{62}$$

Figure 2 shows the graphs of the heat flux distribution, and Fig. 3 presents distribution of temperatures of the metal strip along the axis at different points in time. Under conditions of a quasi-steady boundary layer, the strip heats up quite intensively in time.

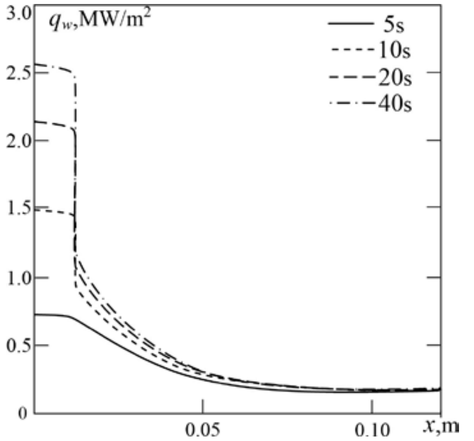

Fig. 2 - The distribution of the heat flux at the boundary of the metal strip at different points in time

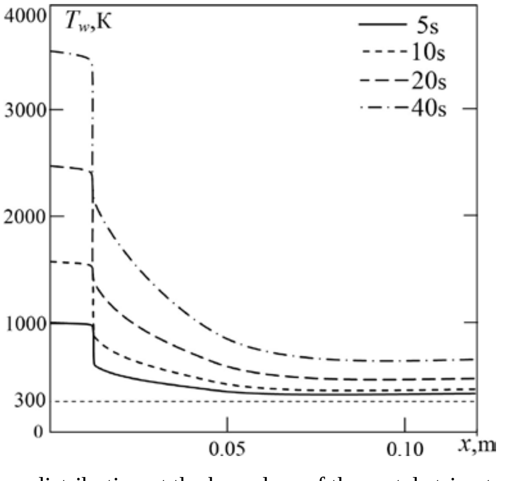

Fig. 3 - The temperature distribution at the boundary of the metal strip at various points in time

4. CONCLUSIONS

The problem of coupled heat transfer between a high-temperature gasdynamic boundary layer and an anisotropic strip with a thermal conductivity tensor in the general form is formulated. For an incompressible boundary layer near the critical point of a blunt wedge, an analytical solution is obtained to determine the components of the velocity, density, temperature vector, and heat fluxes to the anisotropic strip. An analytical solution to the problem is obtained for an anisotropic strip with arbitrary heat fluxes at the boundaries. We obtained closed-form solution, in the form of the Fredholm integral equation of the second kind, to the entire complex problem of coupled heat transfer.

Numerous results of computational experiments have been obtained. The numerical implementation confirmed the adequacy of mathematical modeling and the solution method, despite the simplifications made.

In particular, when solving the conjugate problem, it was assumed that the boundary layer is quasi-steady (steady at each moment of time), and the thermal conductivity problem in the anisotropic strip is non-stationary. In addition, the flow is symmetrical, but the thermal conductivity in the strip is not.

In this case, the gas is considered to be perfect, the equation of state of which satisfies the Mendeleev's-Clapeyron equation Cl, and the thicknesses of the dynamic and thermal boundary layers are taken equal (i.e., the Prandtl number is equal to 1), the viscosity and thermal conductivity of the gas were determined by the Sutherland's formula.

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